Dr Oliver Mathematics
AQA Mathematics
2016 June Paper 1: Calculator
1 hour 30 minutes

The total number of marks available is 70.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You must write down all the stages in your working.

1. Increase £190 by 35%. (3)

Solution

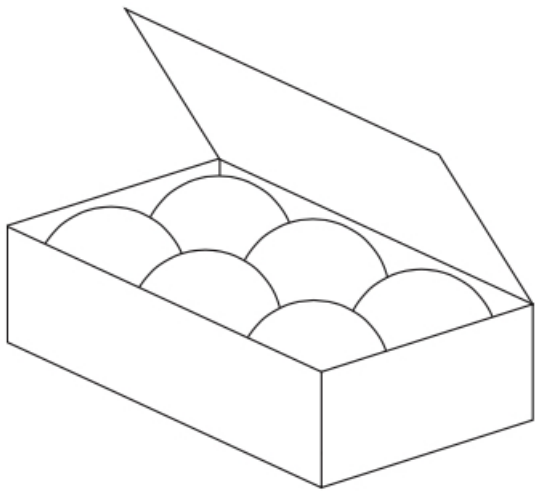
Well,

$$10\% \leftrightarrow 19$$
$$5\% \leftrightarrow 9.5.$$

So,

$$\begin{aligned} \text{new total} &= 190 + (3 \times 10\%) + 5\% \\ &= 190 + 57 + 9.5 \\ &= \underline{\underline{£256.50}}. \end{aligned}$$

2. Six balls just fit inside a box as shown. (3)



- The balls each have a diameter of 5 cm.
- The box is a cuboid.

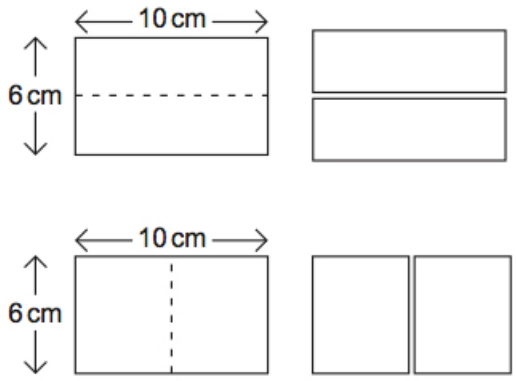
Work out the volume of the box.

Solution

Well,

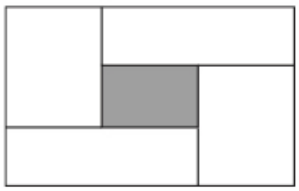
$$\begin{aligned} \text{volume} &= (3 \times 5) \times (2 \times 5) \times 5 \\ &= 15 \times 10 \times 5 \\ &= \underline{\underline{750 \text{ cm}^3}}. \end{aligned}$$

3. Two 10 cm by 6 cm rectangles are cut in half as shown. (3)



Not drawn accurately

The four pieces are joined together, without overlap, as shown.



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Work out the perimeter of the shaded rectangle.

Solution

Now, the first is cut into 10 cm by 3 cm and the second is cut into 5 cm by 6 cm. So, that makes the four pieces (5+10) cm by (6+3) cm, i.e., 15 cm by 9 cm.
The length of the shaded rectangle is

$$15 - (5 + 5) = 5$$

and the breadth of the shaded rectangle is

$$9 - (3 + 3) = 3.$$

Hence,

$$\text{perimeter} = 2(5 + 3)$$

$$= \underline{\underline{16 \text{ cm}}}.$$

4. A bag has only red, white, blue, and yellow counters.
A counter is taken from the bag at random.

Here are some of the probabilities.

Colour	Red	White	Blue	Yellow
Probability	0.1		0.3	

(a) The probability of taking a white counter is twice the probability of taking a yellow counter. (2)

Complete the table.

Solution

Well,

$$\text{P(white or yellow)} = 1 - (0.1 + 0.3) = 0.6$$

and that means

$$\text{P(white)} = \tfrac{2}{3} \times 0.6 = 0.4.$$

Hence,

Colour	Red	White	Blue	Yellow
Probability	0.1	0.4	0.3	0.2

(b) There are 500 counters in the bag altogether. (2)

Complete the table.

Colour	Red	White	Blue	Yellow	Total
Counters in the bag	50				500

Solution

Well,

$$0.1 \leftrightarrow 50$$
$$0.2 \leftrightarrow 100$$
$$0.3 \leftrightarrow 150$$
$$0.4 \leftrightarrow 200,$$

and so we have

Colour	Red	White	Blue	Yellow	Total
Counters in the bag	50	200	150	100	500

(c) All of the yellow counters are taken out of the bag. (2)

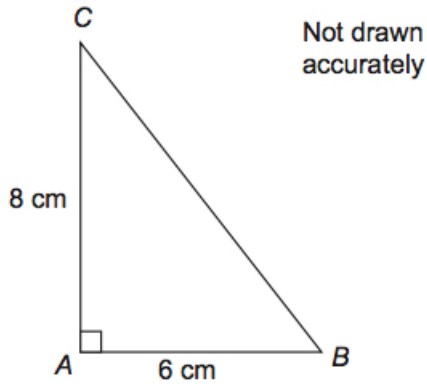
Work out the probability of taking a red counter at random from the bag now.

Solution

Well, that leaves 400 counters and 50 are red:

$$\frac{50}{400} = \frac{1}{8}.$$

5. Work out length BC. (3)

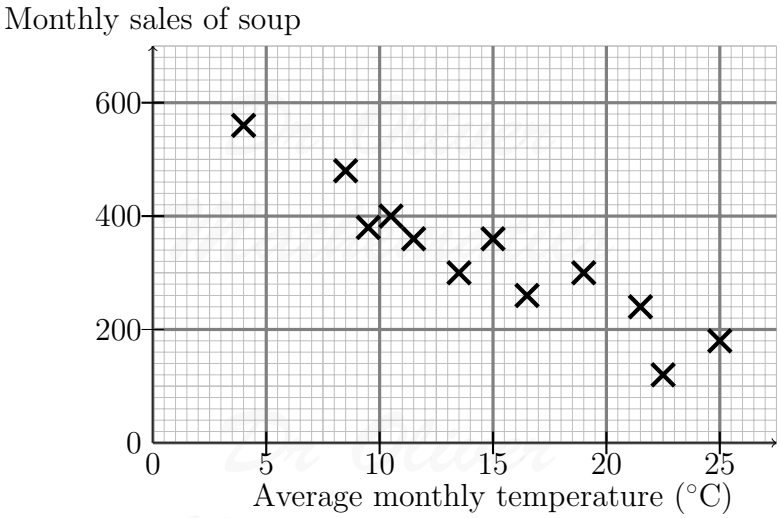


Solution

Pythagoras' theorem:

$$\begin{aligned} BC^2 = AB^2 + AC^2 &\Rightarrow BC^2 = 6^2 + 8^2 \\ &\Rightarrow BC^2 = 36 + 64 \\ &\Rightarrow BC^2 = 100 \\ &\Rightarrow \underline{\underline{BC = 10 \text{ cm}}}. \end{aligned}$$

6. A café owner records the average monthly temperature and the monthly sales of soup over a year.



(a) The scatter graph shows negative correlation. (1)

Write down the relationship between average monthly temperature and monthly sales of soup.

Solution

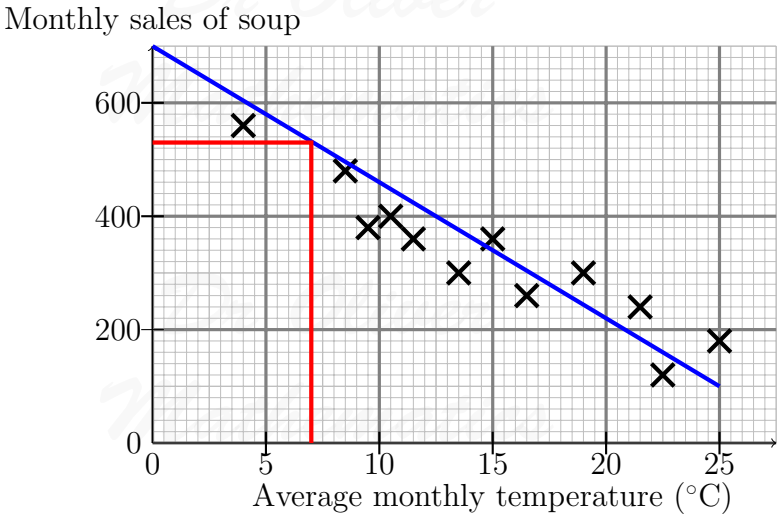
E.g., as the temperature goes up, the monthly sales go down.

(b) The average monthly temperature for the next month is predicted to be 7°C. (2)

Use the graph to estimate the sales of soup that month.
You **must** show your working.

Solution

We draw the line of best fit:



He expects to sell 530 can of soup.

7. Dwayne Pipes uses this formula to work out the cost of a plumbing job in pounds. (4)

$$\text{Cost of job} = 35 \times \text{number of hours} + 40$$

Ivor Wrench uses this formula to work out the cost of a plumbing job in pounds.

$$\text{Cost of job} = 40 \times \text{number of hours} + 17.5$$

A job of x hours costs the same with Dwayne and Ivor.

Set up and solve an equation to work out x.

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Solution

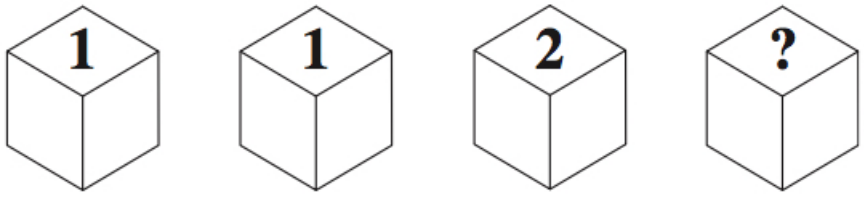
Let C be the cost. Then

$$35x + 40 = C \quad (1)$$
$$40x + 17.5 = C \quad (2).$$

Do (2) − (1):

$$5x - 22.5 = 0 \Rightarrow 5x = 22.5$$
$$\Rightarrow \underline{\underline{x = 4.5}}.$$

8. (a) The scores on four ordinary, six-sided dice are put in order. (1)



The median of the **four** scores is 0.5 **less** than the mean of the four scores.

Circle the value of the fourth score.

2 3 4 5 6

Solution

2 3 $\underline{\underline{4}}$ 5 6

(b) The dice are rolled again. (2)
The median of the scores is 0.5 **less** than the range.

Work out a possible set of scores.

Solution

Well,

$$\text{range} = \tfrac{1}{2}(\text{sum of the middle two values} + 1)$$

and so a possible set of scores is

$$\underline{\underline{1,\ 3,\ 4,\ 5}}.$$

9. (a) Simplify fully (1)

$$\frac{w^3 \times w^4}{w^2}.$$

Solution

Well,

$$\begin{aligned}\frac{w^3 \times w^4}{w^2} &= \frac{w^{3+4}}{w^2}\\ &= \frac{w^7}{w^2}\\ &= w^{7-2}\\ &= \underline{\underline{w^5}}.\end{aligned}$$

(b) Simplify fully (2)

$$2x^2y^3 \times 4xy^2.$$

Solution

$$2x^2y^3 \times 4xy^2 = \underline{\underline{8x^3y^5}}.$$

(c) Simplify fully (2)

$$12a^4b^5 \div 2a^2b.$$

Solution

Now,

$$12a^4b^5 \div 2a^2b = \frac{12a^4b^5}{2a^2b}$$
$$= \underline{\underline{6a^2b^4}}.$$

10. (a) Work out (2)

$$(3 \times 10^5) \times (6 \times 10^{-2}).$$

Write your answer in standard form.

Solution

Well,

$$(3 \times 10^5) \times (6 \times 10^{-2}) = (3 \times 6) \times (10^5 \times 10^{-2})$$
$$= 18 \times 10^3$$
$$= \underline{\underline{1.8 \times 10^4}}.$$

(b) Work out (2)

$$(8 \times 10^4 + 4 \times 10^4) \div 24.$$

Write your answer in standard form.

Solution

Now,

$$(8 \times 10^4 + 4 \times 10^4) \div 24 = (12 \times 10^4) \div 24$$
$$= 0.5 \times 10^4$$
$$= \underline{\underline{5 \times 10^3}}.$$

11. Describe fully the **single** transformation that maps shape A to shape B. (3)

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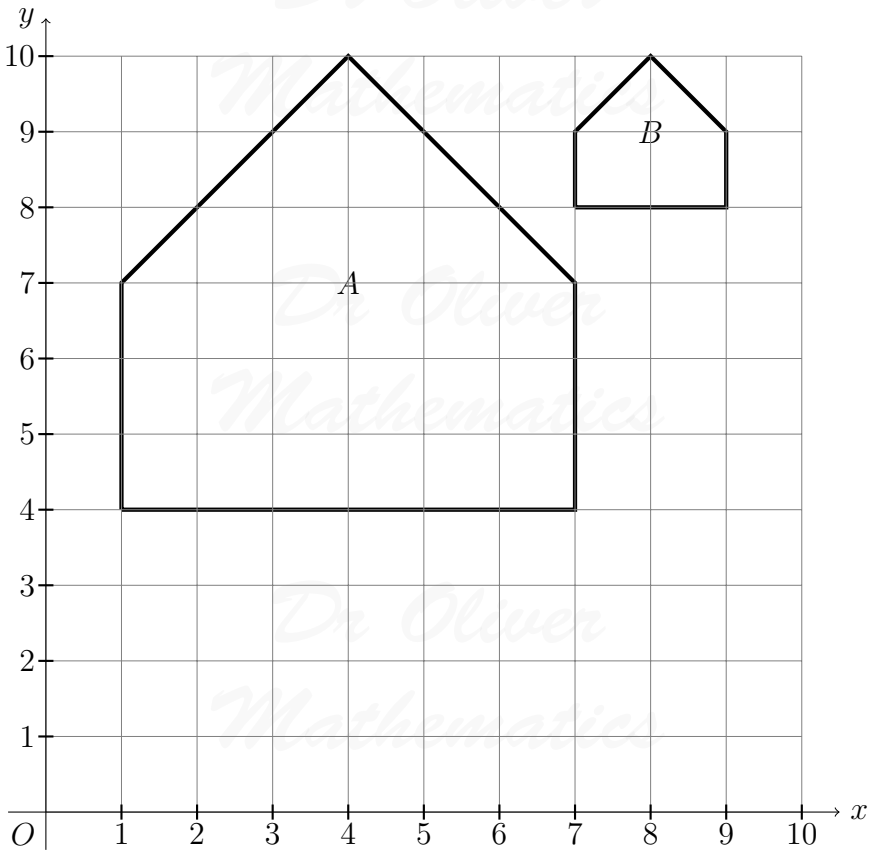

Solution

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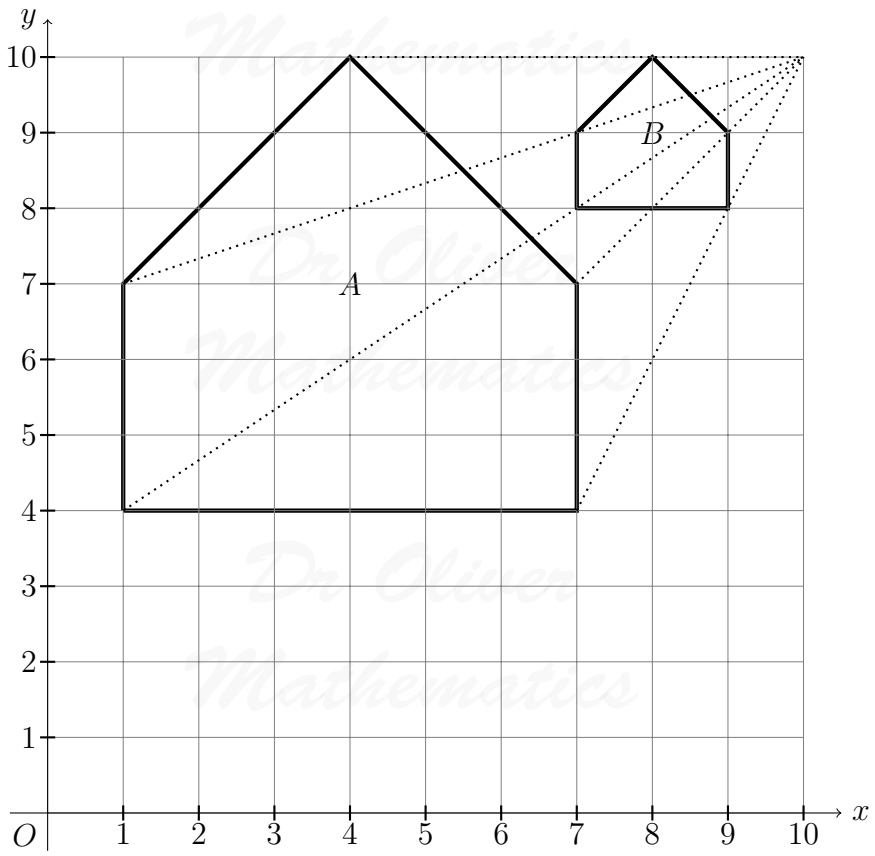

E.g., enlargement, scale factor $\frac{1}{3}$, centre $(10, 10)$.

12. (a) O is the centre of the circle. (1)

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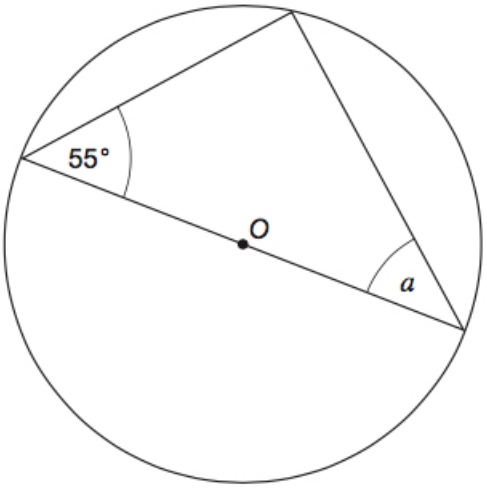


Not drawn accurately

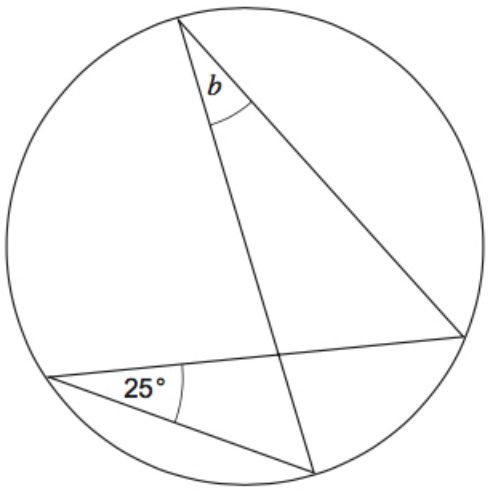
Work out the size of angle a.

Solution

The angle is 90° and so

$$55 + 90 + a = 180 \Rightarrow 145 + a = 180$$
$$\Rightarrow \underline{\underline{a = 35}}.$$

(b) Work out the size of angle b. (1)



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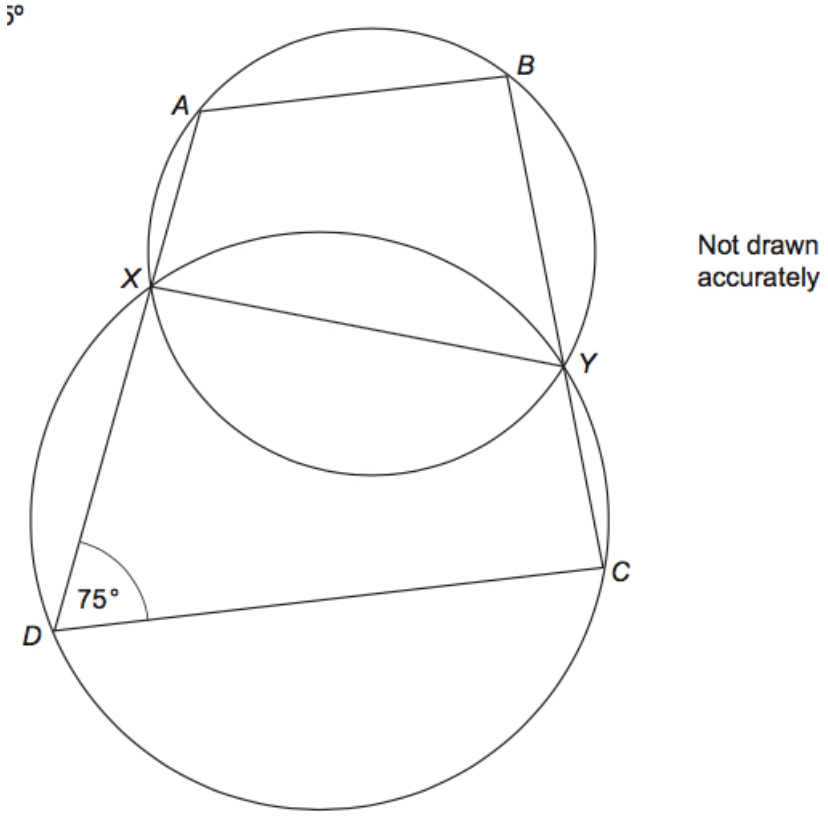
Solution

$\underline{\underline{b = 25}}$.

(c) Two circles, with different radii, intersect at X and Y. (3)



- A and B are points on the smaller circle.
- C and D are points on the larger circle.
- AXD and BYC are straight lines.
- Angle $XDC = 75°$.

Show that AB is parallel to DC.

Solution

Well, $\angle XYC = 180 - 75 = 105°$ (cyclic quadrilateral)
$\angle XYB = 180 - 105 = 75°$ (straight line)
They are $\underline{\underline{\text{parallel}}}$ as interior angles add up to $180°$ (supplementary angles)

13. (a) Expand and simplify (2)

$$(6x - 1)(2x + 3).$$

Solution

Well,

$\times$	$6x$	-1
$2x$	$12x^2$	$-2x$
$+3$	$+18x$	-3

so

$$(6x-1)(2x+3) = \underline{\underline{12x^2+16x-3}}.$$

(b) Solve (3)

$$4x^2 + x - 3 = 0.$$

Solution

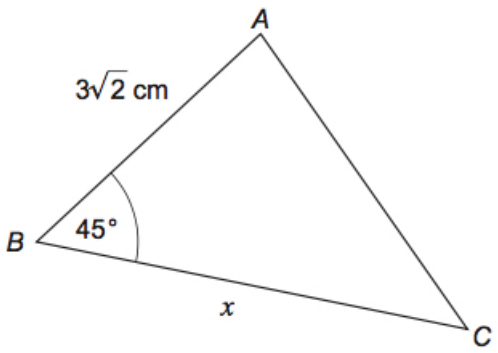
Now,

$$\left.\begin{array}{ll} \text{add to:} & +1 \\ \text{multiply to:} & (+4)\times(-3) = -12 \end{array}\right\} +4,\ -3$$

e.g.,

$$\begin{aligned} 4x^2 + x - 3 = 0 &\Rightarrow 4x^2 + 4x - 3x - 3 = 0 \\ &\Rightarrow 4x(x+1) - 3(x+1) = 0 \\ &\Rightarrow (4x-3)(x+1) = 0 \\ &\Rightarrow 4x - 3 = 0 \text{ or } x + 1 = 0 \\ &\Rightarrow \underline{\underline{x = \tfrac{3}{4} \text{ or } x = -1}}. \end{aligned}$$

14. ABC is a triangle. (2)



- $AB = 3\sqrt{2}$ cm.
- Angle $ABC = 45°$.
- The area of ABC is 12 cm^2.
- You are given that $\sin 45° = \frac{1}{\sqrt{2}}$.

Work out the length x.

Solution

Well,

$$\begin{aligned} & \text{area} = \tfrac{1}{2} \times AB \times BC \times \sin ABC \\ \Rightarrow \quad & 12 = \tfrac{1}{2} \times 3\sqrt{2} \times x \times \tfrac{1}{\sqrt{2}} \\ \Rightarrow \quad & 12 = \tfrac{3}{2}x \\ \Rightarrow \quad & \underline{\underline{x = 8 \text{ cm}}}. \end{aligned}$$

15. Rearrange (3)

$$y = \frac{3x + 5}{x}$$

to make x the subject.
You **must** show your working.

Solution

Well,

$$\begin{aligned} y = \frac{3x+5}{x} &\Rightarrow xy = 3x+5 \\ &\Rightarrow xy - 3x = 5 \\ &\Rightarrow x(y-3) = 5 \\ &\Rightarrow \underline{\underline{x = \frac{5}{y-3}}}. \end{aligned}$$

16. Solve (3)

$$\tfrac{1}{2}(3x-1) < \tfrac{3}{8}(x+1).$$

Solution

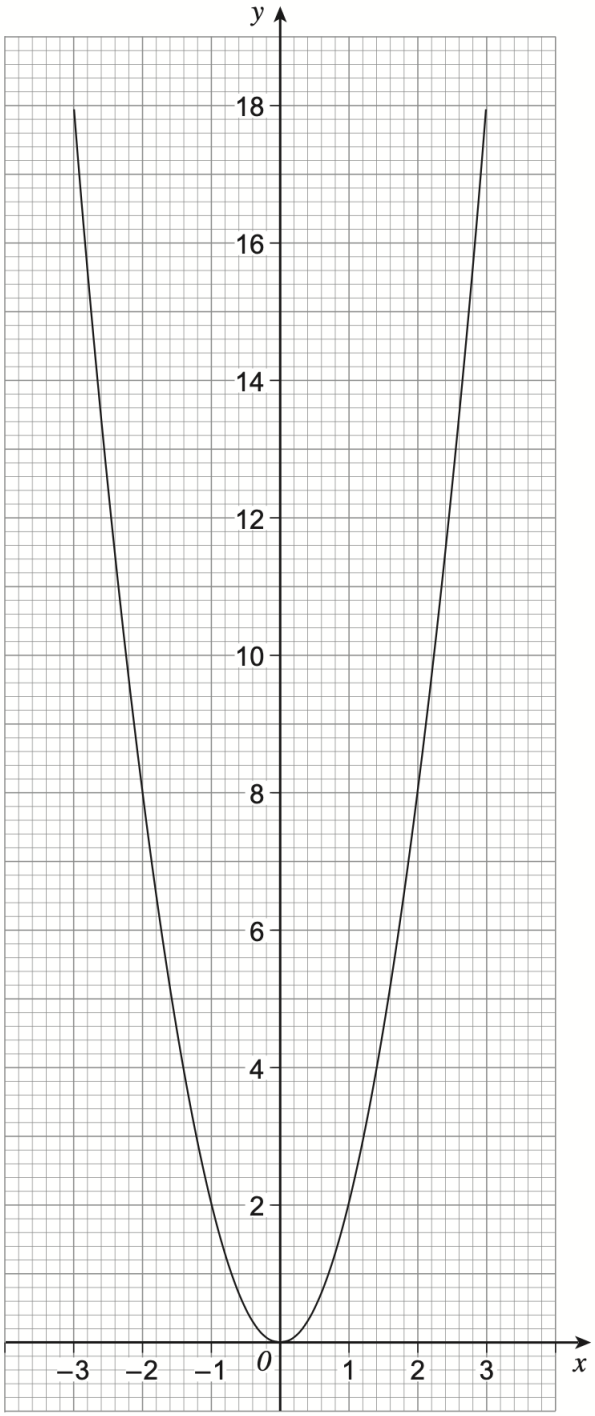
Now,

$$\begin{aligned} \tfrac{1}{2}(3x-1) < \tfrac{3}{8}(x+1) &\Rightarrow \tfrac{3}{2}x - \tfrac{1}{2} < \tfrac{3}{8}x + \tfrac{3}{8} \\ &\Rightarrow \tfrac{12}{8}x - \tfrac{4}{8} < \tfrac{3}{8}x + \tfrac{3}{8} \\ &\Rightarrow \tfrac{9}{8}x < \tfrac{7}{8} \\ &\Rightarrow \underline{\underline{x < \tfrac{7}{9}}}. \end{aligned}$$

17. Here is the graph of

$$y = 2x^2$$

for values of x from -3 to 3.



(a) Use the graph to estimate the solutions to (2)

$$2x^2 = 15.$$

Show clearly how you obtained your answer.

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Solution
Go up the vertical axis to 15 and read off the values of x:

$$\underline{\underline{x = 2.75 \text{ and } x = -2.75}}.$$

(b) Use the graph to estimate the value of $\sqrt{5}$. (2)
Show clearly how you obtained your answer.

Solution
Well,

$$\begin{aligned} y = 10 &\Rightarrow 2x^2 = 10 \\ &\Rightarrow x^2 = 5 \\ &\Rightarrow x = \pm\sqrt{5} \\ &\Rightarrow \underline{\underline{x = \pm 2.2}}. \end{aligned}$$

18. Simplify fully (3)

$$\frac{(5-\sqrt{3})(3-\sqrt{3})}{2}.$$

Give your answer in the form

$$a + b\sqrt{3},$$

where a and b are integers.

Solution
Well,

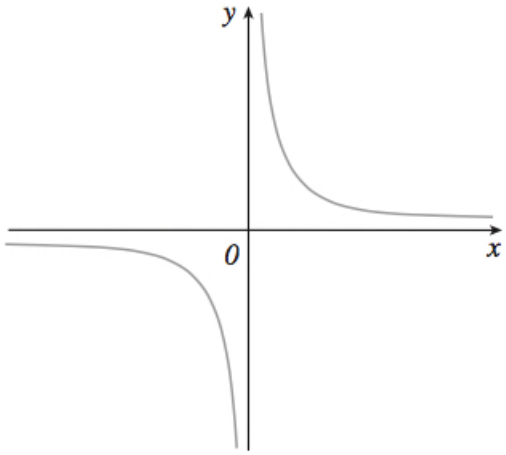
$\times$	5	$-\sqrt{3}$
3	15	$-3\sqrt{3}$
$-\sqrt{3}$	$-5\sqrt{3}$	$+3$

and

$$\begin{aligned} \frac{(5-\sqrt{3})(3-\sqrt{3})}{2} &= \frac{18-8\sqrt{3}}{2} \\ &= \underline{\underline{9-4\sqrt{3}}}; \end{aligned}$$

hence, $\underline{\underline{a = 9}}$ and $\underline{\underline{b = -4}}$.

19. (a) Circle a possible equation for the graph shown below. (1)



$$y = x^3 \quad y = \frac{1}{x} \quad y = \cos x \quad y = \sin x$$

Solution

$$y = x^3 \quad \underline{\underline{y = \frac{1}{x}}} \quad y = \cos x \quad y = \sin x$$

(b) This is the graph of (1)

$$y = 2^x.$$

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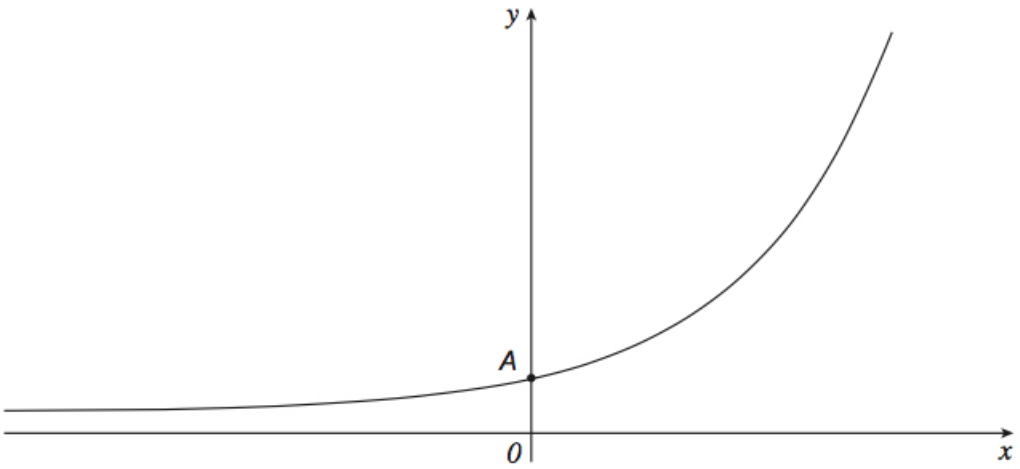
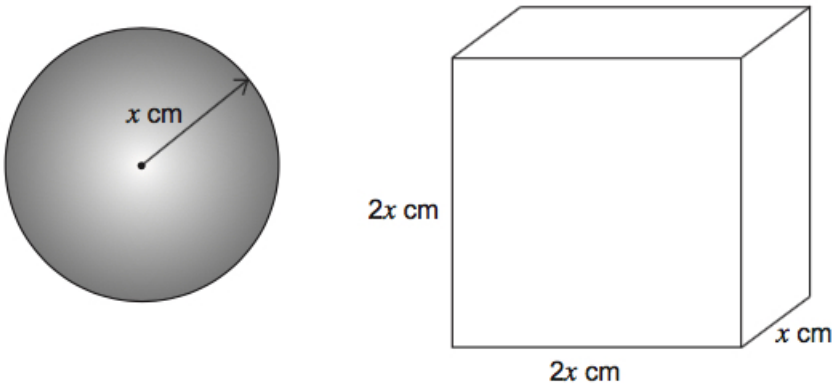


Write down the coordinates of A.

Solution

$A(0, 1)$.

20. A sphere has a radius of x cm. (3)
A cuboid has edges of length x cm, width $2x$ cm, and height $2x$ cm.



Show clearly that the sphere has the larger volume.

Solution

Well,

$$\begin{aligned} \text{cuboid} &= 2x \times 2x \times x \\ &= 4x^3 \\ &< \tfrac{4}{3}\pi x^3 \\ &= \text{sphere;} \end{aligned}$$

hence, the sphere has the larger volume as

$$\tfrac{1}{3}\pi > 1.$$

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