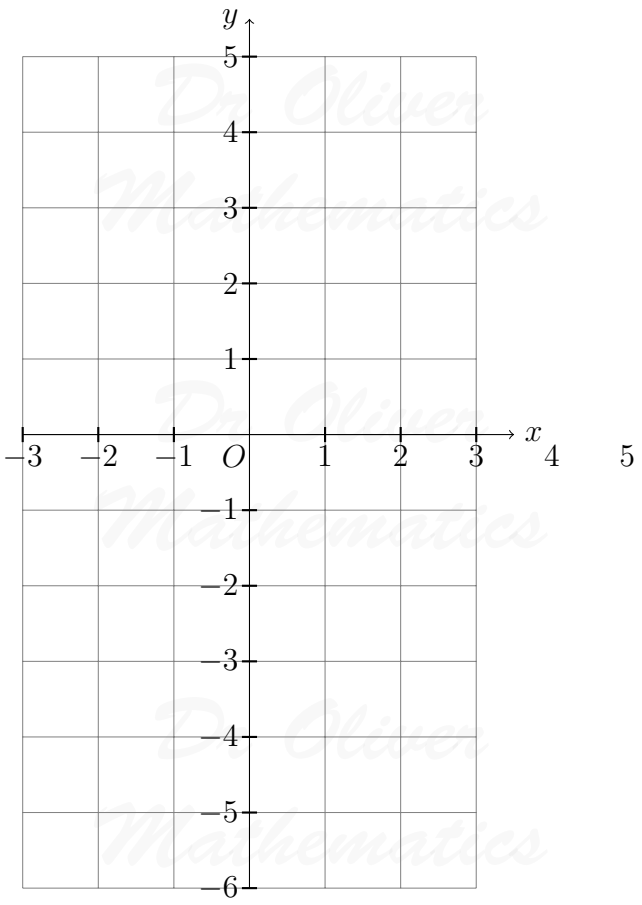
Dr Oliver Mathematics
AQA GCSE Mathematics
2013 November Paper 2: Calculator
2 hours

The total number of marks available is 105.
You must write down all the stages in your working.

1. On the grid, draw the graph of (3)

$$y = 2x - 1$$

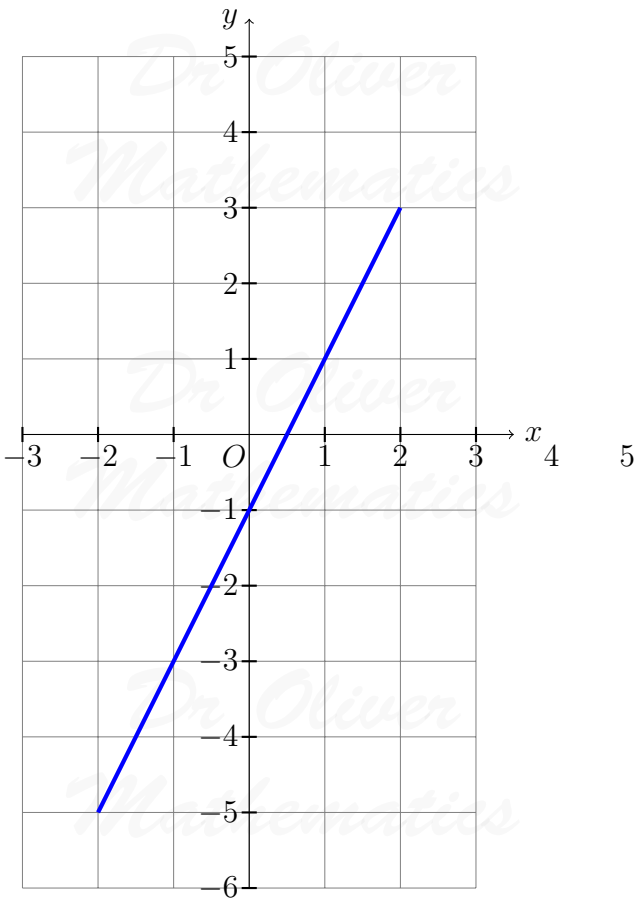
for values of x from -2 to 2.



Solution

We make a table:

x	-2	-1	0	1	2
y	-5	-3	-1	1	3



2. In this question, assume that the car uses the same amount of petrol for each mile it travels.

   (a) A car uses 55 litres of petrol to travel 495 miles. (3)

   How far would the car travel on 80 litres of petrol?

Solution

$$\text{Distance} = \frac{80}{55} \times 495$$
$$= \underline{720 \text{ miles}}.$$

(b) How much petrol would the car use on a trip of 160 miles? (4)
Give your answer to the nearest litre.

Solution

$$\begin{aligned}\text{Litres} &= \frac{55}{495} \times 160 \\ &= 17\tfrac{7}{9} \text{ (exact!)} \\ &= \underline{\underline{18 \text{ litres (nearest whole number)}}}.\end{aligned}$$

3. Decide whether each of these sets of data is discrete or continuous.
Tick the correct box.

(a) The heights of people. (1)

☐ Discrete ☐ Continuous

Solution

Continuous.

(b) The number of coins in a bag. (1)

☐ Discrete ☐ Continuous

Solution

Discrete.

(c) The weights of bicycles. (1)

☐ Discrete ☐ Continuous

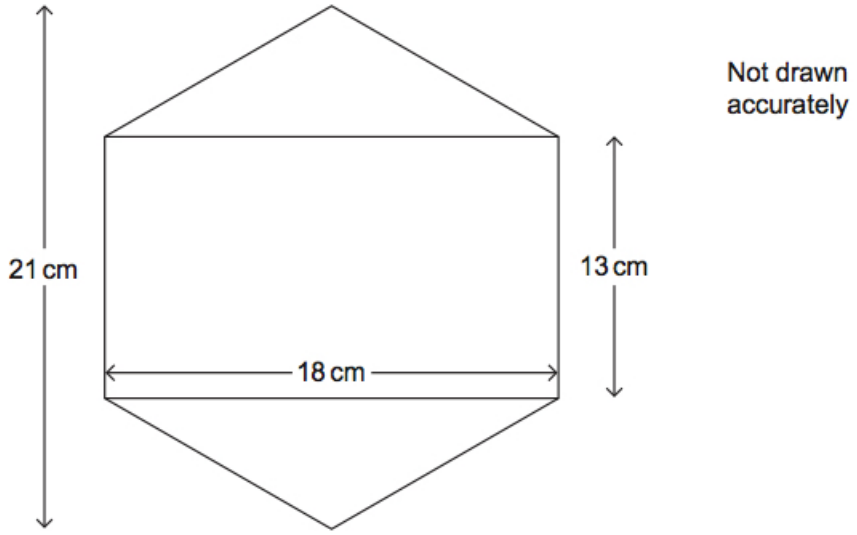
Solution
Continuous.

(d) The shoe sizes of women. (1)

☐ Discrete ☐ Continuous

Solution
Discrete.

4. The hexagon is made from a rectangle and two congruent triangles. (5)



Work out the area of the hexagon.

Solution

$$\tfrac{1}{2}(21 - 13) = 4$$

and so

$$\begin{aligned}\text{area} &= (\tfrac{1}{2} \times 18 \times 4) + (18 \times 13) + (\tfrac{1}{2} \times 18 \times 4) \\ &= 36 + 234 + 36 \\ &= \underline{\underline{306 \text{ cm}^2}}.\end{aligned}$$

5. 20 students choose a sport.

	Tennis	Basketball	Football
Boys	4	3	5
Girls	5	2	1

(a) How many students did **not** choose football? (2)

Solution

$$20 - (5 + 1) = \underline{\underline{14}}.$$

(b) What percentage of the students choose tennis? (3)

Solution

$$\left(\frac{4+5}{20}\right) \times 100\% = \underline{\underline{45\%}}.$$

(c) Considering the boys and the girls separately, compare their relative frequencies of choosing basketball. (3)

Solution

For boys, it is 3 out of 12 and, for girls, it is 2 out of 8.
Thus, boys and girls were equally likely to choose <u>basketball</u>.

6. (a) Multiply out and simplify (3)

$$2(3x + 2) - (x + 7).$$

Solution

$$2(3x+2)-(x+7) = 6x+4-x-7$$
$$= \underline{\underline{5x-3}}.$$

(b) Matt knows the value of a is 6 or 7 and the value of b is -4 or -5. (4)

Work out the largest and smallest possible values of

$$3a-2b.$$

Solution

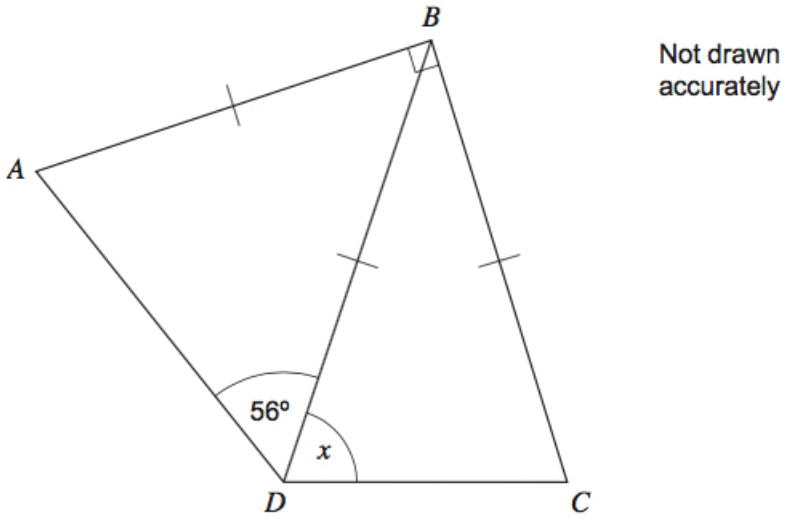
The highest value is

$$3(7)-2(-5) = \underline{\underline{31}}$$

and the lowest value is

$$3(6)-2(-4) = \underline{\underline{26}}.$$

7. Triangles ABD and BCD are isosceles. (4)
Angle ABC is 90°.



Work out the size of angle x.

Solution

$\angle DAB = 56^\circ$ (base angles)
$\angle ABC = 180 - 2 \times 56 = 68^\circ$ (completing the triangle)
$\angle DBC = 90 - 68 = 22^\circ$ ($\angle ABC$ is a right-angle)
$\angle BDC = \frac{1}{2}(180 - 22) = \underline{\underline{79^\circ}}$ (base angles)

8. (a) Rearrange the formula to make w the subject of (2)

$$y = 3w + 8.$$

Solution

$$y = 3w + 8 \Rightarrow 3y = y - 8$$
$$\Rightarrow \underline{\underline{y = \tfrac{1}{3}(y - 8)}}.$$

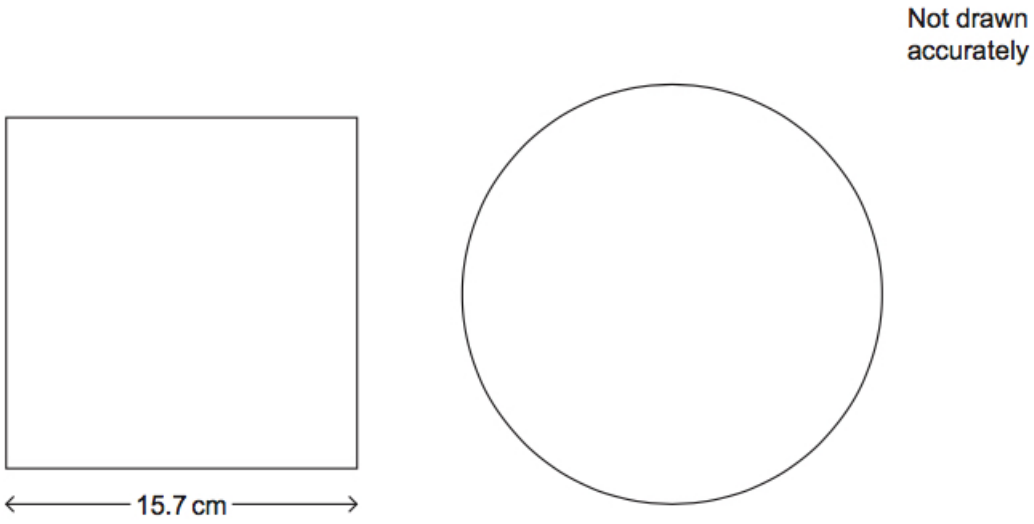
(b) Solve (4)

$$5(x + 4) = 3(x + 7) + 2.$$

Solution

$$5(x + 4) = 3(x + 7) + 2 \Rightarrow 5x + 20 = 3x + 21 + 2$$
$$\Rightarrow 2x = 3$$
$$\Rightarrow \underline{\underline{x = 1\tfrac{1}{2}}}.$$

9. A square of side 15.7 cm is made from a length of wire. (4)
The same length of wire is then made into a circle.



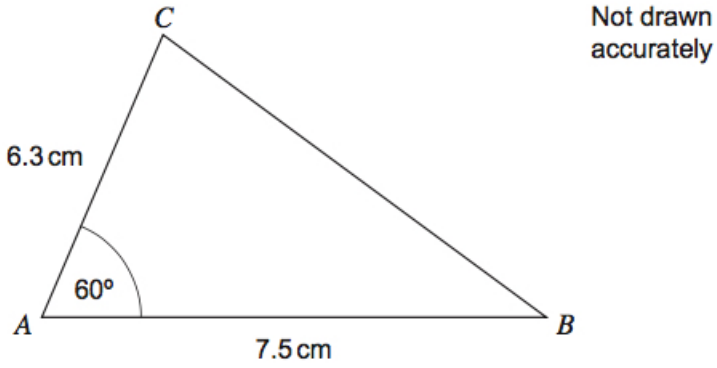
Work out the diameter of the circle.

Solution

Let d be the diameter of the circle. Then

$$4 \times 15.7 = \pi \times d \Rightarrow d = \frac{62.8}{\pi}$$
$$\Rightarrow d = 19.989\,860\,85 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{d = 20.0 \text{ cm (3 sf)}}}.$$

10. The diagram shows a sketch of triangle ABC. (3)



Using ruler and compasses only, make an accurate drawing of triangle ABC.

Solution

Begin with AB, 7.5 cm in length.
Then, opening the compass to 6.3 cm and from A, draw an arc from right above to just past the horizontal line though AB; we will call this point D.
From D, draw an arc from right above to just past the horizontal line though AB — ensure you have gone through A!
Call the intersection the arcs C.
Taking the ruler, draw a straight line between A and C.
Draw a straight line between B and C.

11. The population of England in 2013 is approximately 53 million. (3)
It is predicted that

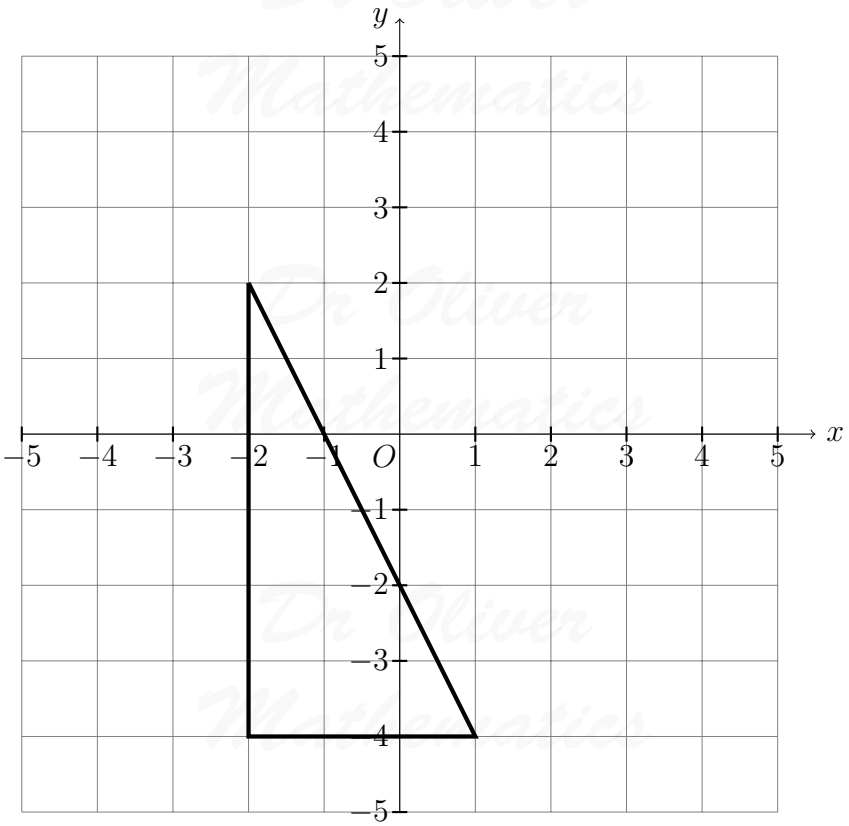
- the population in 2018 will be 4% more than the population in 2013 and
- and the population in 2023 will be 4% more than the population in 2018.

Work out the predicted population of England in 2023.

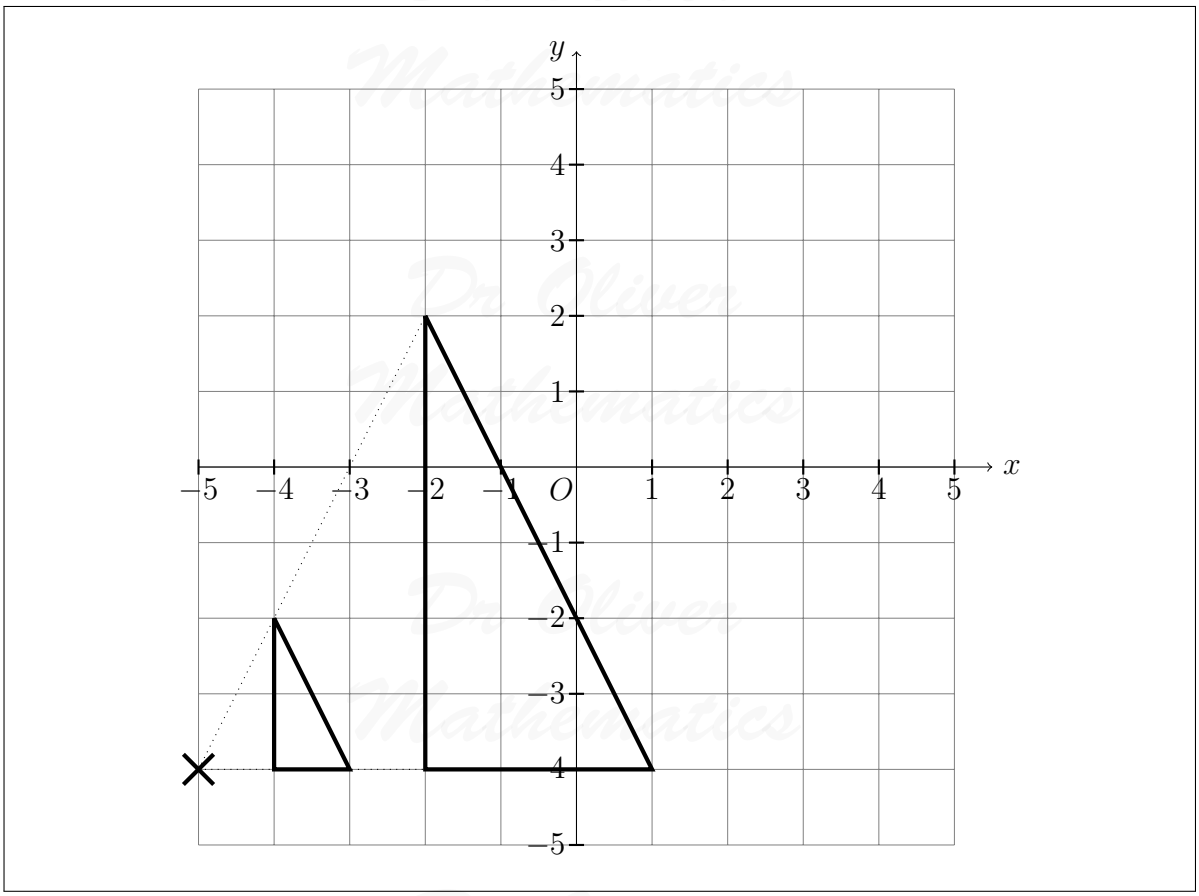
Solution

$$\begin{aligned}\text{Predicted population} &= 1.04 \times 1.04 \times 53\\ &= 57.324\,8\\ &= \underline{\underline{57.3 \text{ million (3 sf)}}}.\end{aligned}$$

12. Enlarge the triangle by scale factor $\frac{1}{3}$ with centre $(-5, -4)$. (2)

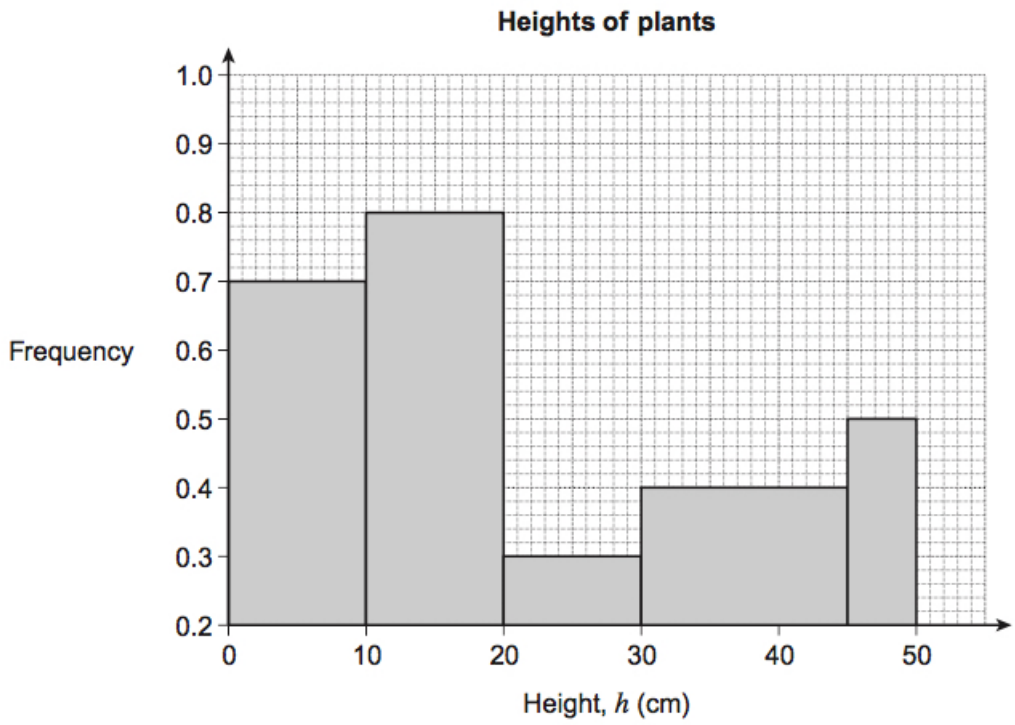


Solution



13. Jon uses this data about the heights of plants (h) to draw the histogram below. (3)

Height, h (cm)	$0 < h \leqslant 10$	$10 < h \leqslant 20$	$20 < h \leqslant 30$	$30 < h \leqslant 45$	$45 < h \leqslant 50$
Frequency	7	8	3	6	5



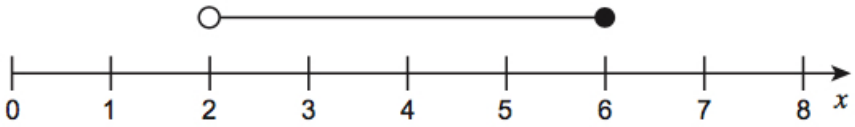
Write down **three different** types of mistake that he has made.

Solution

Height, h (cm)	$0-10$	$10-20$	$20-30$	$30-45$	$45-50$
Frequency	7	8	3	6	5
Width	10	10	10	15	5
Frequency Denisty	0.7	0.8	0.3	0.4	1

E.g., Jon has labelled the vertical axis as *Frequency* and not as *Frequency Density*.
Jon has got the height wrong of his $45 < h \leqslant 50$ — it should be 1.
The vertical scale does not start at 0.

14. (a) Circle the inequality shown by the diagram. (1)

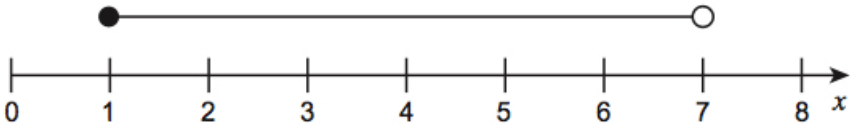


$2 < x < 6$ $2 \leqslant x < 6$ $2 < x \leqslant 6$ $2 \leqslant x \leqslant 6$

Solution

$\underline{\underline{2 < x \leqslant 6}}$.

(b) Write down the integer values satisfied by this diagram. (2)



Solution

The figure represents $1 \leqslant x < 7$ and the integer values are

$\underline{\underline{1,\ 2,\ 3,\ 4,\ 5,\ \text{and } 6}}$.

15. Each number in the grid is double the previous number. (5)
The first **seven** numbers are shown.

1	2	4	8	16
32	64			
				x

Work out the number for the last cell, marked x.
Give your answer in standard form to 3 significant figures.
You must show your working.

Solution

It should be clear that

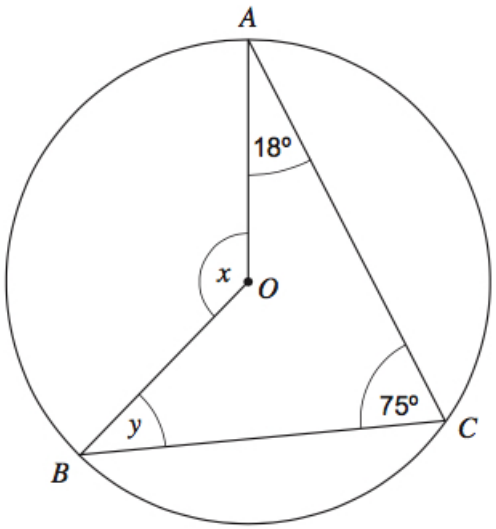
$$x = 2^{25-1} = 2^{24}$$

is the required number and

$$2^{24} = 16\,777\,216$$

and, to 3 significant figures, this is $\underline{\underline{1.68 \times 10^7}}$.

16. The diagram shows a circle, centre O.



Not drawn accurately

(a) Work out the size of angle x. (1)

Solution

$x = 2 \times 75 = \underline{\underline{150^\circ}}$

(b) Work out the size of angle y. (3)

Solution

$360 - 150 = 210^\circ$ (completing the circle)

Now,

$$y + 210 + 18 + 75 = 360 \Rightarrow y + 303 = 360$$
$$\Rightarrow \underline{\underline{y = 57^\circ}}.$$

17. (a) Simplify (3)

$$(2x^5y^4z^6) \times (7x^2y^3z).$$

Solution

$$(2x^5y^4z^6) \times (7x^2y^3z) = \underline{\underline{14x^7y^7z^7}}.$$

(b) Simplify fully (2)

$$\frac{6(x-5)^2}{3(x-5)(x+4)}.$$

Solution

$$\frac{6(x-5)^2}{3(x-5)(x+4)} = \underline{\underline{\frac{2(x-5)}{(x+4)}}}.$$

(c) Factorise (2)

$$(x+1)^2 + 4(x+1).$$

Solution

$$\begin{aligned}(x+1)^2 + 4(x+1) &= (x+1)\left[(x+1)+4\right] \\ &= \underline{\underline{(x+1)(x+5)}}.\end{aligned}$$

(d) Factorise fully (3)

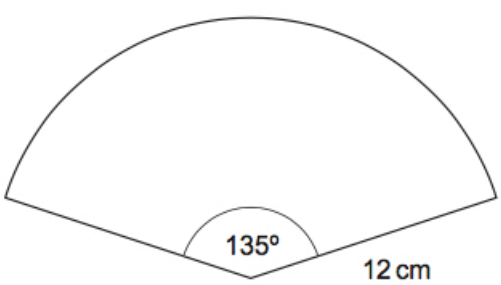
$$2x^2 - 50y^2.$$

Solution

The difference of two squares:

$$\begin{aligned}2x^2 - 50y^2 &= 2\left[x^2 - 25y^2\right] \\ &= 2\left[x^2 - (5y)^2\right] \\ &= \underline{\underline{2(1+5y)(1-5y)}}.\end{aligned}$$

18. The diagram shows a sector of a circle, radius 12 cm. (3)



Not drawn accurately

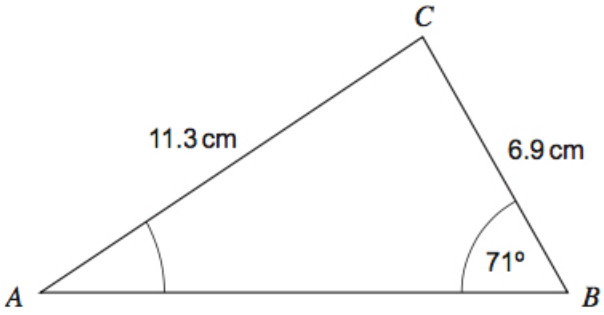
Show that the perimeter of the sector is greater than 52 cm.

Solution

$$\text{Perimeter} = 12 + 12 + \left(\frac{135}{360} \times 2 \times \pi \times 12\right)$$
$$= 52.274\,333\,88 \text{ (FCD)};$$

hence, the perimeter of the sector is greater than 52 cm.

19. Work out the size of angle A. (4)



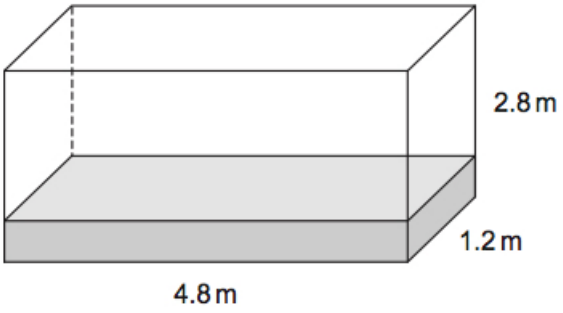
Not drawn accurately

Give your answer to a suitable degree of accuracy.

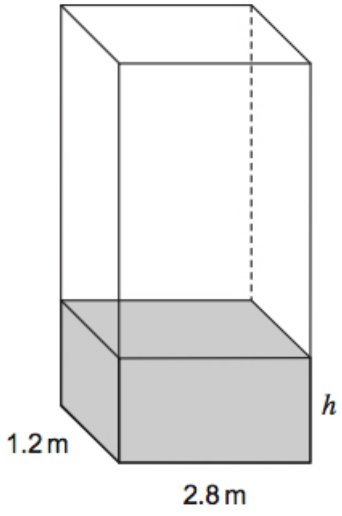
Solution

$$\frac{\sin A}{BC} = \frac{\sin B}{AC} \Rightarrow \frac{\sin A}{6.9} = \frac{\sin 71^\circ}{11.3}$$
$$\Rightarrow \sin A = \frac{6.9 \sin 71^\circ}{11.3}$$
$$\Rightarrow A = 35.264\,514\,69 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{A = 35.3^\circ \text{ (3 sf)}}}.$$

20. The measurements on this tank are exact. (5)
Water is put in the tank to a height of 0.7 m to **the nearest tenth of a metre**.



The tank is now turned on its side as shown.



Work out the minimum height of water in the tank, marked h.
Give your answer to 1 decimal place.

Solution

Well,

$$0.65 \leqslant \text{height} < 0.75$$

and the volume when turned on its side is

$$\begin{aligned} 4.8 \times 1.2 \times \text{height} = 1.2 \times 2.8 \times h &\Rightarrow 5.76\,\text{height} = 3.36\,h \\ &\Rightarrow h = \tfrac{12}{7}\,\text{height} \\ &\Rightarrow h = \tfrac{12}{7} \times 0.65 \\ &\Rightarrow h = 1.114\,285\,714 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{h = 1.1 \text{ cm (1 dp)}}}. \end{aligned}$$

21. n is an integer. (5)

$$S = \tfrac{1}{2}n(n+1).$$

Prove that

$$8S + 1$$

is an odd square number.

Solution

$$\begin{aligned} 8S + 1 &= 8(\tfrac{1}{2}n(n+1)) + 1 \\ &= 4n(n+1) + 1 \\ &= 4n^2 + 4n + 1 \\ &= (2n+1)^2; \end{aligned}$$

hence, it is a <u>square number</u>. Even or odd? Go back to $4n(n+1)+1$: either

$$4 \times \text{even} \times \text{odd} + 1 = \text{odd}$$

or

$$4 \times \text{odd} \times \text{even} + 1 = \text{odd};$$

so it is <u>odd</u>.

22. Robin is firing arrows at a target.
The probability that he hits the target on his xth attempt is

$$\frac{x+2}{x+3}.$$

For example,

$$\text{P(hit on his 5 attempt)} = \frac{7}{8}.$$

(a) Work out the probability that he hits the target with both his 1st and 2nd attempts. (3)

Solution

$$\begin{aligned}\text{P(1st, 2nd)} &= \frac{3}{4} \times \frac{4}{5} \\ &= \underline{\underline{\frac{3}{5}}}.\end{aligned}$$

(b) Work out the probability that he hits the target **exactly** once on his first two attempts. (4)

Solution

$$\begin{aligned}\text{P(not 1st, not 2nd)} &= \frac{1}{4} \times \frac{1}{5} \\ &= \frac{1}{20}\end{aligned}$$

and

$$\begin{aligned}\text{P(exactly once)} &= 1 - \text{P(1st, 2nd)} - \text{P(not 1st, not 2nd)} \\ &= 1 - \frac{3}{5} - \frac{1}{20} \\ &= \underline{\underline{\frac{7}{20}}}.\end{aligned}$$