Dr Oliver Mathematics
Applied Mathematics: Matrices

The total number of marks available is 57.
You must write down all the stages in your working.

1. Given that (3)

$$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix},$$

show that

$$\mathbf{A}^2 - \mathbf{A} = k\mathbf{I}$$

for a suitable value of k, where $\mathbf{I}$ is the 2×2 unit matrix.

2. (a) Calculate $\mathbf{A}^{-1}$ where (3)

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 3 & 1 \\ 2 & 2 & 1 \end{pmatrix}.$$

(b) Hence solve the system of equations (2)

$$\begin{aligned} x + y &= 1 \\ 2x + 3y + z &= 2 \\ 2x + 2y + z &= 1. \end{aligned}$$

3. (a) For the matrix (3)

$$\mathbf{A} = \begin{pmatrix} \lambda & 2 \\ 2 & \lambda - 3 \end{pmatrix},$$

find the values of λ such that the matrix is singular.

(b) Write down the matrix $\mathbf{A}^{-1}$ when $\lambda = 3$. (1)

4. Given that $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ are square matrices where:

$$\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 3 & 5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 & 6 \\ 0 & -3 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} x & 2 \\ 0 & y \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 2 & 7 \\ 12 & -1 \end{pmatrix}.$$

(a) Find $\mathbf{AB}$. (1)

(b) Express $4\mathbf{C} + \mathbf{D}$ as a single matrix. (2)

(c) Given that (2)

$$\mathbf{AB} = 4\mathbf{C} + \mathbf{D},$$

find the values of x and y.

5. Determine k such that the matrix (4)

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & k-2 & -1 \\ 1 & 2 & k \end{pmatrix}$$

does not have an inverse.

6. (a) Find the value(s) of m for which the matrix (3)

$$\begin{pmatrix} m & 1 & 1 \\ 0 & m & -2 \\ 1 & 0 & 1 \end{pmatrix}$$

is singular.

The matrix

$$\mathbf{B} = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & 1 \\ 1 & 0 & -3 \end{pmatrix}.$$

(b) Use elementary row operations to obtain $\mathbf{B}^{-1}$. (4)

(c) Hence, or otherwise, solve the system of equations (2)

$$\begin{aligned} x + y - z &= 3 \\ y + z &= -2 \\ x - 3z &= 7. \end{aligned}$$

7. (a) Given (2)

$$\mathbf{A} = \begin{pmatrix} 1 & -2 \\ 3 & 0 \end{pmatrix},$$

obtain $\mathbf{A}^{-1}$.

(b) Given (3)

$$\mathbf{AB} = \begin{pmatrix} -4 & -3 \\ 6 & -3 \end{pmatrix},$$

find the matrix $\mathbf{B}$.

8. Given

$$\mathbf{M} = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & \lambda \end{pmatrix}:$$

(a) Calculate $\mathbf{M}^2$. (2)

(b) Calculate $\mathbf{M} + \mathbf{M}^2 + \mathbf{M}^3$. (2)

(c) For what values of λ does $\mathbf{M}$ have an inverse? (2)

9. Matrices are given as

$$\mathbf{A} = \begin{pmatrix} 4 & x \\ 0 & 2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 5 & 1 \\ 0 & 1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} y & 3 \\ -1 & 2 \end{pmatrix}.$$

(a) Write (2)

$$\mathbf{A}^2 - 3\mathbf{B}$$

as a single matrix.

(b) (i) Given that $\mathbf{C}$ is non-singular, find $\mathbf{C}^{-1}$, the inverse of $\mathbf{C}$. (2)

(ii) For what value of y would matrix $\mathbf{C}$ be singular? (1)

10. Matrices are given as

$$\mathbf{A} = \begin{pmatrix} 1 & 3 & 4 \\ k & 0 & -1 \\ 5 & 3 & 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 & -10 & 2 \\ -3 & 9 & 0 \\ 0 & -2 & 1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 3 & 2 & -6 \\ 1 & 1 & -2 \\ 2 & 2 & -1 \end{pmatrix}.$$

(a) Calculate $\mathbf{A} + \mathbf{B}$. (1)

(b) Find the determinant of $\mathbf{A}$. (2)

(c) Calculate $\mathbf{BC}$. (1)

(d) Describe the relationship between $\mathbf{B}$ and $\mathbf{C}$. (2)

11. (a) Given matrix (2)

$$\mathbf{A} = \begin{pmatrix} 3 & -5 \\ 1 & -1 \end{pmatrix},$$

find $\mathbf{A}^2$ and show that the inverse of $\mathbf{A}^2$ exists.

(b) Hence, or otherwise, find matrix $\mathbf{B}$ such that (3)

$$\mathbf{A}^2\mathbf{B} = \begin{pmatrix} 4 & 6 \\ 2 & -2 \end{pmatrix}.$$