Dr Oliver Mathematics
GCSE Mathematics
2020 Paper 1H: Non-Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. The first five terms of an arithmetic sequence are (2)

$$1 \qquad 4 \qquad 7 \qquad 10 \qquad 13.$$

Write down an expression, in terms of n, for the nth term of this sequence.

Solution

Let the

$$n\text{th term} = an + b.$$

$$\begin{array}{ccccccc} 1 & & 4 & & 7 & & 10 \\ & 3 & & 3 & & 3 & \end{array}$$

$$\begin{array}{ccccccc} a+b & & 2a+b & & 3a+b & & 4a+b \\ & a & & a & & a & \end{array}$$

We compare terms:

$$a = 3$$

and

$$\begin{aligned} a + b = 10 &\Rightarrow 3 + b = 1 \\ &\Rightarrow b = -2. \end{aligned}$$

Hence,

$$n\text{th term} = \underline{\underline{3n - 2}}.$$

2. Show that (3)

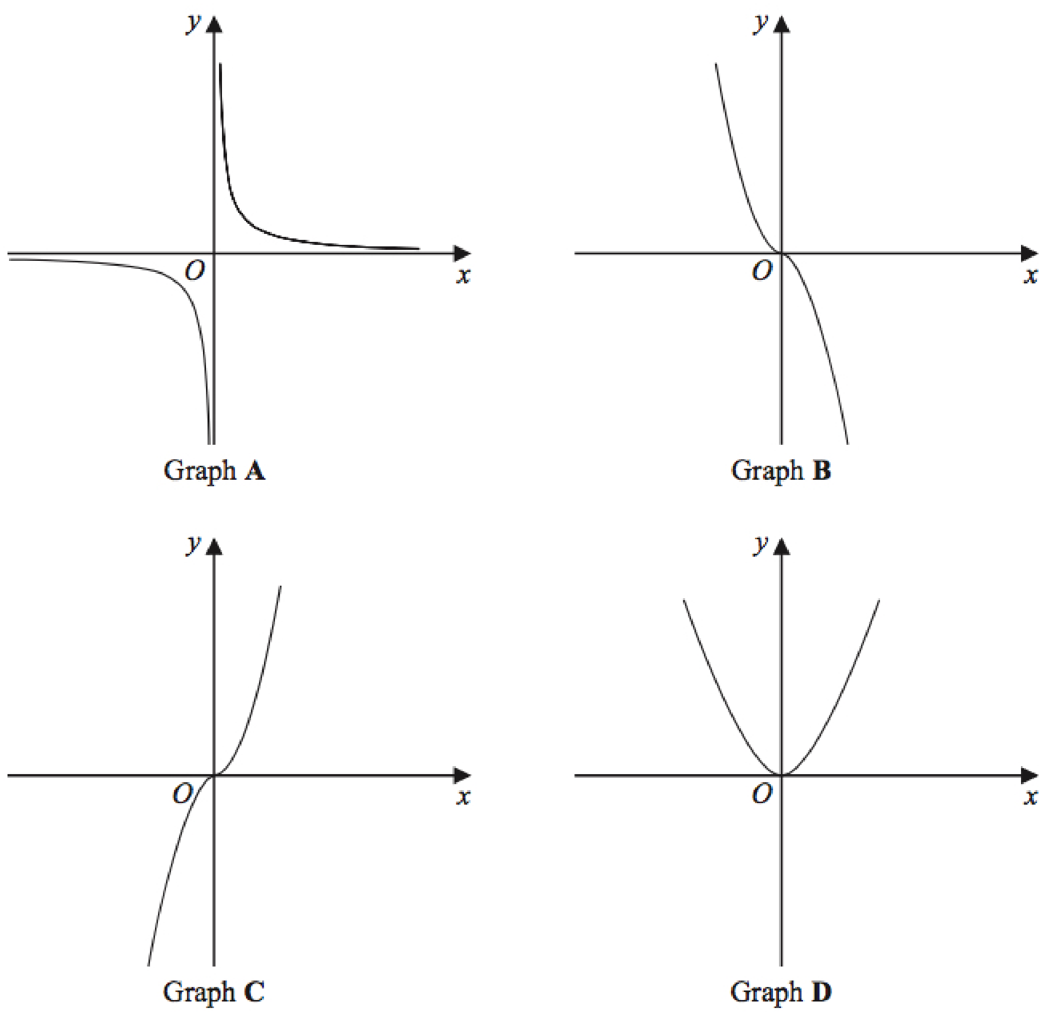
$$2\tfrac{1}{3} \times 3\tfrac{3}{4} = 8\tfrac{3}{4}.$$

Solution

$$\begin{aligned} 2\tfrac{1}{3} \times 3\tfrac{3}{4} &= \tfrac{7}{3} \times \tfrac{15}{4} \\ &= \tfrac{7}{\cancel{3}} \times \tfrac{\cancel{15}\,^{5}}{4} \\ &= \tfrac{35}{4} \\ &= \underline{\underline{8\tfrac{3}{4}}}, \end{aligned}$$

as required.

3. The diagram shows four graphs. (2)



Each of the equations in the table is the equation of one of the graphs.

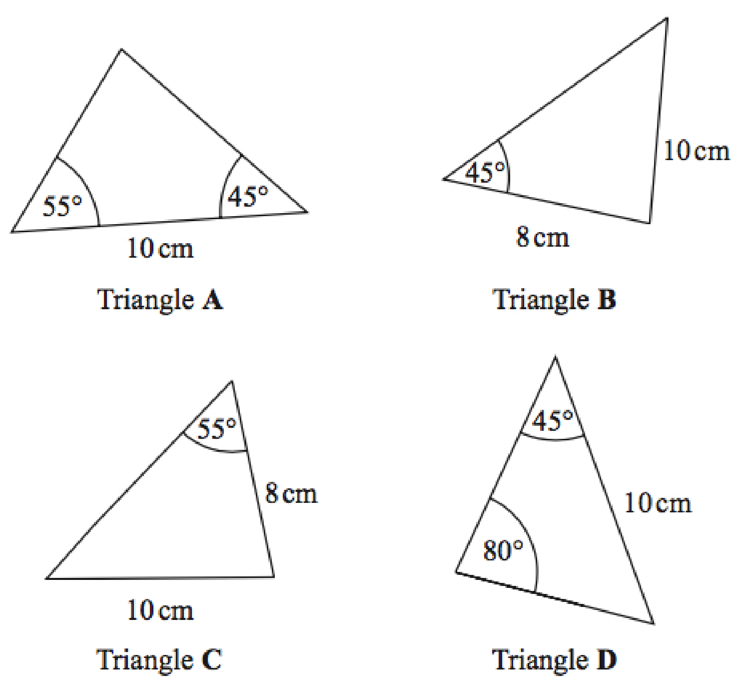
Complete the table.

Equation	Letter of graph
$y = -x^3$	
$y = x^3$	
$y = x^2$	
$y = \dfrac{1}{x}$	

Solution

Equation	Letter of graph
$y = -x^3$	Graph **B**
$y = x^3$	Graph **C**
$y = x^2$	Graph **D**
$y = \dfrac{1}{x}$	Graph **A**

4. The diagram shows four triangles. (1)



Two of these triangles are congruent.

Write down the letters of these two triangles.

Solution

Triangles **A** and **D**.

5. Sean pays £10 for 24 chocolate bars. (3)

He sells all 24 chocolate bars for 50p each.

Work out Sean's percentage profit.

Solution

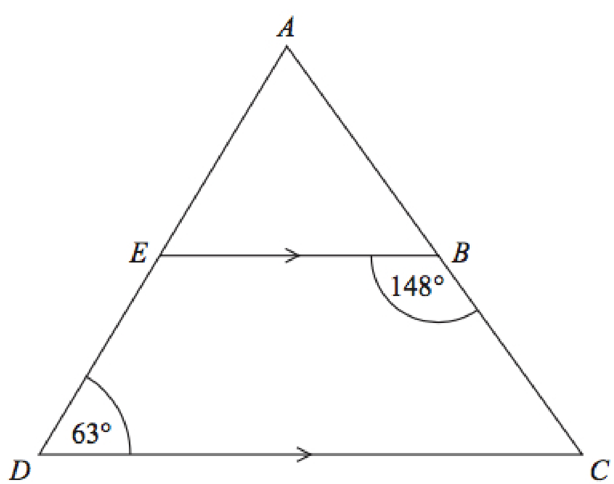
He makes

$$24 \times £0.50 = £12$$

and his percentage profit is

$$\left(\frac{12 - 10}{10}\right) \times 100\% = \left(\frac{2}{10}\right) \times 100\%$$
$$= \underline{\underline{20\%}}.$$

6. ADC is a triangle. (5)



AED and ABC are straight lines.
EB is parallel to DC.

Angle $EBC = 148°$.
Angle $ADC = 63°$.

Work out the size of angle EAB.
You must give a reason for each stage of your working.

Solution

$\angle AEB = 63°$ (corresponding angles)
$\angle ABE = 180 - 148 = 32°$ (supplementary angles)
$\angle EAB = 180 - (63 + 32) = 180 - 95 = \underline{\underline{85°}}$ (completing the triangle)

7. The table shows information about the heights, in cm, of a group of Year 9 girls. (3)

Least height	150 cm
Median	165 cm
Greatest height	170 cm

This stem and leaf diagram shows information about the heights, in cm, of a group of 15 Year 9 boys.

15	8 9 9
16	4 5 7 7 8
17	0 3 4 4 7
18	0 2

Key: 15 | 8 represents 158 cm

Compare the distribution of the heights of the girls with the distribution of the heights of the boys.

Solution

Boys: Well, the

$$\left(\frac{15+1}{2}\right) = 8\text{th}$$

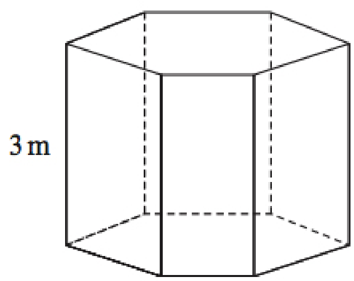
has height 168 cm.

Least height	158 cm
Median	168 cm
Greatest height	182 cm

Average
Since the median for the boys (168) is higher than the median for girls (165), boys are taller on average.

Spread
Since the range for the boys $(182 - 158 = 24)$ is larger than the range the girls $(170 - 150 = 20)$, the girls heights were more consistent.

8. The diagram shows a prism placed on a horizontal floor. (3)



$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

The prism has height 3 m.
The volume of the prism is 18 m^3.

The pressure on the floor due to the prism is 75 newtons/m^2.

Work out the force exerted by the prism on the floor.

Solution
The area of the prism is

$$\frac{18}{3} = 6 \text{ m}^2$$

and the

$$\begin{aligned}\text{pressure} = \frac{\text{force}}{\text{area}} &\Rightarrow 75 = \frac{\text{force}}{6}\\ &\Rightarrow \text{force} = 6 \times 75\\ &\Rightarrow \text{force} = \underline{\underline{450 \text{ N}}}.\end{aligned}$$

9. Write these numbers in order of size. (2)
Start with the smallest number.

$$6.72 \times 10^5 \qquad 67.2 \times 10^{-4} \qquad 672 \times 10^4 \qquad 0.000\,672$$

Solution

Let us put them all in standard form:

$$\begin{aligned} &6.72 \times 10^5\\ 67.2 \times 10^{-4} &= 6.72 \times 10^{-3}\\ 672 \times 10^4 &= 6.72 \times 10^6\\ 0.000\,672 &= 6.72 \times 10^{-4}.\end{aligned}$$

Starting with the smallest number:

$$\underline{\underline{0.000\,672 \qquad 67.2 \times 10^{-4} \qquad 6.72 \times 10^5 \qquad 672 \times 10^4}}.$$

10. Given that (3)

$$\frac{a}{b} = \frac{2}{5} \text{ and } \frac{b}{c} = \frac{3}{4},$$

find $a : b : c$.

Solution

$$\frac{a}{b} = \frac{2}{5} \Rightarrow a = \tfrac{2}{5}b$$

and

$$\frac{b}{c} = \frac{3}{4} \Rightarrow b = \tfrac{3}{4}c.$$

So now

$$\begin{aligned} a &= \tfrac{2}{5}b \\ &= \tfrac{2}{5}(\tfrac{3}{4}c) \\ &= \tfrac{3}{10}c. \end{aligned}$$

Hence,

$$\begin{aligned} a : b : c &= \tfrac{3}{10}c : \tfrac{3}{4}c : c \\ &= \tfrac{3}{10} : \tfrac{3}{4} : 1 \\ &= \underline{\underline{6 : 15 : 20}}. \end{aligned}$$

11. (a) Find the value of (2)

$$\sqrt[4]{81 \times 10^8}.$$

Solution

$$\begin{aligned} \sqrt[4]{81 \times 10^8} &= \sqrt[4]{81} \times \sqrt[4]{10^8} \\ &= \underline{\underline{3 \times 10^2 \text{ or } 300}}. \end{aligned}$$

(b) Find the value of $64^{-\frac{1}{2}}$. (2)

Solution

$$\begin{aligned} 64^{-\frac{1}{2}} &= \frac{1}{64^{\frac{1}{2}}} \\ &= \frac{1}{\sqrt{64}} \\ &= \underline{\underline{\tfrac{1}{8}}}. \end{aligned}$$

(c) Write (2)

$$\frac{3^n}{9^{n-1}}$$

as a power of 3.

Solution

$$\begin{aligned}\frac{3^n}{9^{n-1}} &= \frac{3^n}{(3^2)^{n-1}} \\ &= \frac{3^n}{3^{2n-2}} \\ &= 3^{n-(2n-2)} \\ &= \underline{\underline{3^{2-n}}}.\end{aligned}$$

12. The table gives information about the weekly wages of 80 people.

Wage (£w)	Frequency
$200 < w \leqslant 250$	5
$250 < w \leqslant 300$	10
$300 < w \leqslant 350$	20
$350 < w \leqslant 400$	20
$400 < w \leqslant 450$	15
$450 < w \leqslant 500$	10

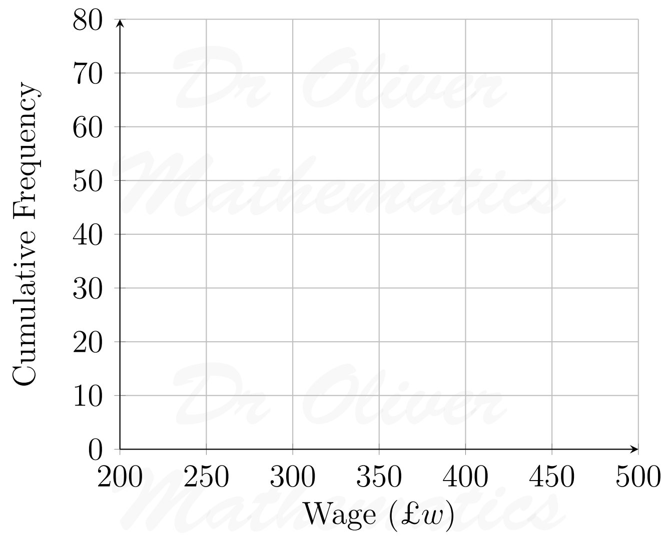
(a) Complete the cumulative frequency table. (1)

Wage (£w)	Cumulative Frequency
$200 < w \leqslant 250$	
$200 < w \leqslant 300$	
$200 < w \leqslant 350$	
$200 < w \leqslant 400$	
$200 < w \leqslant 450$	
$2000 < w \leqslant 500$	

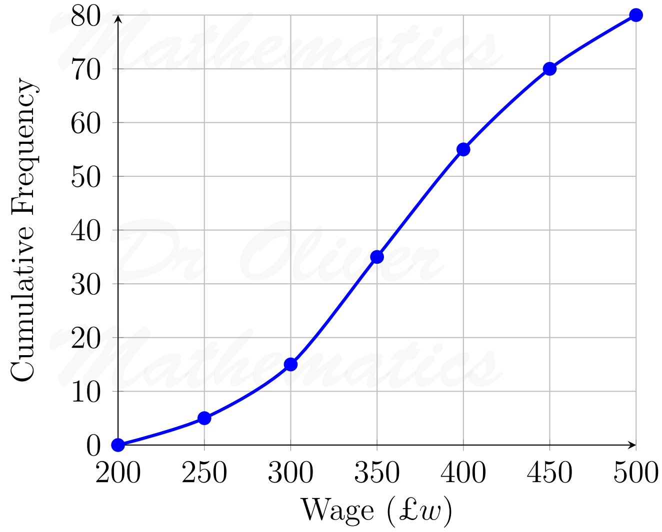
Solution

Wage (£w)	Cumulative Frequency
$200 < w \leqslant 250$	$\underline{\underline{5}}$
$200 < w \leqslant 300$	$5 + 10 = \underline{\underline{15}}$
$200 < w \leqslant 350$	$15 + 20 = \underline{\underline{35}}$
$200 < w \leqslant 400$	$35 + 20 = \underline{\underline{55}}$
$200 < w \leqslant 450$	$55 + 15 = \underline{\underline{70}}$
$2000 < w \leqslant 500$	$70 + 10 = \underline{\underline{80}}$

(b) On the grid below, draw a cumulative frequency graph for your completed table. (2)



Solution



Juan says, "60% of this group of people have a weekly wage of £360 or less."

(c) Is Juan correct? (3)
You must show how you get your answer.

Solution

Correct read-off: Juan is wrong, approximately 39 people have a weekly wage of £360 or less.

13. Liquid **A** and liquid **B** are mixed to make liquid **C**. (3)

Liquid **A** has a density of 70 kg/m^3.
Liquid **A** has a mass of 1400 kg.

Liquid **B** has a density of 280 kg/m^3.
Liquid **B** has a volume of 30 m^3.

Work out the density of liquid **C**.

Solution

In what ratio? Is it 1 : 1, 2 : 5, or ...? We are not told so we will assume that it is 1 : 1.

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

Now, Liquid **A**:

$$70 = \frac{1\,400}{\text{volume}} \Rightarrow \text{volume} = \frac{1\,400}{70}$$
$$\Rightarrow \text{volume} = 20$$

and, Liquid **B**:

$$280 = \frac{\text{mass}}{30} \Rightarrow \text{mass} = 280 \times 30$$
$$\Rightarrow \text{mass} = 8\,400.$$

Hence, the density of liquid **C** is

$$\frac{1\,400 + 8\,400}{20 + 30} = \frac{9\,800}{50}$$
$$= \underline{\underline{196 \text{ kg/m}^3}}.$$

14. Sally plays two games against Martin. (3)
In each game, Sally could win, draw, or lose.

In each game they play,

- the probability that Sally will win against Martin is 0.3,
- the probability that Sally will draw against Martin is 0.1.

Work out the probability that Sally will win **exactly** one of the two games against Martin.

Solution

The probability that Sally will not win against Martin is

$$1 - 0.3 = 0.7.$$

Hence,

$$\begin{aligned}\text{P(exactly one of the two games)} &= \text{P}(S, M) + \text{P}(M, S) \\ &= (0.3 \times 0.7) + (0.7 \times 0.3) \\ &= 0.21 + 0.21 \\ &= \underline{\underline{0.42}}.\end{aligned}$$

15. The straight line L_1 has equation (3)

$$y = 3x - 4.$$

The straight line L_2 is perpendicular to L_1 and passes through the point $(9, 5)$.

Find an equation of line L_2.

Solution

The gradient of L_2 is

$$\frac{-1}{3} = -\tfrac{1}{3}$$

so the equation is $y = -\frac{1}{3}x + c$, for some constant c. Now,

$$\begin{aligned}x = 9, y = 5 &\Rightarrow 5 = -\tfrac{1}{3}(9) + c \\ &\Rightarrow 5 = -3 + c \\ &\Rightarrow c = 8.\end{aligned}$$

Hence, an equation of line L_2 is

$$\underline{\underline{y = -\tfrac{1}{3}x + 8}}.$$

16. Shirley wants to find an estimate for the number of bees in her hive.

On Monday she catches 90 of the bees.
She puts a mark on each bee and returns them to her hive.

On Tuesday she catches 120 of the bees.

She finds that 20 of these bees have been marked.

(a) Work out an estimate for the total number of bees in her hive. (3)

Solution

Let x be the total number of bees in her hive. Now,

$$\begin{aligned}\frac{20}{120} = \frac{90}{x} &\Rightarrow x = \frac{90 \times 120}{20} \\ &\Rightarrow x = \frac{90 \times \cancel{120}^{6}}{\cancel{20}} \\ &\Rightarrow x = 90 \times 6 \\ &\Rightarrow \underline{\underline{x = 540}}.\end{aligned}$$

Shirley assumes that none of the marks had rubbed off between Monday and Tuesday.

(b) If Shirley's assumption is wrong, explain what effect this would have on your answer to part (a). (1)

Solution

If any of the marks fall off, Shirley will have <u>over-estimated</u> the number of bees.

17. Make f the subject of the formula (4)

$$d = \frac{3(1-f)}{f-4}.$$

Solution

$$\begin{aligned}d = \frac{3(1-f)}{f-4} &\Rightarrow d(f-4) = 3(1-f) \\ &\Rightarrow df - 4d = 3 - 3f \\ &\Rightarrow df + 3f- = 3 + 4d \\ &\Rightarrow f(d+3) = 3 + 4d \\ &\Rightarrow \underline{\underline{f = \frac{3+4d}{d+3}}}.\end{aligned}$$

18. x is proportional to $\sqrt{y}$, where $y > 0$. (3)

y is increased by 44%.

Work out the percentage increase in x.

Solution

Well,

$$x \propto \sqrt{y} \Rightarrow x = k\sqrt{y},$$

where k is some constant. Now, if y is increased by 44%, that means it is now $1.44y$. Hence,

$$\begin{aligned} x &= k\sqrt{1.44y} \\ &= k \times \sqrt{1.44} \times \sqrt{y} \\ &= k \times 1.2 \times \sqrt{y} \\ &= 1.2(k\sqrt{y}); \end{aligned}$$

the percentage increase in x is $\underline{\underline{20\%}}$.

19. f and g are functions such that

$$\mathrm{f}(x) = \frac{12}{\sqrt{x}} \text{ and } \mathrm{g}(x) = 3(2x + 1).$$

(a) Find g(5). (1)

Solution

$$\begin{aligned} \mathrm{g}(5) &= 3(2 \times 5 + 1) \\ &= 3(10 + 1) \\ &= 3 \times 11 \\ &= \underline{\underline{33}}. \end{aligned}$$

(b) Find g f(9). (2)

Solution

$$\begin{aligned} \mathrm{g}\,\mathrm{f}(9) &= \mathrm{g}(\mathrm{f}(9)) \\ &= \mathrm{g}\left(\frac{12}{\sqrt{9}}\right) \\ &= \mathrm{g}\left(\frac{12}{3}\right) \\ &= \mathrm{g}\,(4) \\ &= 3(2 \times 4 + 1) \\ &= 3(8 + 1) \\ &= 3 \times 9 \\ &= \underline{\underline{27}}. \end{aligned}$$

(c) Find $\mathrm{g}^{-1}(6)$. (2)

Solution

$$\begin{aligned} 3(2x + 1) = 6 &\Rightarrow 2x + 1 = 2 \\ &\Rightarrow 2x = 1 \\ &\Rightarrow \underline{\underline{x = \tfrac{1}{2}}}. \end{aligned}$$

20. Show that (4)

$$\frac{\sqrt{180} - 2\sqrt{5}}{5\sqrt{5} - 5}$$

can be written in the form

$$a + \frac{\sqrt{5}}{b},$$

where a and b are integers.

Solution

Well,

$$\begin{aligned} \sqrt{180} &= \sqrt{36 \times 5} \\ &= \sqrt{36} \times \sqrt{5} \\ &= 6\sqrt{5}. \end{aligned}$$

Now,

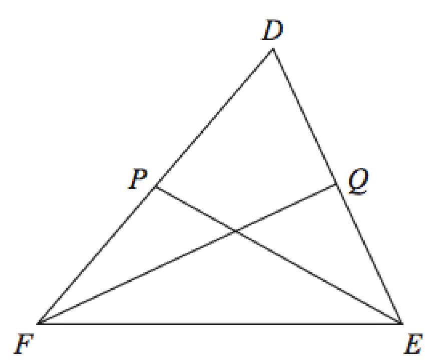
$$\begin{aligned}\frac{\sqrt{180}-2\sqrt{5}}{5\sqrt{5}-5} &= \frac{6\sqrt{5}-2\sqrt{5}}{5(\sqrt{5}-1)} \\ &= \frac{4\sqrt{5}}{5(\sqrt{5}-1)} \\ &= \frac{4\sqrt{5}}{5(\sqrt{5}-1)} \times \frac{\sqrt{5}+1}{\sqrt{5}+1}\end{aligned}$$

$\times$	$\sqrt{5}$	$+1$
$\sqrt{5}$	5	$+\sqrt{5}$
-1	$-\sqrt{5}$	-1

$$\begin{aligned} &= \frac{20+4\sqrt{5}}{5(5-1)} \\ &= \frac{20+4\sqrt{5}}{20} \\ &= 1+\frac{\sqrt{5}}{5};\end{aligned}$$

hence, $\underline{\underline{a=1}}$ and $\underline{\underline{b=5}}$.

21. DEF is a triangle. (4)



P is the midpoint of FD.
Q is the midpoint of DE.

$\overrightarrow{FD} = \mathbf{a}$ and $\overrightarrow{FE} = \mathbf{b}$.

Use a vector method to prove that PQ is parallel to FE.

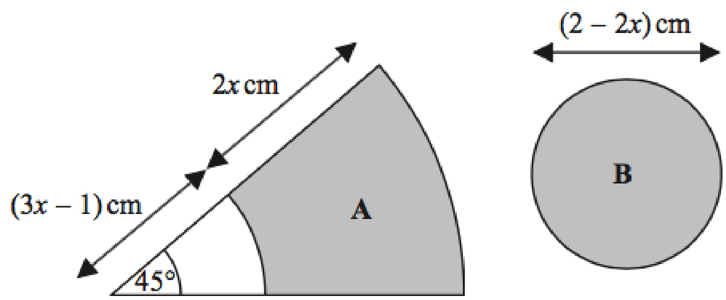
Solution

$$\begin{aligned}\overrightarrow{PQ} &= \overrightarrow{PD} + \overrightarrow{DQ} \\ &= \tfrac{1}{2}\overrightarrow{FD} + \tfrac{1}{2}\overrightarrow{DE} \\ &= \tfrac{1}{2}\overrightarrow{FD} + \tfrac{1}{2}(\overrightarrow{DF} + \overrightarrow{FE}) \\ &= \tfrac{1}{2}\mathbf{a} + \tfrac{1}{2}(-\mathbf{a} + \mathbf{b}) \\ &= \tfrac{1}{2}\mathbf{b} \\ &= \tfrac{1}{2}\overrightarrow{FE}.\end{aligned}$$

Hence, PQ is parallel to FE as $FE = 2PQ$.

22. The diagram shows two shaded shapes, **A** and **B**. (5)

Shape **A** is formed by removing a sector of a circle with radius $(3x-1)$ cm from a sector of the circle with radius $(5x-1)$ cm.
Shape **B** is a circle of diameter $(2-2x)$ cm.



The area of shape **A** is equal to the area of shape **B**.

Find the value of x.
You must show all your working.

Solution

Shape **A**:

$$\text{Shape } \mathbf{A} = \frac{45}{360} \times \pi \times [(5x-1)^2 - (3x-1)^2]$$

$\times$	$5x$	-1
$5x$	$25x^2$	$-5x$
-1	$-5x$	$+1$

$\times$	$3x$	-1
$3x$	$9x^2$	$-3x$
-1	$-3x$	$+1$

$$\begin{aligned} &= \frac{45}{360} \times \pi \times [(25x^2 - 10x + 1) - (9x^2 - 6x + 1)] \\ &= \frac{45}{360} \times \pi \times (16x^2 - 4x) \\ &= \tfrac{1}{8}(16x^2 - 4x)\pi \end{aligned}$$

Shape **B**:

$$\text{Shape } \mathbf{B} = \pi(1-x)^2.$$

Now,

$$\begin{aligned} \tfrac{1}{8}(16x^2 - 4x)\pi = (1-x)^2\pi &\Rightarrow 16x^2 - 4x = 8(1 - 2x + x^2) \\ &\Rightarrow 16x^2 - 4x = 8 - 16x + 8x^2 \\ &\Rightarrow 8x^2 + 12x - 8 = 0 \\ &\Rightarrow 4(2x^2 + 3x - 2) = 0 \end{aligned}$$

$$\left.\begin{array}{ll} \text{add to:} & +3 \\ \text{multiply to:} & (+2) \times (-2) = -4 \end{array}\right\} + 4, \ -1$$

$$\begin{aligned} &\Rightarrow 4[2x^2 + 4x - x - 2] = 0 \\ &\Rightarrow 4[2x(x+2) - 1(x+2)] = 0 \\ &\Rightarrow 4(2x-1)(x+2) = 0 \\ &\Rightarrow 2x - 1 = 0 \text{ or } x + 2 = 0 \\ &\Rightarrow x = \tfrac{1}{2} \text{ or } x = -2; \end{aligned}$$

but $x \neq -2$ (why?). Hence, $x = \frac{1}{2}$.

23. There are four types of cards in a game. (3)

Each card has a black circle or a white circle or a black triangle or a white triangle.

Number of cards with a black shape : number of cards with a white shape = 3 : 5.

Number of cards with a circle : number of cards with a triangle = 2 : 7.

Express the total number of cards with a black shape as a fraction of the total number of cards with a triangle.

Solution

$$3 + 5 = 8 \text{ and } 2 + 7 = 9$$

so we will imagine

$$8 \times 9 = 72$$

cards (it is a common multiple). Now,

$$\begin{aligned}\text{number of cards with a black shape} &= \tfrac{3}{8} \times 72 \\ &= 3 \times 9 \\ &= 27\end{aligned}$$

and

$$\text{number of cards with a white shape} = 72 - 27 = 45.$$

Next,

$$\begin{aligned}\text{number of cards with a circle} &= \tfrac{2}{9} \times 72 \\ &= 2 \times 8 \\ &= 16\end{aligned}$$

and

$$\text{number of cards with a triangle} = 72 - 16 = 56.$$

Hence, the total number of cards with a black shape as a fraction of the total number of cards with a triangle is $\underline{\underline{\frac{27}{56}}}$.