Dr Oliver Mathematics
Mathematics Standard Grade: Credit Level
2008 Paper 1: Non-Calculator
55 minutes

The total number of marks available is 39.
You must write down all the stages in your working.

1. Evaluate (2)

$$24.7 - 0.63 \times 30.$$

Solution

$$\begin{aligned} 24.7 - (0.63 \times 30) &= 24.7 - (6.3 \times 3) \\ &= 24.7 - 18.9 \\ &= \underline{\underline{5.8}}. \end{aligned}$$

2. Factorise fully (2)

$$5x^2 - 45.$$

Solution

$$5x^2 - 45 = 5(x^2 - 9)$$

$$\left.\begin{array}{ll} \text{add to:} & 0 \\ \text{multiply to:} & -9 \end{array}\right\} -3,\ +3$$

$$= \underline{\underline{5(x-3)(x+3)}}.$$

3. (2)

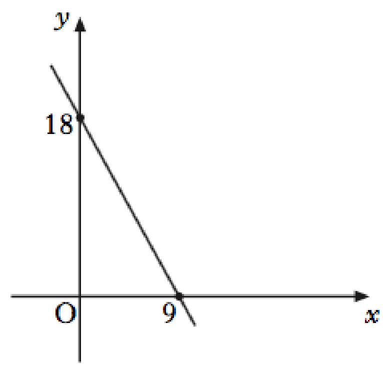
$$W = BH^2.$$

Change the subject of the formula to H.

Solution

$$W = BH^2 \Rightarrow H^2 = \frac{W}{B}$$
$$\Rightarrow \underline{\underline{H = \pm\sqrt{\frac{W}{B}}}}.$$

4. A straight line cuts the x-axis at the point $(9, 0)$ and the y-axis at the point $(0, 18)$, as shown. (3)



Find the equation of this line.

Solution

$$\text{Gradient} = \frac{18 - 0}{0 - 9}$$
$$= -2$$

and the equation of this line is

$$y - 18 = -2(x - 0) \Rightarrow y - 18 = -2x$$
$$\Rightarrow \underline{\underline{y = -2x + 18}}.$$

5. Express as a single fraction in its simplest form (2)

$$\frac{1}{p} + \frac{2}{p + 5}.$$

Solution

$$\begin{aligned}\frac{1}{p}+\frac{2}{p+5} &= \frac{(p+5)}{p(p+5)}+\frac{2p}{p(p+5)}\\ &= \frac{(p+5)+2p}{p(p+5)}\\ &= \underline{\underline{\frac{3p+5}{p(p+5)}}}.\end{aligned}$$

6. Jane enters a two-part race.

   (a) She cycles for 2 hours at a speed of $(x+8)$ kilometres per hour. (1)
   Write down an expression in x for the distance cycled.

Solution

$$\begin{aligned}\text{Cycle} &= 2\times(x+8)\\ &= \underline{\underline{2(x+8)\text{ km}}}.\end{aligned}$$

   (b) She then runs for 30 minutes at a speed of x kilometres per hour. (1)
   Write down an expression in x for the distance run.

Solution

$$\begin{aligned}\text{Run} &= \tfrac{1}{2}\times x\\ &= \underline{\underline{\tfrac{1}{2}x\text{ km}}}.\end{aligned}$$

   (c) The **total** distance of the race is 46 kilometres. (3)
   Calculate Jane's **running** speed.

Solution

$$\begin{aligned} 2(x+8)+\tfrac{1}{2}x = 46 &\Rightarrow 2x+16+\tfrac{1}{2}x = 46 \\ &\Rightarrow \tfrac{5}{2}x = 30 \\ &\Rightarrow \tfrac{1}{2}x = 6 \\ &\Rightarrow \underline{\underline{x = 12}}. \end{aligned}$$

7. The 4th term of each number pattern below is the **mean** of the previous three terms.

(a) When the first three terms are 1, 6, and 8, calculate the 4th term. (1)

Solution

$$\begin{aligned} \text{4th term} &= \frac{1+6+8}{3} \\ &= \frac{15}{3} \\ &= \underline{\underline{5}}. \end{aligned}$$

(b) When the first three terms are x, $(x+7)$ and $(x+11)$, calculate the 4th term. (1)

Solution

$$\begin{aligned} \text{4th term} &= \frac{x+(x+7)+(x+11)}{3} \\ &= \frac{3x+18}{3} \\ &= \frac{3(x+6)}{3} \\ &= \underline{\underline{x+6}}. \end{aligned}$$

(c) When the first, second and fourth terms are (2)

$$-2x, \quad (x+5), \quad \ldots\ldots, \quad (2x+4),$$

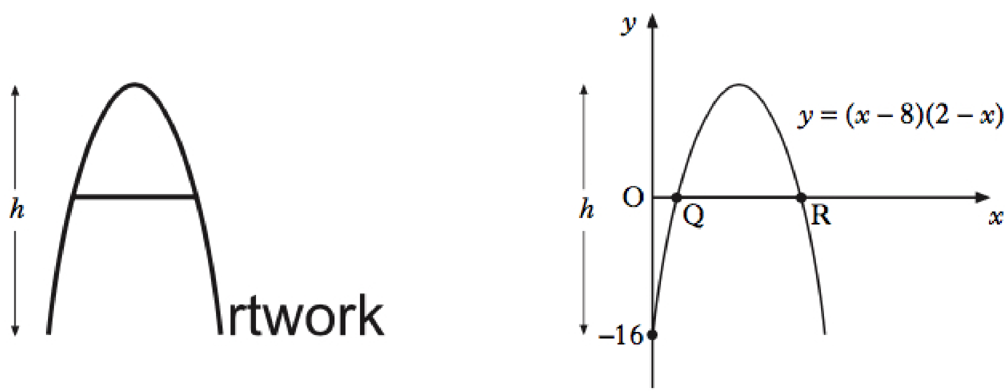
calculate the 3rd term.

Solution

$$\frac{-2x + (x+5) + \text{3rd term}}{3} = 2x + 4 \Rightarrow -x + 5 + \text{3rd term} = 3(2x+4)$$
$$\Rightarrow -x + 5 + \text{3rd term} = 6x + 12$$
$$\Rightarrow \underline{\underline{\text{3rd term} = 7x + 7}}.$$

8. The curved part of the letter A in the *Artwork* logo is in the shape of a parabola. The equation of this parabola is

$$y = (x - 8)(2 - x).$$



(a) Write down the coordinates of Q and R. (2)

Solution
$\underline{\underline{Q(2, 0)}}$ and $\underline{\underline{R(8, 0)}}$

(b) Calculate the height, h, of the letter A. (3)

Solution
The vertex:

$$x = \frac{2+8}{2} = 5$$

and

$$\begin{aligned} y &= (5-8)(2-5) \\ &= (-3) \times (-3) \\ &= 9 \end{aligned}$$

and the height is

$$9 - (-16) = \underline{\underline{25}}.$$

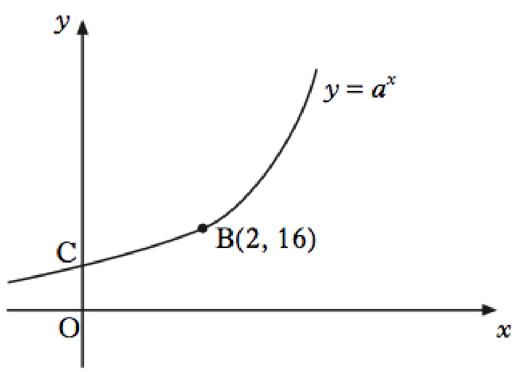
9. Simplify (2)

$$m^3 \times \sqrt{m}.$$

Solution

$$\begin{aligned} m^3 \times \sqrt{m} &= m^3 \times m^{\frac{1}{2}} \\ &= \underline{\underline{m^{\frac{7}{2}}}}. \end{aligned}$$

10. Part of the graph of $y = a^x$, where $a > 0$, is shown below.



The graph cuts the y-axis at C.

(a) Write down the coordinates of C. (1)

Solution

$$x = 0 \Rightarrow y = 1$$

and so the coordinates is $\underline{\underline{C(0, 1)}}$.

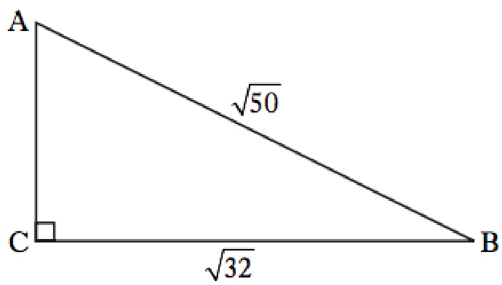
B is the point $(2, 16)$.

(b) Calculate the value of a. (2)

Solution

$$16 = a^2 \Rightarrow a^2 = 4^2$$
$$\Rightarrow \underline{\underline{a = 4}}.$$

11. A right-angled triangle has dimensions as shown. (3)



Calculate the length of AC, leaving your answer as a surd **in its simplest form**.

Solution

$$\begin{aligned} AC &= \sqrt{AB^2 - BC^2} \\ &= \sqrt{(\sqrt{50})^2 - (\sqrt{32})^2} \\ &= \sqrt{50 - 32} \\ &= \sqrt{18} \\ &= \sqrt{9 \times 2} \\ &= \sqrt{9} \times \sqrt{2} \\ &= \underline{\underline{3\sqrt{2}}}. \end{aligned}$$

12. Given that (3)

$$x^2 - 10x + 18 = (x - a)^2 + b,$$

find the values of a and b.

Solution

$$\begin{aligned}(x^2 - 10x) + 18 &= [(x^2 - 10x + 25) - 25] + 18 \\ &= (x-5)^2 - 7;\end{aligned}$$

hence, $\underline{\underline{a = 5}}$ and $\underline{\underline{b = -7}}$.

13. A new fraction is obtained by adding x to the numerator and denominator of the fraction $\frac{17}{24}$. (3)
This new fraction is equivalent to $\frac{2}{3}$.
Calculate the value of x.

Solution

$$\begin{aligned}\frac{17+x}{24+x} = \frac{2}{3} &\Rightarrow 3(17+x) = 2(24+x) \\ &\Rightarrow 51 + 3x = 48 + 2x \\ &\Rightarrow \underline{\underline{x = -3}}.\end{aligned}$$