Dr Oliver Mathematics
GCSE Mathematics
2006 November Paper 6H: Calculator
2 hours

The total number of marks available is 100.
You must write down all the stages in your working.

1. (a) Use your calculator to work out (2)

$$\frac{\sqrt{19.2 + 2.6^2}}{2.7 \times 1.5}.$$

Write down all the figures on your calculator display.

Solution

$$\frac{\sqrt{19.2 + 2.6^2}}{2.7 \times 1.5} = \frac{\sqrt{25.96}}{4.05} = \underline{\underline{1.258\,048\,316 \text{ (FCD)}}}.$$

(b) Write your answer to part (a) correct to 3 significant figures. (1)

Solution

$\underline{\underline{1.26 \text{ (3 sf)}}}$.

2. (a) Simplify $p^7 \times p^2$. (1)

Solution

$$p^7 \times p^2 = \underline{\underline{p^9}}.$$

(b) Simplify $\dfrac{q^8}{q^3}$. (1)

Solution

$$\frac{q^8}{q^3} = \underline{\underline{q^5}}.$$

(c) Simplify $(t^3)^4$. (1)

Solution

$$(t^3)^4 = \underline{\underline{t^{12}}}.$$

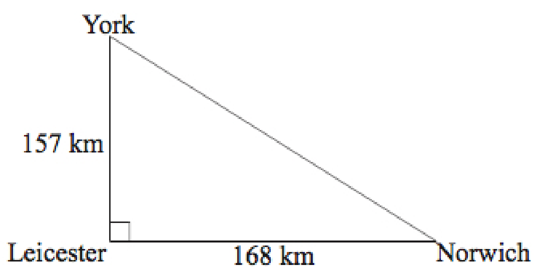
(d) Expand and simplify (2)

$$2(3m + 4) + 3(m - 5).$$

Solution

$$\begin{aligned} 2(3m + 4) + 3(m - 5) &= 6m + 8 + 3m - 15 \\ &= \underline{\underline{9m - 7}}. \end{aligned}$$

3. The diagram shows three cities. (3)
Norwich is 168 km due east of Leicester.
York is 157 km due north of Leicester.



Calculate the distance between Norwich and York.
Give your answer correct to the nearest kilometre.

Solution

$$\begin{aligned} NY &= \sqrt{168^2 + 157^2} \\ &= 229.941\,296\,9 \text{ (FCD)} \\ &= \underline{\underline{230 \text{ km (nearest kilometre)}}}. \end{aligned}$$

4. A DIY store bought 1750 boxes of nails. (3)
Barry took 25 of these boxes and counted the number of nails in each.
The table shows his results.

Number of nails	Number of boxes
14	2
15	9
16	8
17	4
18	2

The numbers of nails in the 25 boxes are typical of the numbers of nails in the 1750 boxes.
Work out an estimate for how many of the 1 750 boxes contain 16 nails.

Solution

$$\frac{8}{25} \times 1\,750 = \underline{\underline{560 \text{ boxes}}}.$$

5. (a) The equation (4)

$$x^3 + 4x^2 = 100$$

has a solution between 3 and 4.
Use a trial and improvement method to find this solution.
Give your answer correct to one decimal place.
You must show **all** your working.

Solution

You must be in TABLE mode; on my calculator (Casio fx-991) it is Mode 3.
F(X)= and you type in $X^3 + 4X^2$; then you press $\boxed{=}$.
Start? and you enter 3; then you press $\boxed{=}$.
End? and you enter 4; then you press $\boxed{=}$.
Step? and enter 0.05 – 1 decimal place divided by 2; then you press $\boxed{=}$.

x	f(x)	Comment
3.6	98.496	too low
3.65	101.91	too high

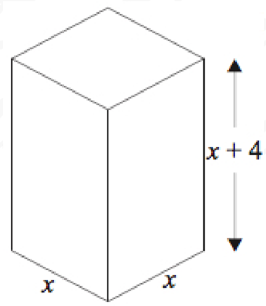
Clearly,

$$3.6 < x < 3.65$$

and the answer is

$$\underline{\underline{x = 3.6 \text{ (1 dp)}}}.$$

The diagram shows a cuboid.



The base of the cuboid is a square of side x cm.
The height of the cuboid is $(x + 4)$ cm.
The volume of the cuboid is 100 cm^3.

(b) (i) Show that (2)

$$x^3 + 4x^2 = 100.$$

Solution

$$x \cdot x \cdot (x + 4) = 100 \Rightarrow x^2(x + 4) = 100$$
$$\Rightarrow \underline{\underline{x^3 + 4x^2 = 100}},$$

as required.

(ii) Use your answer to part (a) to write down the height of the cuboid, correct to 1 decimal place.

Solution

$$4 + 3.6 = \underline{\underline{7.6 \text{ cm}}}.$$

6. The price of all rail season tickets to London increased by 4%.

(a) The price of a rail season ticket from Cambridge to London increased by £121.60. Work out the price before this increase. (2)

Solution

$$\frac{121.60}{4} \times 100 = \underline{\underline{£3\,040}}.$$

(b) After the increase, the price of a rail season ticket from Brighton to London was £2 828.80. (3)
Work out the price before this increase.

Solution

$$\frac{2\,828.80}{1.04} = \underline{\underline{£2\,720}}.$$

7. The table shows information about the ages of the 240 people at a club.

Age (t years)	Frequency
$15 \leqslant t < 20$	95
$20 \leqslant t < 25$	90
$25 \leqslant t < 30$	35
$30 \leqslant t < 35$	15
$35 \leqslant t < 40$	5

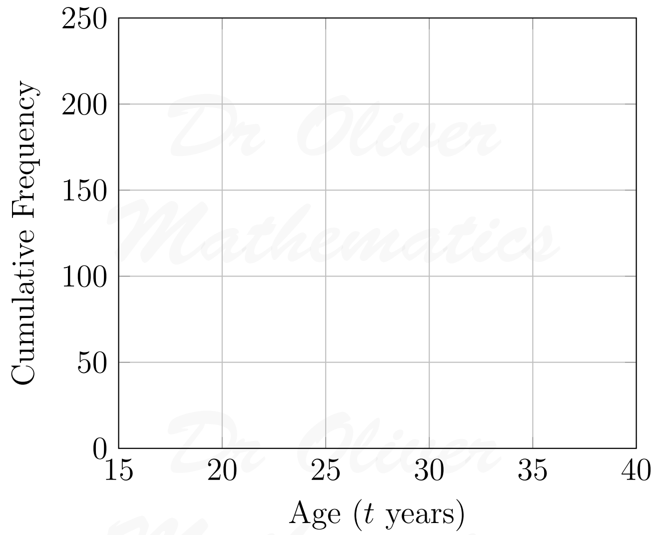
(a) Complete the cumulative frequency table. (1)

Age (t years)	Cumulative Frequency
$15 \leqslant t < 20$	95
$15 \leqslant t < 25$	
$15 \leqslant t < 30$	
$15 \leqslant t < 35$	
$15 \leqslant t < 40$	

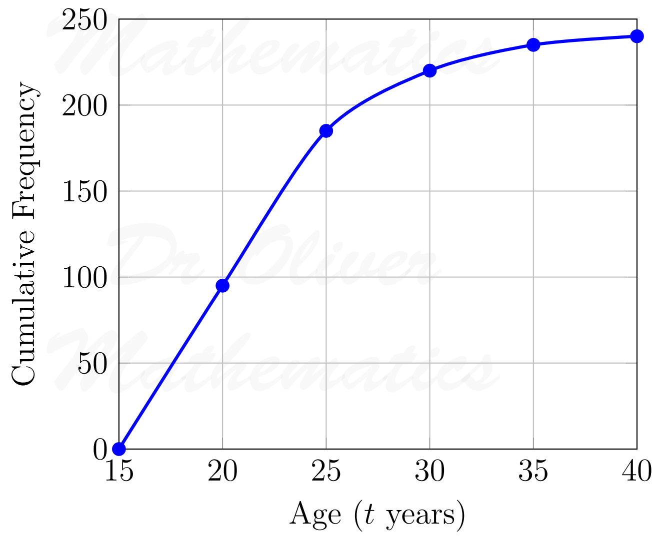
Solution

Age (t years)	Cumulative Frequency
$15 \leqslant t < 20$	$\underline{\underline{95}}$
$15 \leqslant t < 25$	$95 + 90 = \underline{\underline{185}}$
$15 \leqslant t < 30$	$185 + 35 = \underline{\underline{220}}$
$15 \leqslant t < 35$	$220 + 15 = \underline{\underline{235}}$
$15 \leqslant t < 40$	$235 + 5 = \underline{\underline{240}}$

(b) On the grid, draw the cumulative frequency graph for your table. (2)

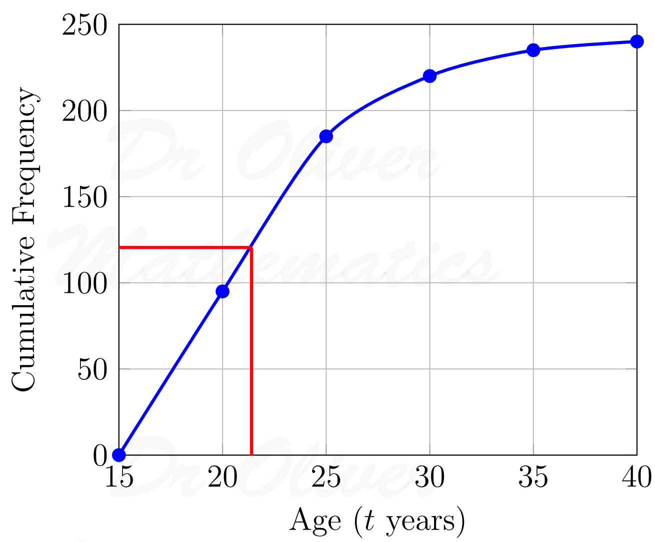


Solution



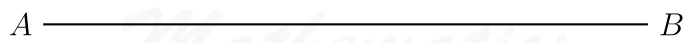
(c) Use your graph to find an estimate for the median age of the people. (1)

Solution

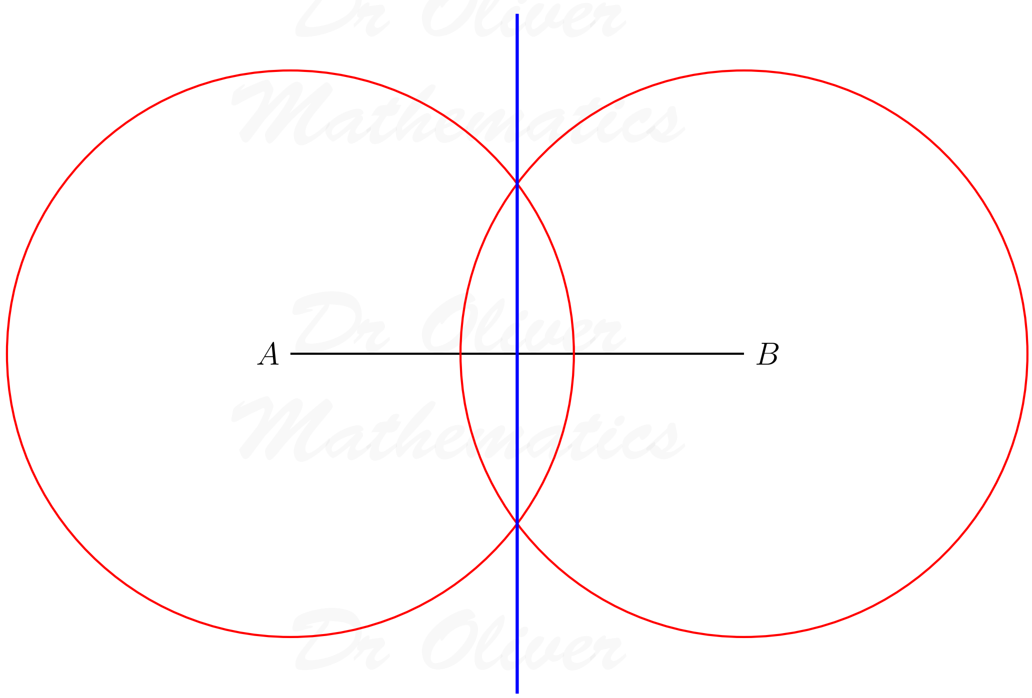


Use your graph to find a read-off from CF = 120 or CF = 120.5: approximately 21.4.

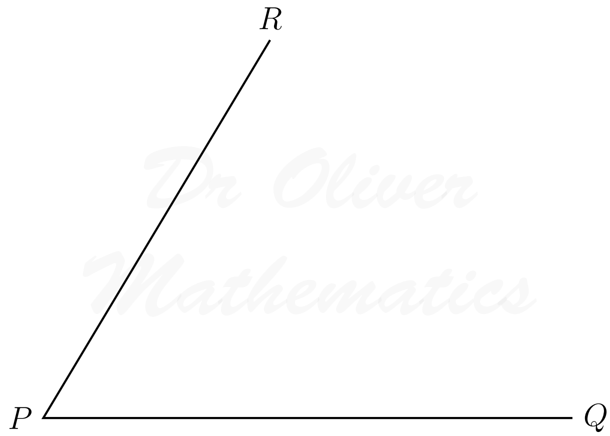
8. (a) Use ruler and compasses to construct the perpendicular bisector of the line AB. You must show all your construction lines. (2)



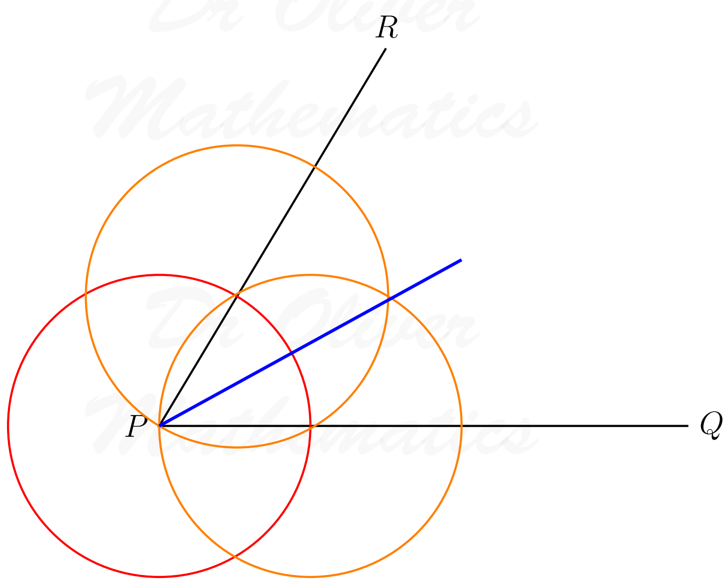
Solution



(b) Use ruler and compasses to construct the bisector of angle RPQ. (2)
You must show all your construction lines.



Solution



9. When you are h feet above sea level, you can see d miles to the horizon, where

$$d = \sqrt{\frac{3h}{2}}.$$

(a) Calculate the value of d when $h = 8.4 \times 10^3$. (2)
Give your answer in standard form correct to 3 significant figures.

Solution

$$\begin{aligned} d &= \sqrt{\frac{3 \times 8.4 \times 10^3}{2}} \\ &= 112.249\,721\,6 \text{ (FCD)} \\ &= \underline{\underline{1.12 \times 10^2 \text{ m (3 sf)}}}. \end{aligned}$$

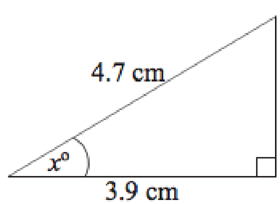
(b) Make h the subject of the formula (2)

$$d = \sqrt{\frac{3h}{2}}.$$

Solution

$$\begin{aligned} d = \sqrt{\frac{3h}{2}} &\Rightarrow d^2 = \frac{3h}{2} \\ &\Rightarrow \underline{\underline{h = \tfrac{2}{3}d^2}}. \end{aligned}$$

10. Work out the value of x. (3)



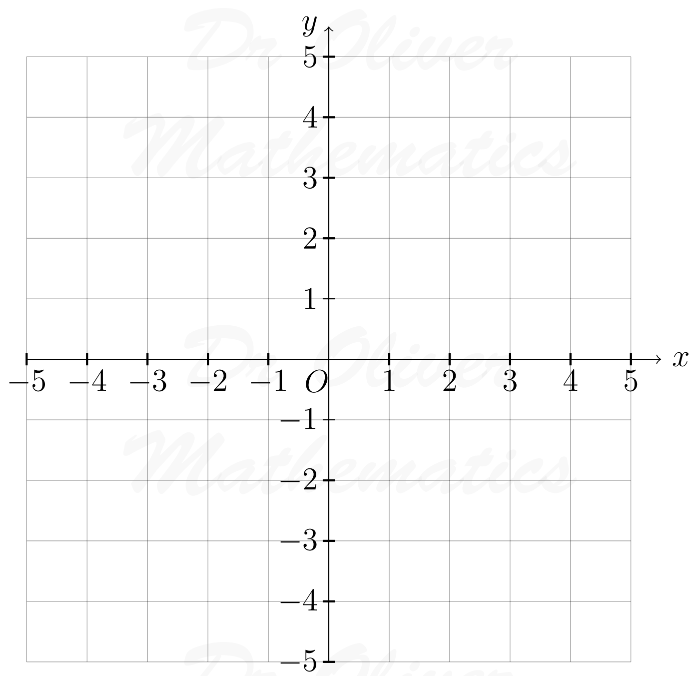
Give your answer correct to 1 decimal place.

Solution

$$\cos = \frac{\text{adj}}{\text{hyp}} \Rightarrow x^\circ = \cos^{-1}\frac{3.9}{4.7}$$
$$\Rightarrow x^\circ = 33.923\,112 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{x^\circ = 33.9^\circ \text{ (1 dp)}}}.$$

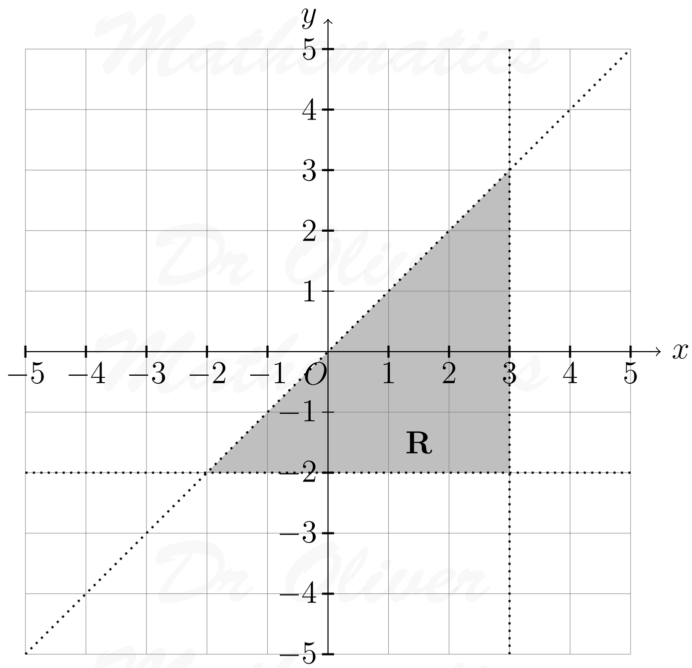
11. On the grid, show by shading, the region which satisfies all three of the inequalities: (4)

$$x < 3,\ y > 2, \text{ and } y < x.$$

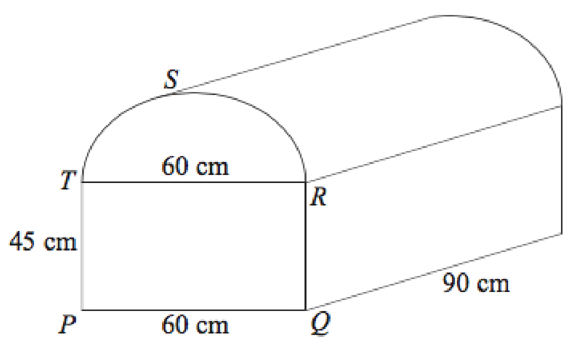


Label the region **R**.

Solution



12. The diagram shows a prism of length 90 cm. (5)

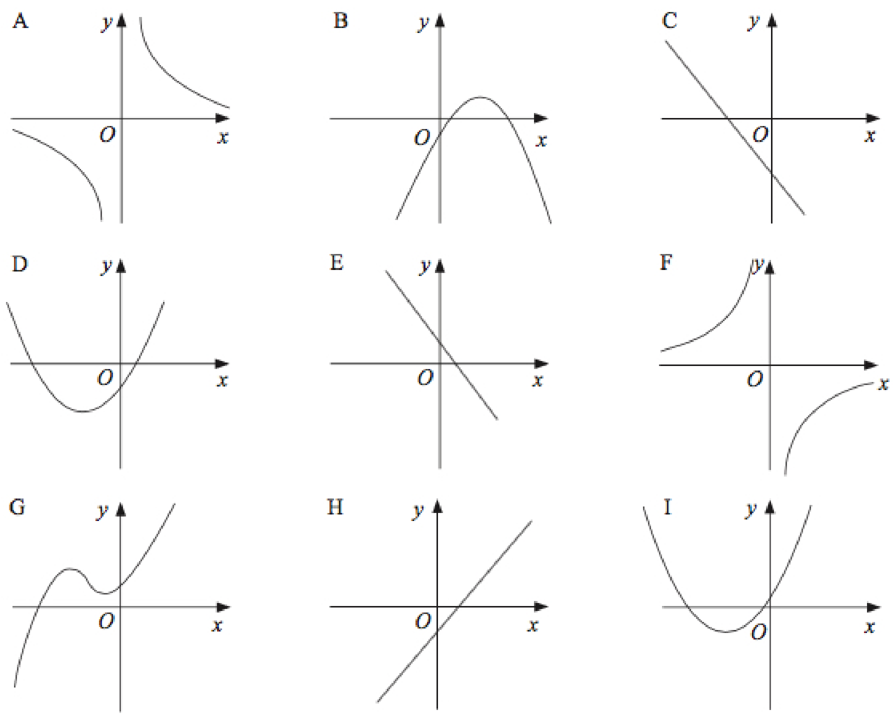


The cross section, $PQRST$, of the prism is a semi-circle above a rectangle.
$PQRT$ is a rectangle.
RST is a semi-circle with diameter RT.
$PQ = RT = 60$ cm.
$PT = QR = 45$ cm.
Calculate the volume of the prism.
Give your answer correct to 3 significant figures.
State the units of your answer.

Solution

$$\begin{aligned}\text{Volume} &= \text{cuboid} + \text{half a cylinder}\\ &= (45 \times 60 \times 90) + (\tfrac{1}{2} \times \pi \times 30^2 \times 90)\\ &= 370\,234.502\,5 \text{ (FCD)}\\ &= \underline{\underline{370\,000 \text{ cm}^3 \text{ (3 sf)}}}.\end{aligned}$$

13. Here are nine graphs.



Write down the letter of the graph which could have the equation

(a) $y = 1 - 3x$, (1)

Solution

$\underline{\underline{E}}$.

(b) $y = \dfrac{1}{x}$, (1)

Solution

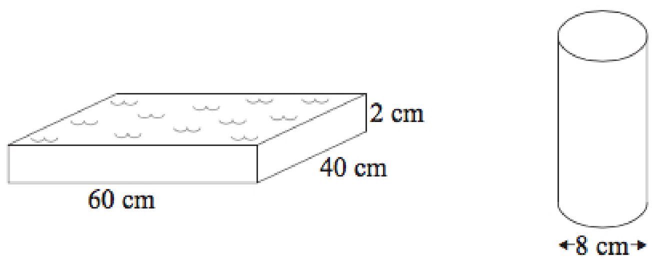
$\underline{\underline{A}}$.

(c) $y = 2x^2 + 7x + 3$. (1)

Solution

$\underline{\underline{I}}$.

14. A rectangular tray has length 60 cm, width 40 cm and depth 2 cm. (5)
It is full of water.
The water is poured into an empty cylinder of diameter 8 cm.



Calculate the depth, in cm, of water in the cylinder. Give your answer correct to 3 significant figures.

Solution

The volume in the tray is

$$2 \times 40 \times 60 = 4\,800 \text{ cm}^3$$

and the depth of water is

$$\begin{aligned}\frac{4\,800}{\pi \times 4^2} &= 95.492\,965\,86 \text{ (FCD)} \\ &= \underline{\underline{95.5 \text{ cm (3 sf)}}}.\end{aligned}$$

15. A school has 450 students. (3)
Each student studies one of Greek or Spanish or German or French.
The table shows the number of students who study each of these languages.

Language	Number of students
Greek	45
Spanish	121
German	98
French	186

An inspector wants to look at the work of a stratified sample of 70 of these students. Find the number of students studying each of these languages that should be in the sample.

Solution

Language	Number of students	Sample
Greek	45	$\frac{45}{450} \times 70 = 7$
Spanish	121	$\frac{121}{450} \times 70 = 18.82\ldots$
German	98	$\frac{98}{450} \times 70 = 15.24\ldots$
French	186	$\frac{186}{450} \times 70 = 28.93\ldots$

He should choose $\underline{\underline{7}}$ from Greek, $\underline{\underline{19}}$ from Spanish, $\underline{\underline{15}}$ from German, and $\underline{\underline{29}}$ from French.

16. A ball falls vertically after being dropped.
The ball falls a distance d metres in a time of t seconds.
d is directly proportional to the square of t.
The ball falls 20 metres in a time of 2 seconds.

(a) Find a formula for d in terms of t. (3)

Solution

$$d \propto t^2 \Rightarrow d = kt^2$$

for some constant d. Now,

$$20 = k \times 2^2 \Rightarrow k = 5$$

and

$$\underline{\underline{d = 5t^2}}.$$

(b) Calculate the distance the ball falls in 3 seconds. (1)

Solution

$$d = 5 \times 3^2 = \underline{\underline{45 \text{ m}}}.$$

(c) Calculate the time the ball takes to fall 605 m. (3)

Solution

$$605 = 5t^2 \Rightarrow t^2 = 121$$
$$\Rightarrow \underline{\underline{t = 11 \text{ s}}}.$$

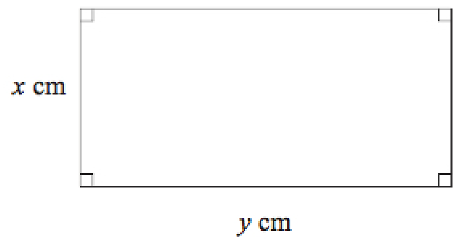
17. Gwen bought a new car. (3)
Each year, the value of her car depreciated by 9%.
Calculate the number of years after which the value of her car was 47% of its value when new.

Solution

$$\begin{aligned}
1 \text{ year} &\Rightarrow \text{cost} = 0.91 \\
2 \text{ years} &\Rightarrow \text{cost} = 0.91^2 = 0.828\,1 \\
3 \text{ years} &\Rightarrow \text{cost} = 0.91^3 = 0.753\,571 \\
4 \text{ years} &\Rightarrow \text{cost} = 0.91^4 = 0.685\,749\,61 \\
5 \text{ years} &\Rightarrow \text{cost} = 0.91^5 = 0.624\,032\,145\,1 \\
6 \text{ years} &\Rightarrow \text{cost} = 0.91^6 = 0.567\,869\,252 \\
7 \text{ years} &\Rightarrow \text{cost} = 0.91^7 = 0.516\,761\,019\,4 \\
8 \text{ years} &\Rightarrow \text{cost} = 0.9^8 = 0.470\,252\,527\,6;
\end{aligned}$$

it is after $\underline{\underline{\text{8 years}}}$.

18. The diagram shows a rectangle.
The width of the rectangle is x cm and its length is y cm.



The perimeter of the rectangle is 10 cm.

(a) Show that $x + y = 5$. (1)

Solution

$$2(x + y) = 10 \Rightarrow \underline{\underline{x + y = 5}},$$

as required.

The length of a diagonal of the rectangle is 4 cm.

(b) Show that (3)

$$2x^2 - 10x + 9 = 0.$$

Solution

$$\begin{aligned} x^2 + y^2 = 4^2 &\Rightarrow x^2 + (5 - x)^2 = 16 \\ &\Rightarrow x^2 + (25 - 10x + x^2) - 16 = 0 \\ &\Rightarrow \underline{\underline{2x^2 - 10x + 9 = 0}}, \end{aligned}$$

as required.

(c) Solve the equation (3)

$$2x^2 - 10x + 9 = 0$$

to find the possible values of x.
Give your answers correct to 3 significant figures.

Solution

$a = 2$, $b = -10$, and $c = 9$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{10 \pm \sqrt{(-10)^2 - 4 \times 2 \times 9}}{2 \times 2}$$
$$\Rightarrow x = \frac{10 \pm \sqrt{28}}{4}$$
$$\Rightarrow x = 1.177\,124\,344 \text{ or } 3.822\,875\,656 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{x = 1.18 \text{ or } 3.82 \text{ (3 sf)}}}.$$

19. (4)

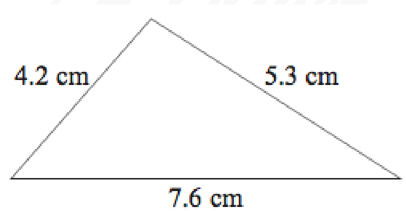
$$\frac{x}{x+c} = \frac{p}{q}.$$

Make x the subject of the formula.

Solution

$$\frac{x}{x+c} = \frac{p}{q} \Rightarrow qx = p(x+c)$$
$$\Rightarrow qx = px + cp$$
$$\Rightarrow qx - px = cp$$
$$\Rightarrow x(q-p) = cp$$
$$\Rightarrow \underline{\underline{x = \frac{cp}{q-p}}}.$$

20. The lengths of the sides of a triangle are 4.2 cm, 5.3 cm, and 7.6 cm.



(a) Calculate the size of the largest angle of the triangle. (3)
Give your answer correct to 1 decimal place.

Solution

$$\cos\angle A = \frac{4.2^2 + 5.3^2 - 7.6^2}{2 \times 4.2 \times 5.3} \Rightarrow \cos\angle A = -\tfrac{401}{1\,484}$$
$$\Rightarrow \angle A = 105.677\,098\,7 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{\angle A = 105.7^\circ \text{ (1 dp)}}}.$$

(b) Calculate the area of the triangle. (3)
Give your answer correct to 3 significant figures.

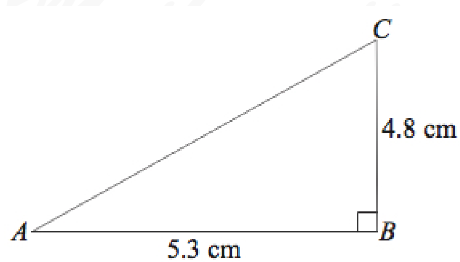
Solution

$$s = \tfrac{1}{2}(4.2 + 5.3 + 7.6) = 8.55$$

and

$$\begin{aligned} \text{area} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{8.55 \times 4.35 \times 3.25 \times 0.95} \\ &= 10.715\,962\,1 \text{ (FCD)} \\ &= \underline{\underline{10.7 \text{ cm}^2 \text{ (3 sf)}}}. \end{aligned}$$

21. In triangle ABC, angle $ABC = 90^\circ$.



$AB = 5.3$ cm, correct to 2 significant figures.
$BC = 4.8$ cm, correct to 2 significant figures.
The base, AB, of the triangle is horizontal.

(a) (i) Calculate the lower bound for the gradient of the line AC. (3)

Solution

The opposite side is

$$4.75 \leqslant y < 4.85$$

and the adjacent side is

$$5.25 \leqslant x < 5.35.$$

Now,

$$\begin{aligned}\text{Lower bound} &= \frac{4.75}{5.35} \\ &= \underline{\underline{0.887\,850\,467\,3 \quad \text{(FCD)}}}.\end{aligned}$$

(ii) Calculate the upper bound for the gradient of the line AC.

Solution

$$\begin{aligned}\text{Upper bound} &= \frac{4.85}{5.25} \\ &= \underline{\underline{0.923\,809\,523\,8 \quad \text{(FCD)}}}.\end{aligned}$$

(b) Use your answers to part (a) to give the gradient of the line AC to an appropriate degree of accuracy. (2)
You must explain your answer.

Solution

Degree of accuracy	Lower bound	Upper bound
1 sf	0.9	0.9
2 sf	0.89	0.92

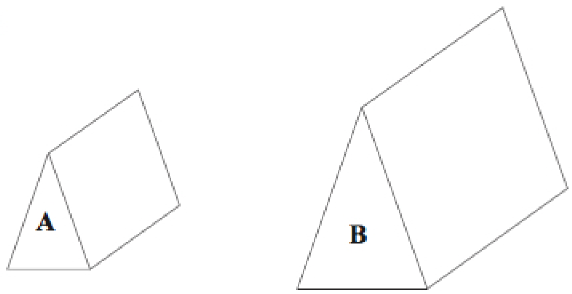
Since the lower and upper bound agree to 1 significant figure but not to 2 significant figures, the answer is $\underline{\underline{0.9}}$.

22. The probability that any piece of buttered toast will land buttered side down when it is dropped is 0.62. (4)
Two pieces of buttered toast are to be dropped, one after the other.
Calculate the probability that exactly one piece of buttered toast will land buttered side down.

Solution

$$\begin{aligned}\text{Probability} &= 2 \times (1 - 0.62) \times 0.62 \\ &= 2 \times 0.38 \times 0.62 \\ &= \underline{\underline{0.471\,2}}.\end{aligned}$$

23. Two prisms, **A** and **B**, are mathematically similar. (4)



The volume of prism **A** is $12\,000$ cm^3.
The volume of prism **B** is $49\,152$ cm^3.
The total surface area of prism **B** is $9\,728$ cm^2.
Calculate the total surface area of prism **A**.

Solution

The volume scale ratio (VSR) from **B** to **A** is

$$\frac{12\,000}{49\,152} = \tfrac{125}{512}$$

and length scale ratio (LSR) is

$$\sqrt[3]{\tfrac{125}{512}} = \tfrac{5}{8}.$$

Finally,

$$\begin{aligned}\text{area of } \mathbf{A} &= 9\,728 \times (\tfrac{5}{8})^2 \\ &= \underline{\underline{3\,800 \text{ cm}^2}}.\end{aligned}$$