Dr Oliver Mathematics
AQA GCSE Mathematics
2019 November Paper 3: Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. Circle the relative frequency that represents 13 successes out of 50 trials. (1)

$$0.13 \quad 26 \quad 13:50 \quad 0.26$$

Solution

Now,

$$13 \text{ out of } 50 = \tfrac{13}{50} = 0.26$$

so

$$0.13 \quad 26 \quad 13:50 \quad \underline{\underline{0.26}}$$

2. The equation of a straight line is (1)

$$2y = 3x + 5.$$

Circle the gradient of the line.

$$\tfrac{2}{3} \quad \tfrac{3}{2} \quad 3 \quad 5$$

Solution

$$2y = 3x + 5 \Rightarrow y = \tfrac{3}{2}x + \tfrac{5}{2}$$

so

$$\tfrac{2}{3} \quad \underline{\underline{\tfrac{3}{2}}} \quad 3 \quad 5$$

3. (1)

$$(2x - 4)(3x + 5)$$

is expanded and simplified.
Circle the term which is part of the answer.

$$2x \quad -2x \quad 22x \quad -22x$$

Solution

$\times$	$2x$	-4
$3x$	$6x^2$	$-12x$
$+5$	$+10x$	-20

so

$$2x \quad \underline{\underline{-2x}} \quad 22x \quad -22x$$

4. When rounded to 3 significant figures, (1)

$$x = 6.37.$$

Circle the correct error interval.

$$6.365 \leqslant x < 6.375 \quad 6.36 \leqslant x < 6.38 \quad 6.369 \leqslant x < 6.379 \quad 6.365 \leqslant x < 6.3749$$

Solution

$$\underline{\underline{6.365 \leqslant x < 6.375}} \quad 6.36 \leqslant x < 6.38 \quad 6.369 \leqslant x < 6.379 \quad 6.365 \leqslant x < 6.3749$$

5. Solve the simultaneous equations (3)

$$7x + 2y = 36$$
$$3x + 2y = 16.$$

Solution

$$7x + 2y = 36 \quad (1)$$
$$3x + 2y = 16 \quad (2)$$

Do (1) − (2):

$$4x = 20 \Rightarrow \underline{\underline{x = 5}}.$$

Substitute into (1):

$$\begin{aligned} 7(5) + 2y = 36 &\Rightarrow 35 + 2y = 36 \\ &\Rightarrow 2y = 1 \\ &\Rightarrow \underline{\underline{y = \tfrac{1}{2}}}. \end{aligned}$$

6. (a) Tom is tiling a wall. (3)

He needs to buy at least 100 tiles.
The tiles are sold in large packs and small packs.

Large pack	40 tiles	£18
Small pack	28 tiles	£14

Special offer

25% reduction when you buy 3 or more **large** packs

Work out the cheapest cost for Tom to buy the packs of tiles he needs.

Solution

2 large, 1 small:

$$2 \times 18 + 14 = 50.$$

3 large

$$0.75 \times 3 \times 18 = 40.5.$$

So, he needs to buy 3 large packs and this will cost him £40.50.

(b) Tom is also tiling a floor. (1)

- The floor is a rectangle with length 600 cm and width 240 cm.

- Each tile is a square with side 40 cm.

Tom uses this method to work out the number of tiles he needs.

Number of tiles that will fit along the length	=	600 ÷ 40
	=	15
Number of tiles that will fit along the width	=	240 ÷ 40
	=	6
Total number of tiles needed	=	15 + 6
	=	21

Give a reason why Tom's method is wrong.

Solution

He has to multiply:

$$15 \times 6 = 90.$$

7. An equilateral triangle has side length 16 metres. (3)
Using ruler and compasses only, construct a scale drawing of the triangle.
Use the scale 1 centimetre represents 2 metres.

Solution

- Draw a line which is 8 cm.
- Call your left-hand point A and call your left-hand point B.
- Open your compasses to 8 cm.
- At A, place your compasses at one end of the straight line.
- Draw a faint (but visible!) arc from about 12 o'clock down to above 3 o'clock.
- At B, place your compasses at one end of the straight line.
- Draw a faint (but visible!) arc from about 9 o'clock down to above 12 o'clock.
- Draw two straight lines.

8. In a choir there are 35 men and 48 women.

- The probability that a man chosen at random wears glasses is $\frac{2}{5}$.
- The probability that a woman chosen at random wears glasses is $\frac{3}{8}$.

(a) Work out the number of people in the choir who wear glasses. (3)

Solution

$$\begin{aligned}\text{Number of people} &= (35 \times \tfrac{2}{5}) + (48 \times \tfrac{3}{8}) \\ &= 14 + 18 \\ &= \underline{\underline{32}}.\end{aligned}$$

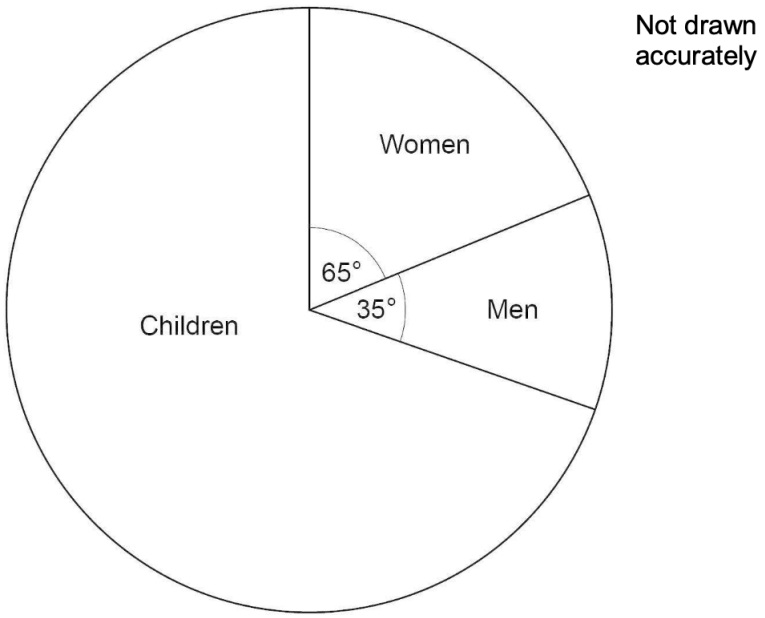
(b) A person is chosen at random from the choir. (2)

Work out the probability that the person does **not** wear glasses.

Solution

$$\begin{aligned}\text{P(does not wear glasses)} &= 1 - \text{P(does wear glasses)} \\ &= 1 - \tfrac{32}{35+48} \\ &= \underline{\underline{\tfrac{51}{83}}}.\end{aligned}$$

9. The pie chart shows information about people at a theme park. (3)



There were 450 **more** women than men.

Work out the number of children.

Solution

Let the number of people be x. Now,

$$\left(\frac{65}{360}\times x\right)-\left(\frac{35}{360}\times x\right)=450 \Rightarrow \frac{30}{360}\times x=450$$
$$\Rightarrow \tfrac{1}{12}x=450$$
$$\Rightarrow x=5\,400.$$

Finally,

$$\text{number of children}=\left(1-\frac{65+35}{360}\right)\times 5\,400$$
$$\Rightarrow \tfrac{260}{360}\times 5\,400$$
$$\Rightarrow \underline{\underline{3\,900}}.$$

10. (1)

$$\text{Density}=\frac{\text{mass}}{\text{volume}}.$$

The mass is divided by 2 and the volume is multiplied by 4.
What happens to the density?
Circle your answer.

$\times 2$ $\quad \div 2$ $\quad \times 8$ $\quad \div 8$

Solution

Well,

$$\text{new density}=\frac{\frac{1}{2}\times \text{mass}}{4\times \text{volume}}$$
$$=\frac{\text{mass}}{8\times \text{volume}}$$
$$=\tfrac{1}{8}\times\frac{\text{mass}}{\text{volume}}$$

so

$\times 2$ $\quad \div 2$ $\quad \times 8$ $\quad \underline{\underline{\div 8}}$

11. Work out (3)

cube root of 512 : reciprocal of 0.4.

Give your answer in the form $n : 1$.

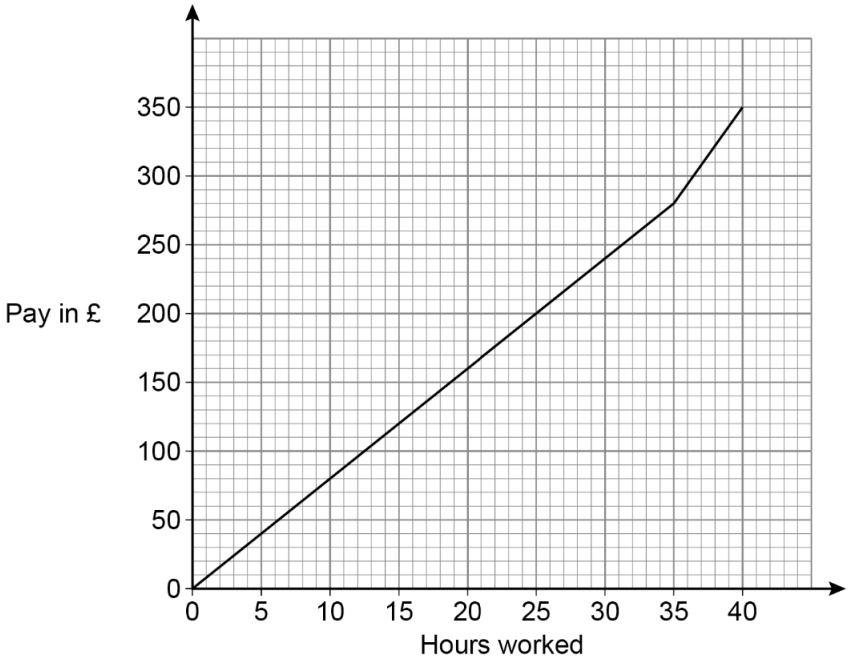
Solution

$$\text{cube root of } 512 : \text{reciprocal of } 0.4 = 8 : 2.5$$
$$= \frac{16}{5} : 1.$$

12. The graph shows how much Molly is paid for working for up to 40 hours. (3)

She receives

- a basic rate of pay for the first 35 hours worked and
- a higher rate of pay for the next 5 hours worked.



Work out the difference between the higher rate of pay and the basic rate of pay.
Give your answer in £ per hour.

Solution

Basic rate:

$$\begin{aligned} m &= \frac{280-0}{35-0} \\ &= 8. \end{aligned}$$

Higher rate:

$$\begin{aligned} m &= \frac{350-280}{40-35} \\ &= \frac{70}{5} \\ &= 14. \end{aligned}$$

Hence, the difference is

$$14 - 8 = \underline{\underline{£6 \text{ an hour}}}.$$

13. Naga states a hypothesis:

"Most people read more than 100 books a year."

She asks a sample of five people in a book club how many books they read last month. The table shows the results.

	Lynn	Ali	Paul	Chen	Ruth
Number of books	10	11	8	10	13

(a) Show how Naga could use the data to support her hypothesis. (2)

Solution

Multiply total by 12:

	Lynn	Ali	Paul	Chen	Ruth
Number of books	10	11	8	10	13
Total	120	132	96	120	156

So, given that 4 out of 5 read more than 100 books, her hypothesis $\underline{\underline{\text{makes sense}}}$.

(b) Give two reasons why this sample should **not** be used to support her hypothesis. (2)

Solution

E.g., the sample is biased, the sample is small, they may not read at the same rate in other months.

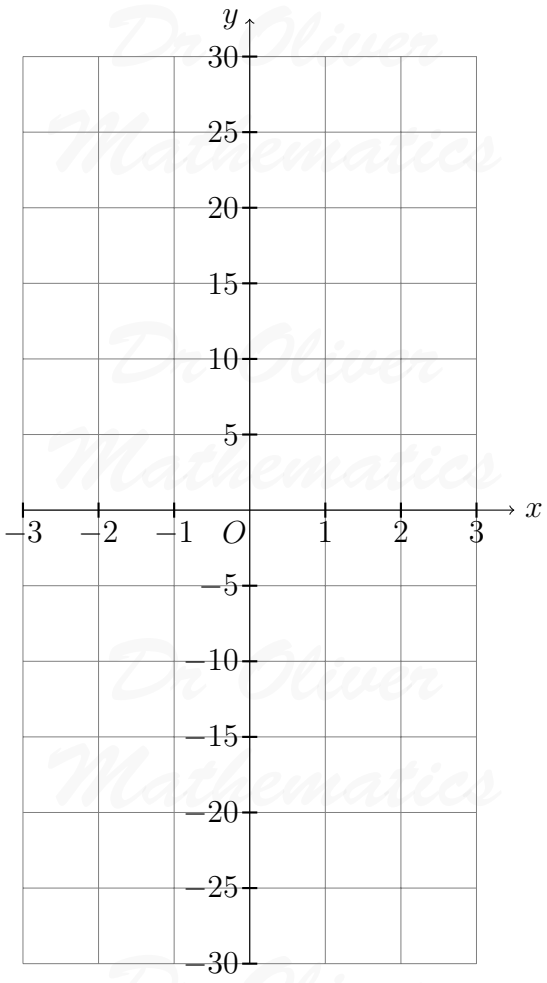
14. A graph has equation (3)

$$y = x^3 + a,$$

where a is an integer.

The graph passes through the point $(3, 29)$.

Draw the graph for values of x from -3 to 3.



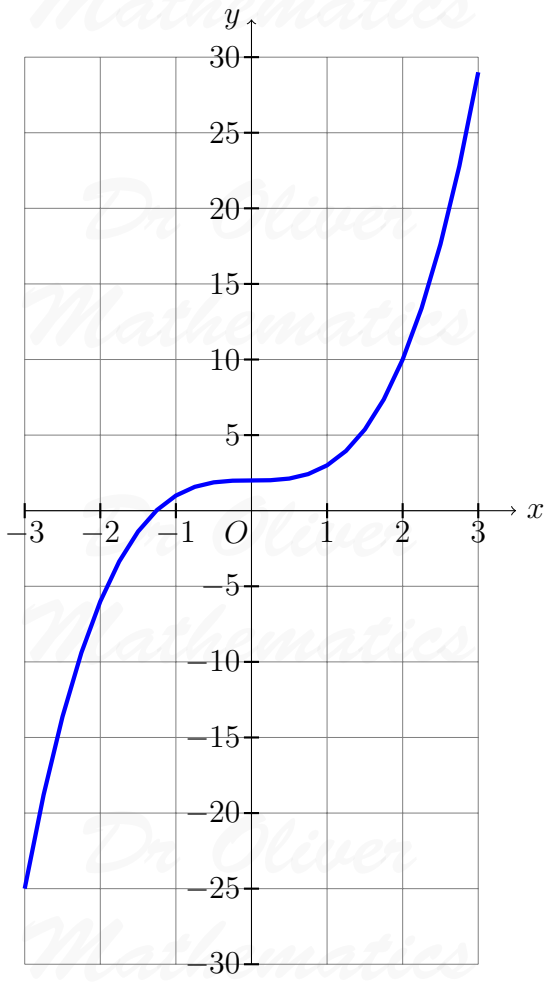
Solution

Well,

$$x = 3, y = 29 \Rightarrow 29 = 27 + 2$$
$$\Rightarrow a = 2$$

and the equation is

$$y = x^3 + 2.$$



15. When you earn money you pay income tax. (4)
The amount you pay depends on how much you earn that year.
You pay

- 0% on the first £12 500 you earn,

- 20% on the next £37 500 you earn, and
- 40% on the next £112 500 you earn

One year, Kim paid £9 260 income tax.

Work out how much she earned that year.

Solution

Well,

$$0.2 \times 37\,500 = 7\,500.$$

Now,

$$9\,260 - 7\,500 = 1\,760$$

and

$$\frac{1\,760}{0.4} = 4\,400.$$

Finally,

$$\begin{aligned} \text{her earnings} &= 12\,500 + 37\,500 + 4\,400 \\ &= \underline{\underline{£54\,400}}. \end{aligned}$$

16. A building company employs

- 2 labourers,
- 14 joiners,
- 9 electricians, and
- 8 plumbers.

For a job, the company needs one of each type of worker.

(a) In how many ways can the company choose the four workers? (2)

Solution

$$2 \times 14 \times 9 \times 8 = \underline{\underline{2\,016}}.$$

(b) One labourer and two plumbers are on holiday. (2)

In how many ways can the company now choose the four workers?

Solution

$$1 \times 14 \times 9 \times 6 = \underline{\underline{756}}.$$

17. (2)

$$\mathrm{f}(x) = 3x^2 - 4x + 8, \text{ for all values of } x.$$

Jenny says,

"$\mathrm{f}(10)$ must equal $2 \times \mathrm{f}(5)$, because 10 is 2×5."

Is Jenny correct?
Show working to support your answer.

Solution

Well,

$$\begin{aligned} \mathrm{f}(10) &= 3(10^2) - 4(10) + 8 \\ &= 300 - 40 + 8 \\ &= 268 \end{aligned}$$

whereas

$$\begin{aligned} 2\,\mathrm{f}(5) &= 2[3(5^2) - 4(5) + 8] \\ &= 2(75 - 20 + 8) \\ &= 2 \times 63 \\ &= 126. \end{aligned}$$

So, $\underline{\underline{\mathrm{f}(10) \neq 2\,\mathrm{f}(5)}}$.

18. Work out the **two** roots of (1)

$$(7x + 1)(2x - 3) = 0.$$

Circle **both** roots.

$$-\tfrac{1}{7} \quad \tfrac{1}{7} \quad -\tfrac{3}{2} \quad \tfrac{3}{2}$$

Solution

Now,

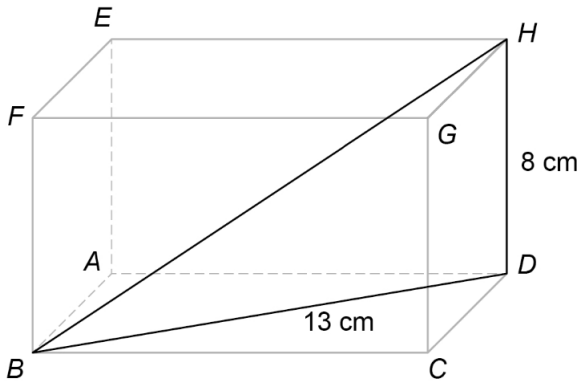
$$(7x+1)(2x-3)=0 \Rightarrow 7x+1=0 \text{ or } 2x-3=0$$
$$\Rightarrow x=-\tfrac{1}{7} \text{ or } x=\tfrac{3}{2}$$

so

$$\underline{\underline{-\tfrac{1}{7}}} \quad \tfrac{1}{7} \quad -\tfrac{3}{2} \quad \underline{\underline{\tfrac{3}{2}}}$$

19. Here is a cuboid.

- $DH = 8$ cm.
- $DB = 13$ cm.



(a) Work out the size of angle DBH. (2)

Solution

$$\tan = \frac{\text{opp}}{\text{adj}} \Rightarrow \tan DBH = \frac{8}{13}$$
$$\Rightarrow \angle DBH = 31.607\,502\,25 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{\angle DBH = 31.6^\circ \text{ (3 sf)}}}.$$

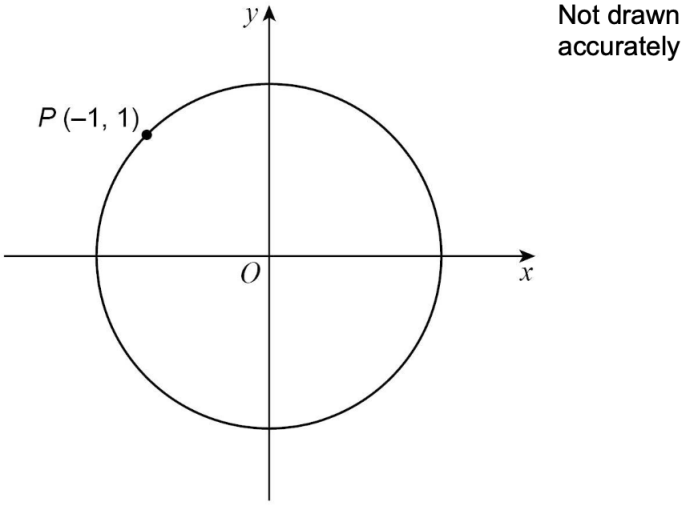
(b) Using your answer to part (a), work out the size of angle ECG. (1)

Solution

It is the complementary angle:

$$\begin{aligned}\angle ECG &= 90 - 31.607\ldots \\ &= 58.392\,497\,75 \text{ (FCD)} \\ &= \underline{\underline{58.4^\circ \text{ (3 sf)}}}.\end{aligned}$$

20. $P(-1, 1)$ is a point on the circle, centre O, radius r. (1)



Work out the value of r.
Circle your answer.

$$1 \quad 2 \quad \sqrt{2} \quad 2\sqrt{2}$$

Solution

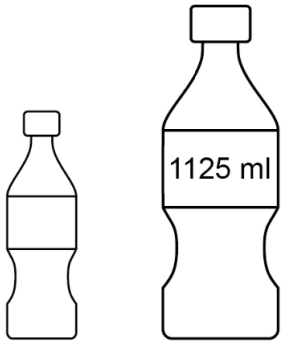
Well,

$$\begin{aligned}OP &= \sqrt{(1-0)^2 + (-1-0)^2} \\ &= \sqrt{1+1} \\ &= \sqrt{2}\end{aligned}$$

so

$$1 \quad 2 \quad \underline{\underline{\sqrt{2}}} \quad 2\sqrt{2}$$

21. Juice is sold in small bottles and large bottles. (4)
The volume of the large bottle is 1 125 ml.



Volume of small bottle : volume of large bottle = 2 : 5.

A café has small glasses and large glasses.

Volume of small glass : volume of large glass = 4 : 7.

A small bottle fills 6 small glasses with no juice left over.

How many large glasses can be filled by a large bottle?
You **must** show your working.

Solution

Now,

$$\begin{aligned} & \text{volume of small bottle} : \text{volume of large bottle} = 2 : 5 \\ \Rightarrow \quad & \frac{\text{volume of small bottle}}{\text{volume of large bottle}} = \frac{2}{5} \\ \Rightarrow \quad & \frac{\text{volume of small bottle}}{1\,125} = \frac{2}{5} \\ \Rightarrow \quad & \text{volume of small bottle} = \frac{2}{5} \times 1\,125 \\ \Rightarrow \quad & \text{volume of small bottle} = 450 \text{ ml}. \end{aligned}$$

Next, a small bottle fills 6 small glasses which means each small bottle contains

$$\frac{450}{6} = 75 \text{ ml}.$$

Moreover,

$$\begin{aligned} & \text{volume of small glass} : \text{volume of large glass} = 4 : 7 \\ \Rightarrow \quad & \frac{\text{volume of large glass}}{\text{volume of small glass}} = \frac{7}{4} \\ \Rightarrow \quad & \frac{\text{volume of large glass}}{75} = \frac{7}{4} \\ \Rightarrow \quad & \text{volume of large glass} = \frac{7}{4} \times 75 \\ \Rightarrow \quad & \text{volume of large glass} = 131.25 \text{ ml.} \end{aligned}$$

Finally,

$$\frac{1\,125}{131.25} = 8\tfrac{4}{7},$$

so he can fill 8 glasses (with some to spare).

22. The **only** solution to (2)

$$x^2 + bx + c = 0$$

is $x = 5$.

Work out the values of b and c.

Solution

Now,

$\times$	x	-5
x	x^2	$-5x$
-5	$-5x$	$+25$

so

$$(x-5)^2 = 0 \Rightarrow x^2 - 10x + 25 = 0.$$

Hence,

$$b = -10 \text{ and } c = 25.$$

23. (1)

$$x : y = \tfrac{1}{4} : \tfrac{2}{3}$$

What is x as a fraction of y?
Circle your answer.

$\frac{8}{3}$ $\frac{1}{6}$ $\frac{3}{7}$ $\frac{3}{8}$

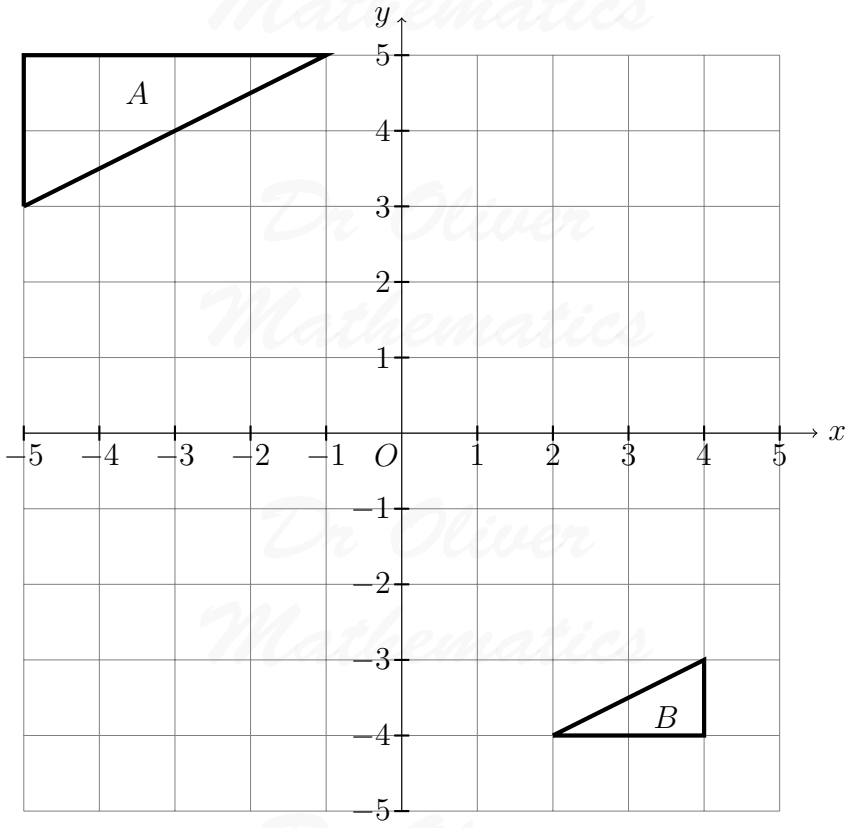
Solution

$$x : y = \frac{1}{4} : \frac{2}{3} \Rightarrow \frac{x}{y} = \frac{\frac{1}{4}}{\frac{2}{3}}$$
$$\Rightarrow \frac{x}{y} = \frac{3}{8}$$

so

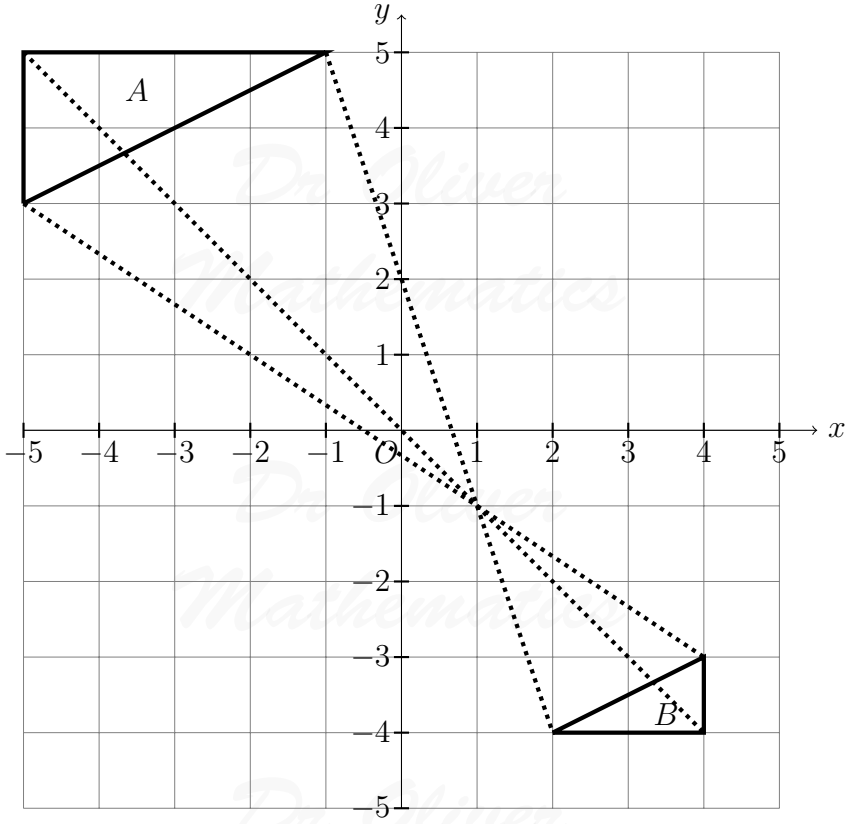
$\frac{8}{3}$ $\frac{1}{6}$ $\frac{3}{7}$ $\underline{\underline{\frac{3}{8}}}$

24. Shape A and shape B are shown on the grid. (3)



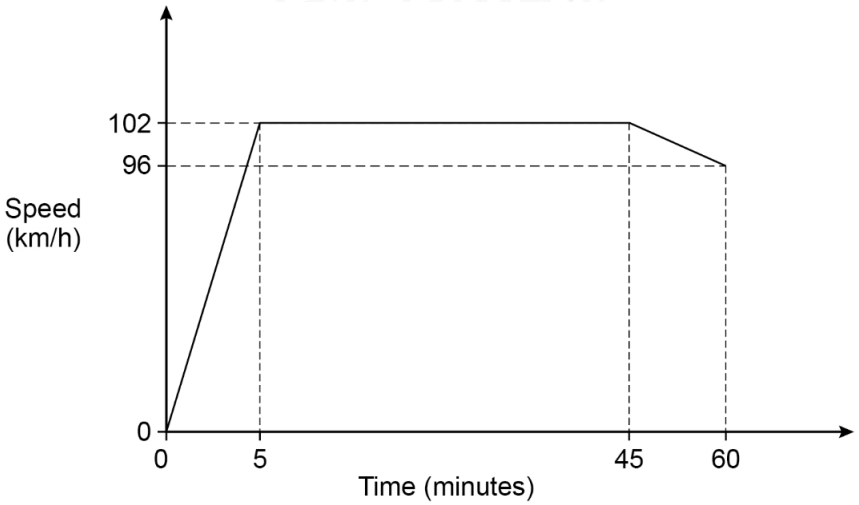
Describe the single transformation that maps shape A to shape B.

Solution



Hence, it is an enlargement, centre $(1, -1)$, scale factor $-\frac{1}{2}$.

25. Here is a sketch of a speed-time graph for the first part of a journey. (4)



The total distance for the journey is 130 kilometres.

How far is left to travel?

Solution

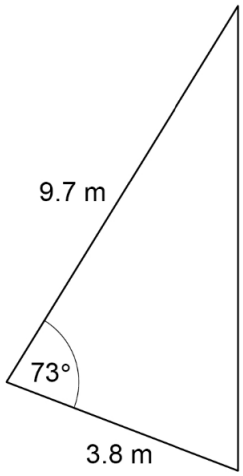
Well, let us convert the times to hours:

$$\begin{aligned}\text{distance} &= (\tfrac{1}{2} \times \tfrac{1}{12} \times 102) + (\tfrac{2}{3} \times 102) + [\tfrac{1}{2} \times (102 + 96) \times \tfrac{1}{14}] \\ &= 4\tfrac{1}{4} + 68 + 24\tfrac{3}{4} \\ &= 97;\end{aligned}$$

so, the remaining distance is

$$130 - 97 = \underline{\underline{33 \text{ km}}}.$$

26. Here is a triangular sail.



Not drawn accurately

(a) Vicky needs to buy waterproofing liquid for the sail. (3)

- She will put **3 coats** of liquid on **each** side of the sail.
- A litre of liquid covers 8.5 square metres of sail.

How many 1-litre bottles of liquid does Vicky need?

Solution

Well,

$$\begin{aligned}\text{area} &= \tfrac{1}{2} \times 3.8 \times 9.7 \times \sin 73^\circ \\ &= 17.624\,696\,665 \text{ (FCD)}.\end{aligned}$$

Now, she will put 3 coats of liquid on each side of the sail so

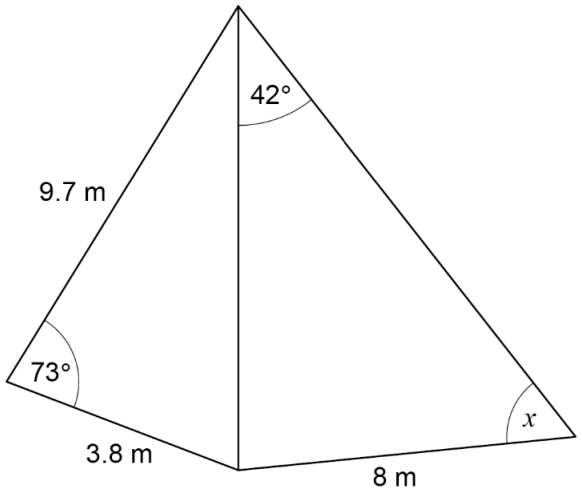
$$\begin{aligned}\text{area of the coats} &= 6 \times 17.624\ldots \\ &= 105.748\,179\,9 \text{ (FCD)}.\end{aligned}$$

She needs

$$\frac{105.748\ldots}{8.5} = 12.440\,962\,34 \text{ (FCD)};$$

So, she needs 13 bottles.

(b) Another sail is joined to the first sail as shown. (5)



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x is an acute angle.

Work out the size of angle x.

Solution

Let us call the side in common d.

Cosine rule:

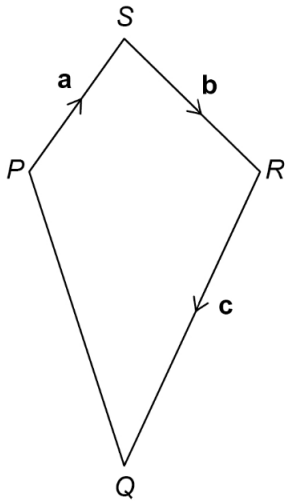
$$\begin{aligned} & d^2 = 3.8^2 + 9.7^2 - 2 \times 3.8 \times 9.7 \times \cos 73^\circ \\ \Rightarrow \quad & d^2 = 86.976\,357\,93 \text{ (FCD)} \\ \Rightarrow \quad & d = 9.326\,111\,619 \text{ (FCD)}. \end{aligned}$$

Sine rule:

$$\begin{aligned} \frac{\sin x}{9.326\ldots} = \frac{\sin 42^\circ}{8} \Rightarrow \sin x &= \frac{9.326\ldots \sin 42^\circ}{8} \\ \Rightarrow x &= 51.265\,001\,61 \text{ (FCD)} \\ \Rightarrow x &= \underline{\underline{51.3^\circ \text{ (3 sf)}}}. \end{aligned}$$

27. Here is quadrilateral $PQRS$. (3)

- $\overrightarrow{PS} = \mathbf{a}$.
- $\overrightarrow{SR} = \mathbf{b}$.
- $\overrightarrow{RQ} = \mathbf{c}$.



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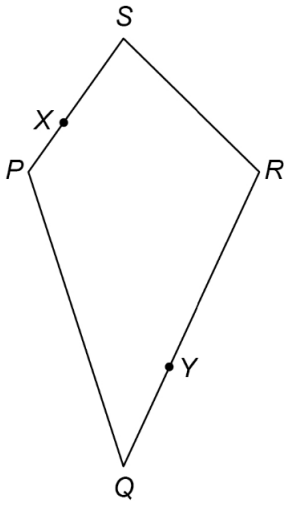
X is a point on PS where

$$PX : XS = 1 : 2.$$

Y is a point on RQ where

$$RY : YQ = 2 : 1.$$



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Is XY parallel to PQ?
Show working to support your answer.

Solution

Well,

$$\begin{aligned}\overrightarrow{PQ} &= \overrightarrow{PS} + \overrightarrow{SR} + \overrightarrow{RQ} \\ &= \mathbf{a} + \mathbf{b} + \mathbf{c}\end{aligned}$$

and

$$\begin{aligned}\overrightarrow{XY} &= \overrightarrow{XS} + \overrightarrow{SR} + \overrightarrow{RY} \\ &= \tfrac{2}{3}\overrightarrow{PS} + \overrightarrow{SR} + \tfrac{2}{3}\overrightarrow{RQ} \\ &= \tfrac{2}{3}\mathbf{a} + \mathbf{b} + \tfrac{2}{3}\mathbf{c}.\end{aligned}$$

No, they are not parallel because $\overrightarrow{PQ}$ is not a scalar multiple of $\overrightarrow{XY}$.

28. (4)

$$\mathrm{f}(x) = 2x - 3 \text{ and } \mathrm{g}(x) = x^2.$$

Show that

$$\mathrm{f}^{-1}(55) = \mathrm{f}\,\mathrm{g}(4).$$

Solution

Well,

$$\begin{aligned}y = 2x - 3 &\Rightarrow y + 3 = 2x \\ &\Rightarrow \tfrac{1}{2}(y + 3) = x\end{aligned}$$

and

$$\mathrm{f}^{-1}(x) = \tfrac{1}{2}(x + 3).$$

Now,

$$\mathrm{f}^{-1}(55) = \tfrac{1}{2}(55 + 3) = 29.$$

Next,

$$\begin{aligned}\mathrm{f}\,\mathrm{g}(4) &= \mathrm{f}(\mathrm{g}(4)) \\ &= \mathrm{f}(16) \\ &= 2(16) - 3 \\ &= 29;\end{aligned}$$

so,

$$\mathrm{f}^{-1}(55) = \mathrm{f}\,\mathrm{g}(4),$$

as required.