

Dr Oliver Mathematics
GCSE Mathematics
2015 Paper 1H: Non-Calculator
1 hour 45 minutes

The total number of marks available is 100.
You must write down all the stages in your working.

1. Here are the heights in centimetres of 20 men.

165 164 176 179 188 178 183 172 180 190
167 159 156 176 173 168 169 182 167 192

- (a) Show this information in an ordered stem and leaf diagram. (3)

Solution

19	0 2
18	0 2 3 8
17	2 3 6 6 8 9
16	4 5 7 7 8 9
15	6 9

Key: 15|6 means 156 cm.

- (b) Work out the percentage of these men with a height greater than 184 cm. (2)

Solution

There are 3 men taller than 184 cm and so the percentage is

$$\frac{3}{20} \times 100\% = \underline{\underline{15\%}}.$$

2. $x = 3$.

- (a) Work out the value of $4x^2$. (1)

Solution

$$4 \times 3^2 = 4 \times 9 = \underline{\underline{36}}.$$

(b) Solve $5x + 4 = 14 + x$.

(2)

Solution

$$\begin{aligned}5x + 4 &= 14 + x \Rightarrow 4x = 10 \\ &\Rightarrow x = \underline{\underline{2\frac{1}{2}}}.\end{aligned}$$

3. Sean works for a town council.

(2)

He wants to find out how often people use the BMX track in the town.

He is going to use a questionnaire.

Design a suitable question for Sean to use in his questionnaire.

Solution

A suitable question with a time frame, e.g., “Did you use the BMX track today/last week/last month? Tick the appropriate box.”

At least three exhaustive and non-overlapping tick boxes (best defined using inequality notation): for example, 0, 1-3, 4-6, 7 or more.

4. Tom is going to buy 25 plants to make a hedge.

(5)

Here is information about the cost of buying the plants.

Kirsty’s Plants: £2.39 each.

Hedge World: Pack of 25, £52.50 plus VAT at 20%.

Tom wants to buy the 25 plants as cheaply as possible.

Should Tom buy the plants from Kirsty’s Plants or from Hedge World?

You must show all your working.

Solution

Kirsty’s Plants:

×	200	30	9
20	4000	600	180
5	1000	150	45

$$25 \times 2.39 = 4000 + 600 + 180 + 1000 + 150 + 45 = \pounds 59.75.$$

Hedge World:

×	52	0.50
1	52	0.50
0.2	10.40	0.10

$$1.2 \times 52.50 = 52 + 0.50 + 10.40 + 0.10 = \pounds 63.$$

Tom should buy the plants from Kirsty's Plants.

5. Jane makes cheese.
The cheese is in the shape of a cuboid.

(3)

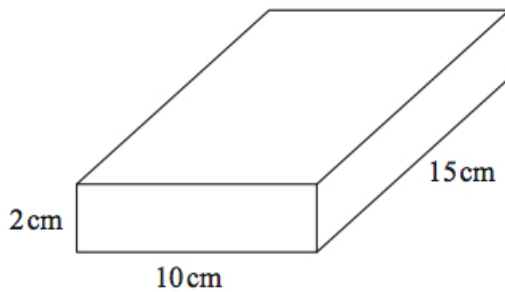


Diagram **NOT** accurately drawn

Jane is going to make a new cheese.
The new cheese will also be in the shape of a cuboid.
The cross section of the cuboid will be a 5 cm by 5 cm square.

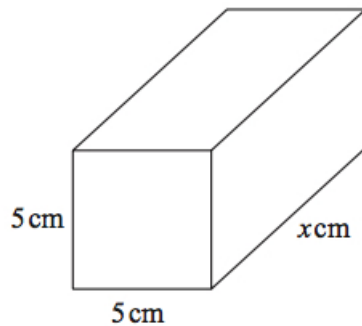


Diagram **NOT** accurately drawn

Jane wants the new cuboid to have the same volume as the 2 cm by 10 cm by 15 cm

cuboid.

Work out the value of x .

Solution

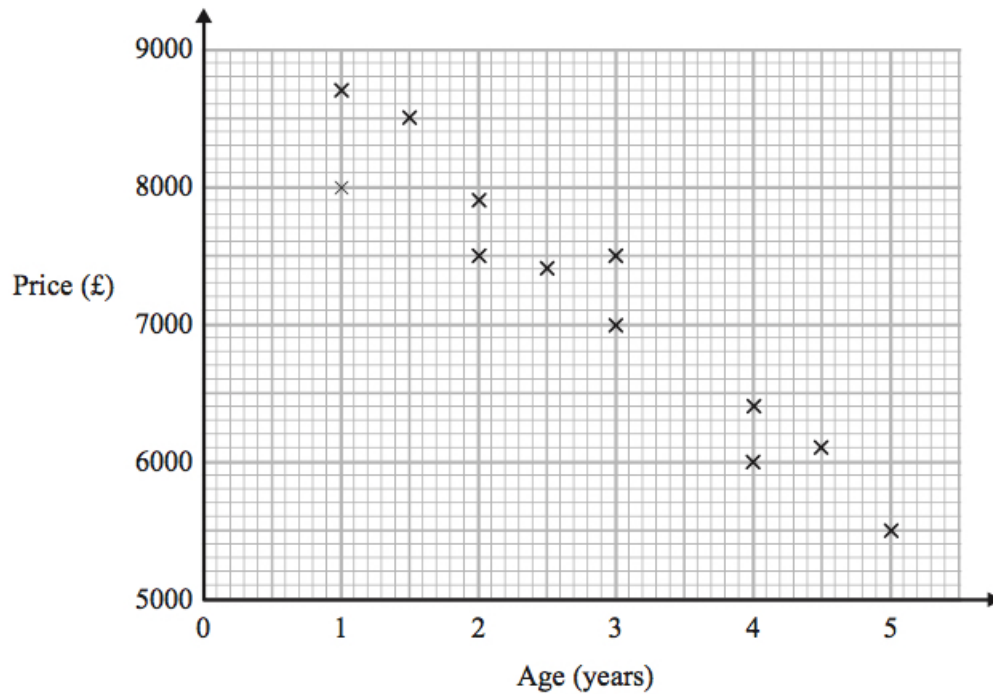
Old cheese:

$$2 \times 10 \times 15 = 20 \times 15 = 300.$$

New cheese:

$$5 \times 5 \times x = 300 \Rightarrow x = \frac{300}{25}$$
$$\Rightarrow \underline{\underline{x = 12.}}$$

6. The scatter graph shows information about the age and the price of each of 12 cars of the same model.



- (a) Describe the relationship between the age of a car and its price.

(1)

Solution

As the car gets older, the cheaper it is.

A different car of the same model is $3\frac{1}{2}$ years old.

(b) Estimate the price of this car.

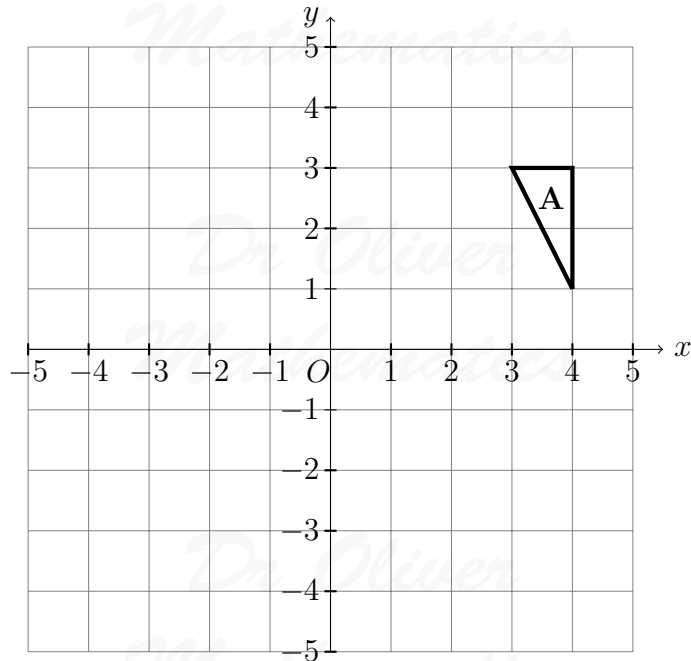
(2)

Solution

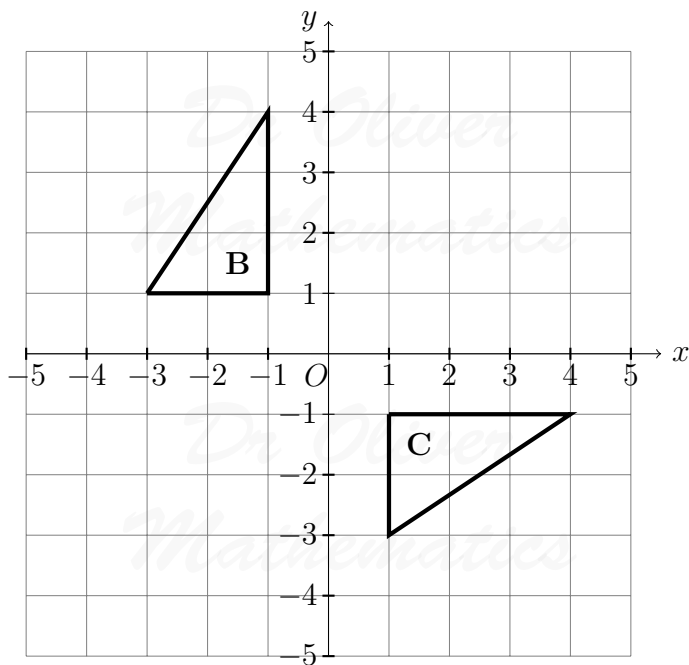
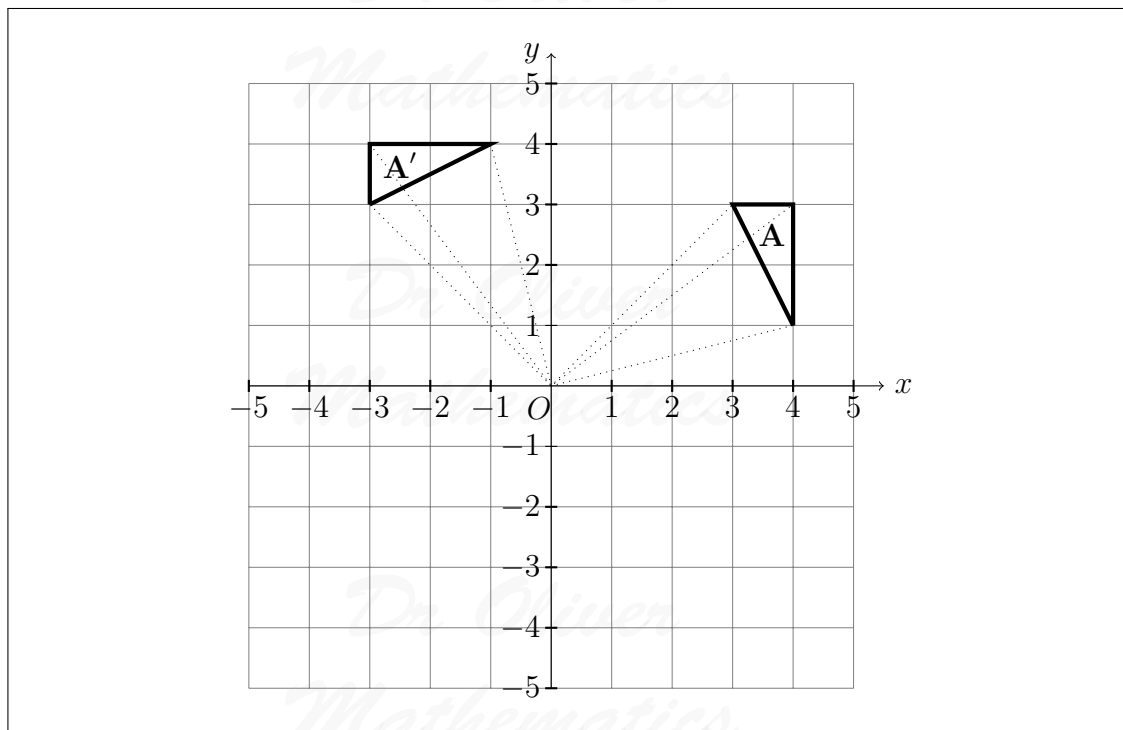
Draw a line of best fit and read-off at Age = 3.5: approximately £6 500.

7. (a) Rotate triangle **A** 90° anticlockwise with centre O .

(2)



Solution



(b) Describe fully the single transformation that maps triangle **B** onto triangle **C**. (2)

Solution

Reflection, in the line $y = x$.

8. (a) Simplify $6g - 5h - 4g + 2h$.

(2)

Solution

$$\begin{aligned}6g - 5h - 4g + 2h &= (6g - 4g) + (-5h + 2h) \\ &= \underline{\underline{2g - 3h}}.\end{aligned}$$

- (b) Factorise $y^2 - 2y$.

(1)

Solution

$$y^2 - 2y = \underline{\underline{y(y - 2)}}.$$

- (c) Simplify fully

(2)

$$\frac{p^3 \times p^4}{p^2}.$$

Solution

$$\begin{aligned}\frac{p^3 \times p^4}{p^2} &= \frac{p^7}{p^2} \\ &= \underline{\underline{p^5}}.\end{aligned}$$

9. John buys some boxes of pencils and some packets of pens for people to use at a conference.

(3)

There are 40 pencils in a box.

There are 15 pens in a packet.

John gives one pencil and one pen to each person at the conference.

He has no pencils left.

He has no pens left.

How many boxes of pencils and how many packets of pens did John buy?

Solution

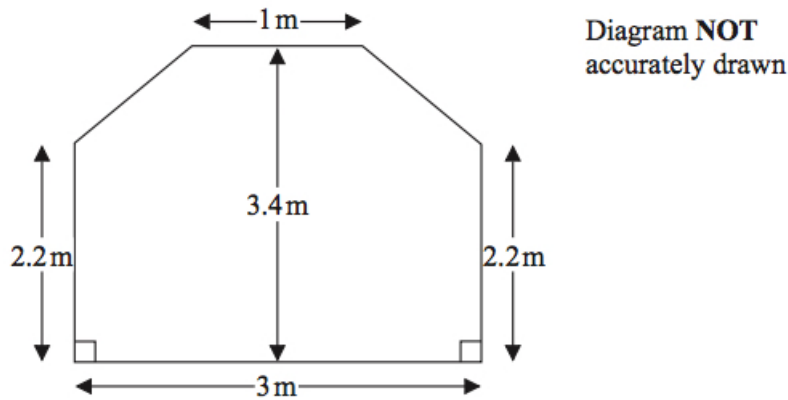
Pencils: $40 = 2^3 \times 5$.

Pens: $15 = 3 \times 5$.

Highest common factor: he buys $2^3 \times 3 \times 5 = 120$ of each which means he has 3 boxes of pencils and 8 packets of pens.

10. The diagram shows the floor plan of Mary's conservatory.

(5)



Mary is going to cover the floor with tiles.

The tiles are sold in packs.

One pack of tiles will cover 2 m^2 .

A pack of tiles normally costs £24.80.

Mary gets a discount of 25% off the cost of the tiles.

Mary has £100.

Does Mary have enough money to buy all the tiles she needs?

You must show all your working.

Solution

$$\begin{aligned} \text{Area} &= \left[\frac{1}{2} \times (2.2 + 3.4) \times 1 \right] + (3.4 \times 1) + \left[\frac{1}{2} \times (2.2 + 3.4) \times 1 \right] \\ &= \left(\frac{1}{2} \times 5.6 \right) + 3.4 + \left(\frac{1}{2} \times 5.6 \right) \\ &= 5.6 + 3.4 \\ &= 9 \text{ m}^2. \end{aligned}$$

Hence, Mary need 5 packs of tiles.

×	20	4	0.8
0.7	14	2.8	0.56
0.05	1	0.2	0.04

Mary gets a discount: each pack costs her

$$14 + 1 + 2.8 + 0.2 + 0.56 + 0.04 = \text{£}18.60$$

and this costs her

$$5 \times 18.60 = \text{£}93;$$

so, Mary has enough money.

11. Karl wants to raise money for charity. He designs a game for people to play. Karl uses a fair 10-sided dice for the game. The dice is numbered from 1 to 10. Each person will roll the dice once. A person wins the game if the dice lands on a multiple of 4. Ali plays the game once.

(a) Work out the probability that Ali will win the game.

(2)

Solution

The only way that Ali can win is by rolling a 4 or 8 and the probability is $\frac{2}{10} = \frac{1}{5}$.

Each person pays 30p to play the game once.

The prize for a win is £1.

Karl thinks that the game will be played 100 times.

(b) Work out an estimate for how much money Karl will raise for charity.

(3)

Solution

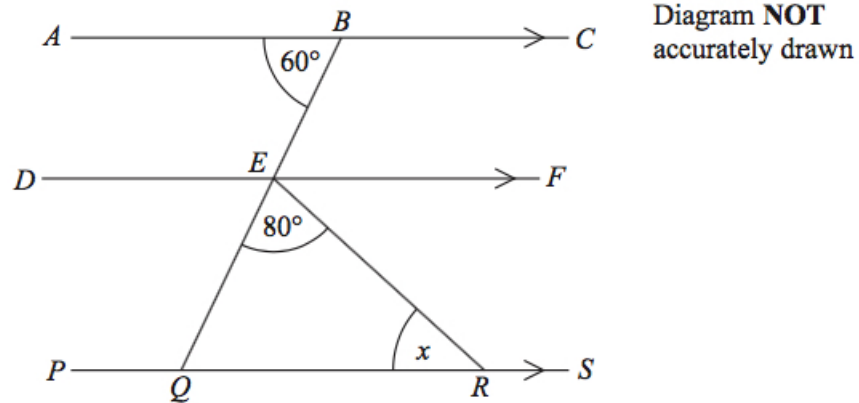
He will take £30 and he will give out

$$\frac{1}{5} \times 100 = \text{£}20;$$

hence, Karl can expect the charity to make £10.

12. ABC , DEF , and $PQRS$ are parallel lines.
 BEQ is a straight line.

(4)



Angle $ABE = 60^\circ$.

Angle $QER = 80^\circ$.

Work out the size of the angle marked x .

Give reasons for each stage of your working.

Solution

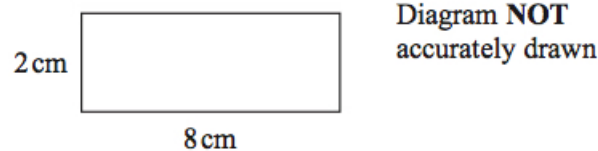
Angle $DEQ = 60^\circ$ (corresponding angles).

$\angle DERQ$ is an interior angle:

$$60 + 80 + x = 180 \Rightarrow \underline{x = 40^\circ}.$$

13. Here is a rectangle.

(5)



The 8-sided shape below is made from 4 of these rectangles and 4 congruent right-angled triangles.

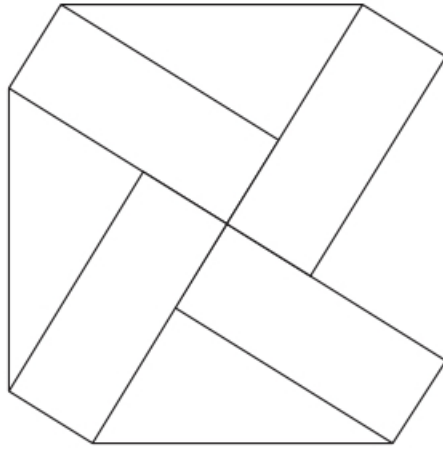


Diagram **NOT**
accurately drawn

Work out the perimeter of the 8-sided shape.
You must show all your working.

Solution

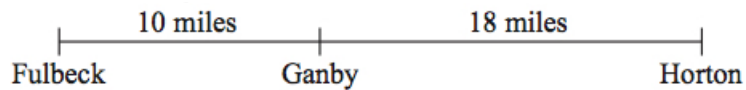
$$\begin{aligned} \text{Hypotenuse} &= \sqrt{8^2 + 6^2} \\ &= \sqrt{100} \\ &= 10, \end{aligned}$$

and the perimeter is

$$\begin{aligned} (4 \times 2) + (4 \times 10) &= 8 + 40 \\ &= \underline{\underline{48 \text{ cm.}}} \end{aligned}$$

14. The distance from Fulbeck to Ganby is 10 miles.
The distance from Ganby to Horton is 18 miles.

(3)



Raksha is going to drive from Fulbeck to Ganby.
Then she will drive from Ganby to Horton.
Raksha leaves Fulbeck at 10 : 00.
She drives from Fulbeck to Ganby at an average speed of 40 mph.

Raksha wants to get to Horton at 10 : 35.

Work out the average speed Raksha must drive at from Ganby to Horton.

Solution

Fulbeck to Ganby:

$$\text{Time} = \frac{10}{\frac{2}{3}} = 15 \text{ minutes}$$

and that means she has 20 minutes to cover the rest of the journey.

Ganby to Horton:

$$\text{Speed} = \frac{18}{\frac{1}{3}} = \underline{\underline{54 \text{ mph.}}}$$

15. A and B are two points.

Point A has coordinates $(-2, 4)$.

Point B has coordinates $(8, 9)$.

C is the midpoint of the line segment AB .

(a) Find the coordinates of C .

(2)

Solution

$$\left(\frac{-2+8}{2}, \frac{4+9}{2}\right) = \underline{\underline{(3, 6\frac{1}{2})}}.$$

D is the point with coordinates $(100, 56)$.

(b) Does point D lie on the straight line that passes through A and B ?

(3)

You must show how you work out your answer.

Solution

$$\begin{aligned} \text{Gradient} &= \frac{9 - 4}{8 - (-2)} \\ &= \frac{1}{2} \end{aligned}$$

and the equation is

$$y - 9 = \frac{1}{2}(x - 8) \Rightarrow y = \frac{1}{2}x + 5.$$

Does $(100, 56)$ lie on the line? Well,

$$y = \frac{1}{2}(100) + 5 = 55;$$

no, it does not.

16. The table shows information about the times taken by 100 people in a fun run.

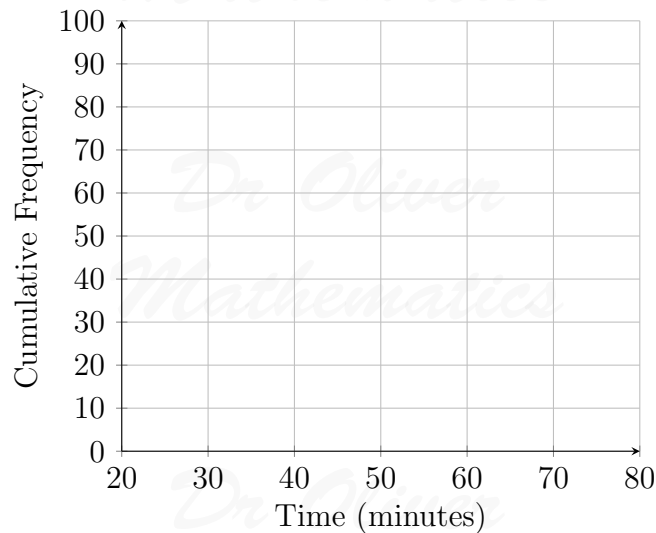
Time (t minutes)	Frequency
$20 < t \leq 30$	4
$30 < t \leq 40$	16
$40 < t \leq 50$	36
$50 < t \leq 60$	24
$60 < t \leq 70$	14
$70 < t \leq 80$	6

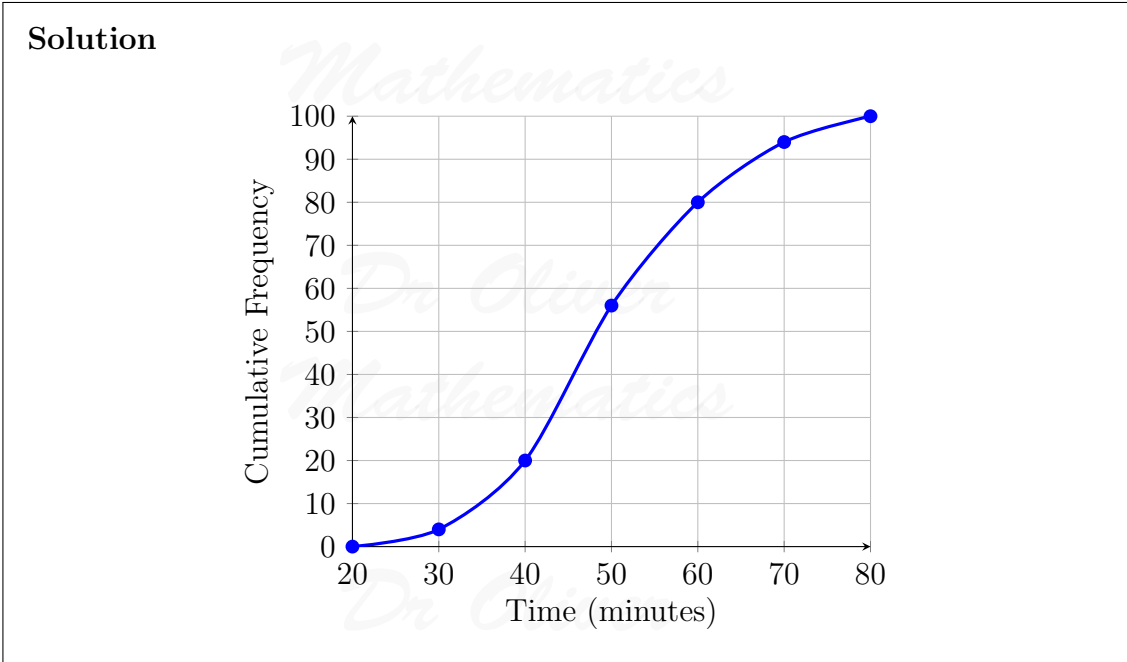
(a) Complete the cumulative frequency table for this information. (1)

Solution

Time (t minutes)	Cumulative Frequency
$20 < t \leq 30$	<u>4</u>
$20 < t \leq 40$	$4 + 16 = \underline{\underline{20}}$
$20 < t \leq 50$	$20 + 36 = \underline{\underline{56}}$
$20 < t \leq 60$	$56 + 24 = \underline{\underline{80}}$
$20 < t \leq 70$	$80 + 14 = \underline{\underline{94}}$
$20 < t \leq 80$	$94 + 6 = \underline{\underline{100}}$

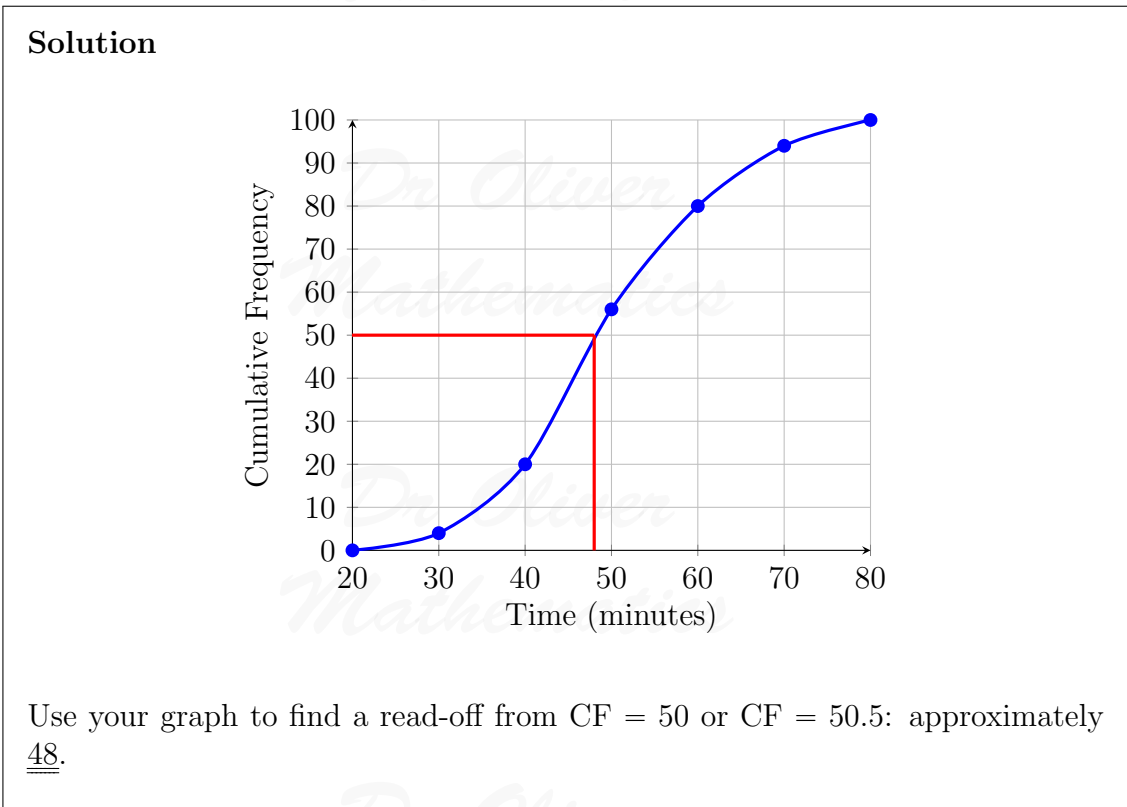
(b) Draw a cumulative frequency graph for your table. (2)





(c) Use your graph to find an estimate for the median time.

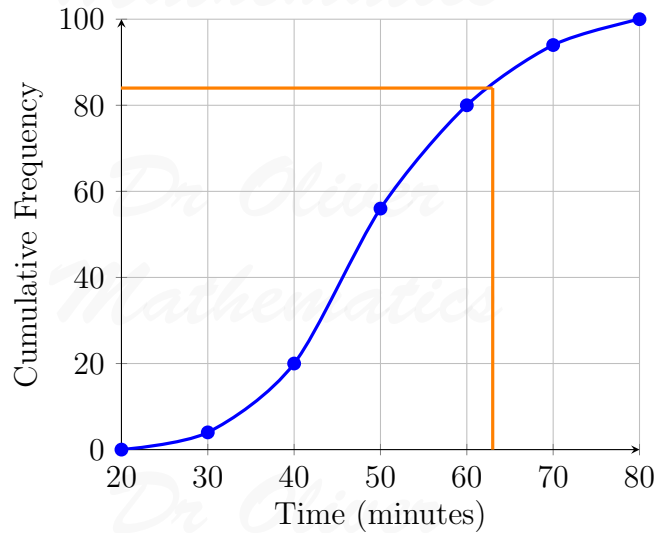
(1)



(d) Use your graph to find an estimate for the number of people who took longer than 63 minutes.

(2)

Solution



Use your graph to find a read-off from Time = 63: approximately 84 and so the estimate for the number of people who took longer than 63 minutes is

$$100 - 84 = \underline{16}.$$

17. $ABCDE$ is a regular pentagon.
 $ACFG$ is a square.

(4)

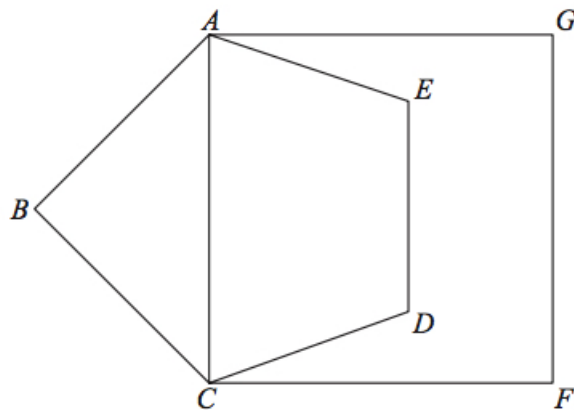


Diagram **NOT** accurately drawn

Work out the size of angle DCF .
 You must show all your working.

Solution

The number of sides in a regular pentagon is 5 and the each angle is

$$\frac{(5 - 2)180}{5} = 108^\circ.$$

Now,

$$\angle BCA = \frac{1}{2}(180 - 108) = \frac{1}{2} \times 72 = 36^\circ$$

and

$$\angle DCA = 108 - 36 = 72^\circ.$$

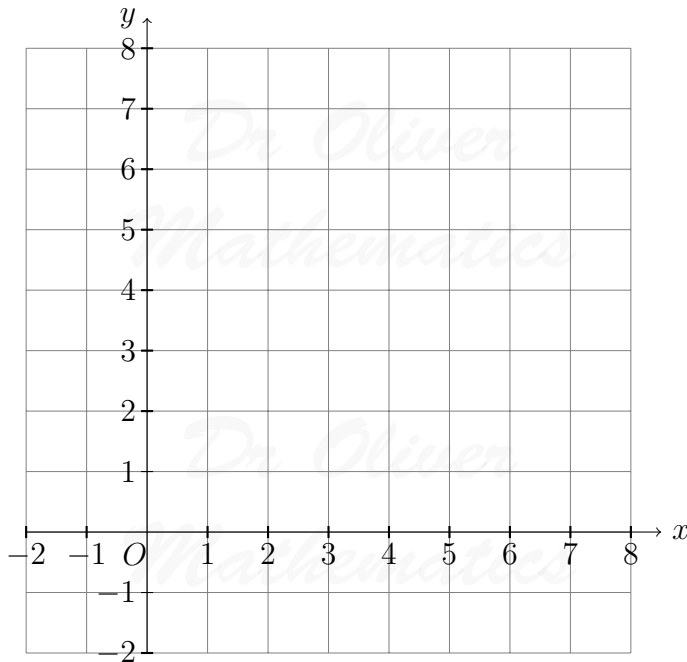
Finally,

$$\angle DCF = 90 - 72 = \underline{\underline{18^\circ}}.$$

18. On the grid show, by shading, the region that satisfies all three of the inequalities:

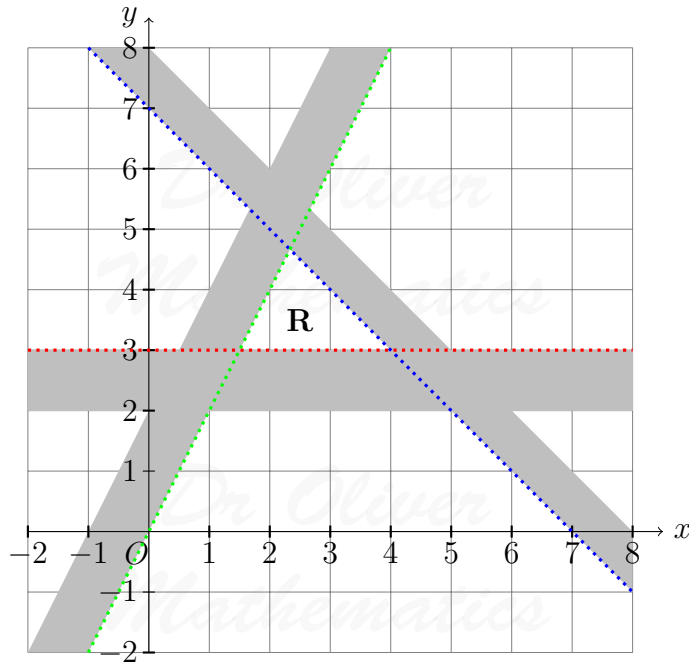
(4)

$$x + y < 7, y < 2x, \text{ and } y > 3.$$



Label the region **R**.

Solution



19. There are n sweets in a bag.
 6 of the sweets are orange.
 The rest of the sweets are yellow.
 Hannah takes at random a sweet from the bag.
 She eats the sweet.
 Hannah then takes at random another sweet from the bag.
 She eats the sweet.
 The probability that Hannah eats two orange sweets is $\frac{1}{3}$.

(a) Show that $n^2 - n - 90 = 0$.

(3)

Solution

$$\begin{aligned} \frac{6}{n} \times \frac{5}{n-1} &= \frac{1}{3} \Rightarrow 90 = n(n-1) \\ &\Rightarrow 90 = n^2 - n \\ &\Rightarrow \underline{\underline{n^2 - n - 90 = 0}}, \end{aligned}$$

as required.

(b) Solve $n^2 - n - 90 = 0$ to find the value of n .

(3)

Solution

$$\begin{aligned}n^2 - n - 90 = 0 &\Rightarrow (n - 10)(n + 9) = 0 \\ &\Rightarrow n = 10 \text{ and } n = -9;\end{aligned}$$

now, $n = -9$ makes no sense (because we are talking about sweets) and so we are left with $n = 10$.

20. Make a the subject of the formula

(4)

$$p = \frac{3a + 5}{4 - a}.$$

Solution

$$\begin{aligned}p = \frac{3a + 5}{4 - a} &\Rightarrow p(4 - a) = 3a + 5 \\ &\Rightarrow 4p - ap = 3a + 5 \\ &\Rightarrow 4p - 5 = 3a + ap \\ &\Rightarrow 4p - 5 = a(3 + p) \\ &\Rightarrow a = \frac{4p - 5}{3 + p}.\end{aligned}$$

21. $x = 0.04\dot{5}$.

(3)

Prove algebraically that x can be written as $\frac{1}{22}$.

Solution

$$\begin{aligned}100x &= 4.54\dot{5} \\ x &= 0.04\dot{5}\end{aligned}$$

Subtract:

$$\begin{aligned}99x = 4.5 &\Rightarrow 99x = \frac{9}{2} \\ &\Rightarrow 11x = \frac{1}{2} \\ &\Rightarrow \underline{\underline{x = \frac{1}{22}}},\end{aligned}$$

as required.

22. (a) Find the value of 2^{-3} .

(1)

Solution

$$2^{-3} = \frac{1}{2^3} = \frac{1}{\underline{\underline{8}}}.$$

$5\sqrt{5}$ can be written in the form 5^k .

- (b) Find the value of k .

(1)

Solution

$$5\sqrt{5} = 5^1 \times 5^{\frac{1}{2}} = 5^{\frac{3}{2}};$$

so, $\underline{\underline{k = \frac{3}{2}}}$.

- (c) Work out the value of $(\sqrt{12} - \sqrt{3})^2$.

(2)

Solution

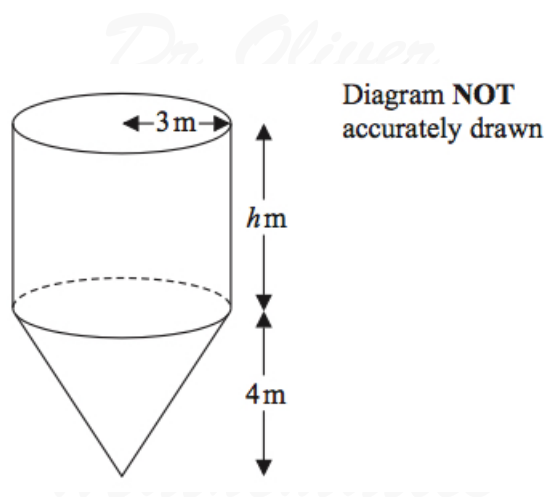
$$\begin{array}{r|rr} \times & \sqrt{12} & -\sqrt{3} \\ \hline \sqrt{12} & 12 & -6 \\ -\sqrt{3} & -6 & +3 \\ \hline \end{array}$$

Hence,

$$(\sqrt{12} - \sqrt{3})^2 = \underline{\underline{3}}.$$

23. The diagram shows a container for grain.

(5)



The container is a cylinder on top of a cone.

The cylinder has a radius of 3 m and a height of h m.

The cone has a base radius of 3 m and a vertical height of 4 m.

The container is empty.

The container is then filled with grain at a constant rate.

After 5 hours the depth of the grain is 6 metres above the vertex of the cone.

After 9 hours the container is full of grain.

Work out the value of h .

Give your answer as a fraction in its simplest form.

You must show all your working.

Solution

$$\begin{aligned} \text{Volume of the cone} &= \frac{1}{3} \times \pi \times 3^2 \times 4 \\ &= 12\pi, \end{aligned}$$

$$\begin{aligned} \text{Volume}_{t=5} &= 12\pi + (\pi \times 3^2 \times 2) \\ &= 12\pi + 18\pi \\ &= 30\pi, \end{aligned}$$

and

$$\begin{aligned} \text{Volume}_{t=9} &= 12\pi + (\pi \times 3^2 \times h) \\ &= 12\pi + 9h\pi \\ &= (12 + 9h)\pi. \end{aligned}$$

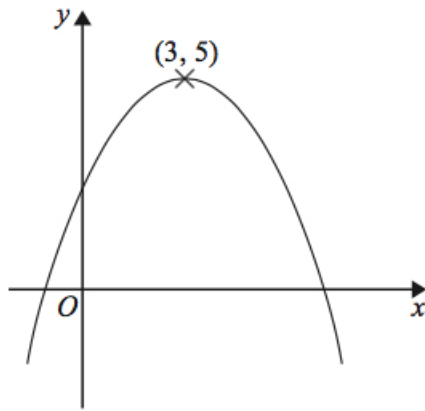
But

$$\begin{aligned} \text{Volume}_{t=9} &= \frac{9}{5} \times 30\pi \\ &= 54\pi, \end{aligned}$$

and so

$$\begin{aligned}(12 + 9h)\pi &= 54\pi \Rightarrow 12 + 9h = 54 \\ \Rightarrow 9h &= 42 \\ \Rightarrow h &= \frac{42}{9} \\ \Rightarrow h &= \underline{\underline{\frac{14}{3}}}\end{aligned}$$

24. The diagram shows part of the curve with equation $y = f(x)$.



The coordinates of the maximum point of the curve are $(3, 5)$.

(a) Write down the coordinates of the maximum point of the curve with equation

(3)

(i) $y = f(x + 3)$,

Solution

$(0, 5)$

(ii) $y = 2f(x)$,

Solution

$(3, 10)$

(iii) $y = f(3x)$.

Solution

$(1, 5)$

The curve with equation $y = f(x)$ is transformed to give the curve with equation $y = f(x) - 4$.

(b) Describe the transformation.

(1)

Solution

It is a translation, by $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$.