

Dr Oliver Mathematics

Method of Exhaustion

In this note, we will examine the method of exhaustion.

Example 1

Every cube number is either a multiple of 9, or is 1 more or 1 less than a multiple of 9.

Solution

We will take $3n$, $3n + 1$, and $3n + 2$ as our examples for a^3 .

$$(3n)^3 = 27n^3 = 3(9n^3)$$

which is a multiple of 9;

$$\begin{aligned}(3n + 1)^3 &= 27n^3 + 27n^2 + 9n + 1 \\ &= 9(3n^3 + n^2 + n) + 1,\end{aligned}$$

which is 1 more than a multiple of 9;

$$\begin{aligned}(3n + 2)^3 &= 27n^3 + 54n^2 + 36n + 8 \\ &= 9(3n^3 + 6n^2 + 4n) + 8;\end{aligned}$$

which is 1 less than a multiple of 9.

Here are some examples for you to try.

1. Suppose a and b are even integers. Prove that the sum and difference of a and b are divisible by 2.
2. If n is a positive integer then $(n^7 - n)$ is divisible by 7.