Dr Oliver Mathematics
Mathematics: Higher
2016 Paper 1: Non-Calculator
1 hour 10 minutes

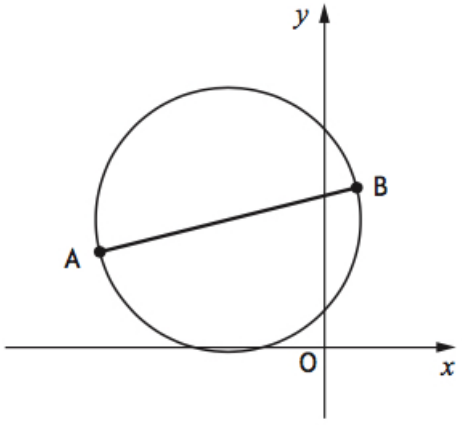
The total number of marks available is 60.
You must write down all the stages in your working.

1. Find the equation of the line passing through the point $(-2, 3)$ which is parallel to the line with equation $y + 4x = 7$. (2)

2. Given that (3)
$$y = 12x^3 + 8\sqrt{x}, \text{ where } x > 0,$$
find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

3. A sequence is defined by the recurrence relation
$$u_{n+1} = \tfrac{1}{3}u_n + 10 \text{ with } u_3 = 6.$$
   (a) Find the value of u_4. (1)
   (b) Explain why this sequence approaches a limit as $n \to \infty$. (1)
   (c) Calculate this limit. (2)

4. A and B are the points $(-7, 3)$ and $(1, 5)$. (3)
AB is a diameter of a circle.



Find the equation of this circle.

5. Find (2)

$$\int 8\cos(4x+1)\,\mathrm{d}x.$$

6. Functions f and g are defined on $\mathbb{R}$, the set of real numbers.
The inverse functions f^{-1} and g^{-1} both exist.

   (a) Given (3)

   $$\mathrm{f}(x) = 3x + 5,$$

   find $\mathrm{f}^{-1}(x)$.

   (b) If (1)

   $$\mathrm{g}(2) = 7,$$

   write down the value of $\mathrm{g}^{-1}(7)$.

7. Three vectors can be expressed as follows:

   - $\overrightarrow{FG} = -2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}$,
   - $\overrightarrow{GH} = 3\mathbf{i} + 9\mathbf{j} - 7\mathbf{k}$, and
   - $\overrightarrow{EH} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$.

   (a) Find $\overrightarrow{FH}$. (2)

   (b) Hence, or otherwise, find $\overrightarrow{FE}$. (2)

8. Show that the line with equation (5)

   $$y = 3x - 5$$

   is a tangent to the circle with equation

   $$x^2 + y^2 + 2x - 4y - 5 = 0$$

   and find the coordinates of the point of contact.

9. (a) Find the x-coordinates of the stationary points on the graph with equation $y = \mathrm{f}(x)$, where (4)

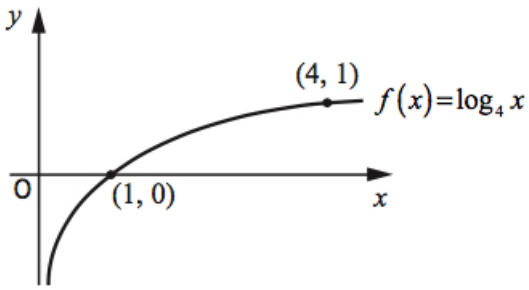
   $$\mathrm{f}(x) = x^3 + 3x^2 - 24x.$$

   (b) Hence determine the range of values of x for which the function f is strictly increasing. (2)

10. The diagram below shows the graph of the function (2)

    $$\mathrm{f}(x) = \log_4 x,$$

    where $x > 0$.



The inverse function, f^{-1}, exists.
Sketch the graph of the inverse function.

11. A and C are the points $(1, 3, -2)$ and $(4, -3, 4)$ respectively.
Point B divides AC in the ratio $1 : 2$.

(a) Find the coordinates of B. (2)

$k\overrightarrow{AC}$ is a vector of magnitude 1, where $k > 0$.

(b) Determine the value of k. (3)

12. The functions f and g are defined on $\mathbb{R}$, the set of real numbers, by

$$\mathrm{f}(x) = 2x^2 - 4x + 5 \text{ and } \mathrm{g}(x) = 3 - x.$$

(a) Given (2)

$$\mathrm{h}(x) = \mathrm{f}(\mathrm{g}(x)),$$

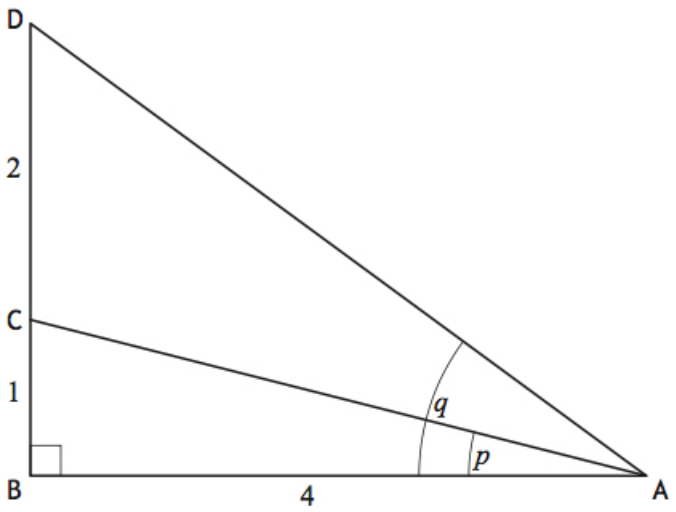
show that

$$\mathrm{h}(x) = 2x^2 - 8x + 11.$$

(b) Express $\mathrm{h}(x)$ in the form (3)

$$p(x + q)^2 + r.$$

13. Triangle ABD is right-angled at B with angles $BAC = p$ and $BAD = q$ and lengths as shown in the diagram below. (5)



Show that the exact value of $\cos(q - p)$ is

$$\frac{19\sqrt{17}}{85}.$$

14. (a) Evaluate (1)

$$\log_5 25.$$

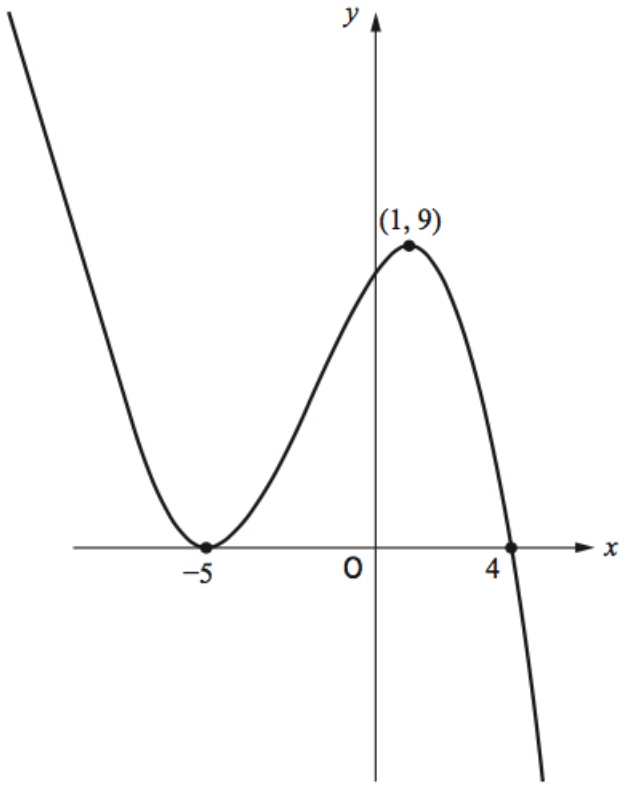
(b) Hence, solve (5)

$$\log_4 x + \log_4(x - 6) = \log_5 25,$$

where $x > 6$.

15. The diagram below shows the graph with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = k(x - a)(x - b)^2.$$



(a) Find the values of a, b, and k. (3)

(b) For the function (1)

$$g(x) = f(x) - d,$$

where d is positive, determine the range of values of d for which $g(x)$ has exactly one real root.