

Dr Oliver Mathematics
Cambridge O Level Additional Mathematics
2011 November Paper 1 Variant 2: Calculator
2 hours

The total number of marks available is 80.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You must write down all the stages in your working.

1. Show that

$$\frac{1}{\tan \theta + \cot \theta} \equiv \sin \theta \cos \theta. \quad (3)$$

2. Find the coordinates of the points where the line $2y = x - 1$ meets the curve

$$x^2 + y^2 = 29. \quad (5)$$

3. (a) Express $\log_x 2$ in terms of a logarithm to base 2. (1)

- (b) Using the result of part (a), and the substitution (4)

$$u = \log_2 x,$$

find the values of x which satisfy

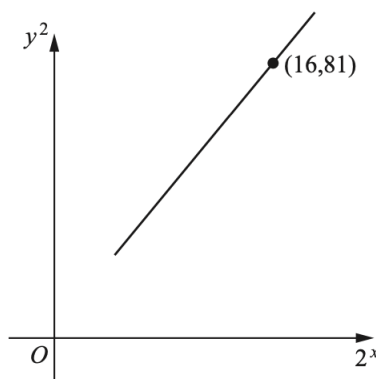
$$\log_2 x = 3 - 2 \log_x 2.$$

4. A curve has equation (6)

$$y = (3x^2 + 15)^{\frac{2}{3}}.$$

Find the equation of the normal to the curve at the point where $x = 2$.

5. Variables x and y are such that, when y^2 is plotted against 2^x , a straight line graph is obtained.



This line has a gradient of 5 and passes through the point (16, 81).

(a) Express y^2 in terms of 2^x . (3)

(b) Find the value of x when $y = 6$. (3)

6. (a) Given that (4)

$$(3 + x)^5 + (3 - x)^5 \equiv A + Bx^2 + Cx^4,$$

find the value of A , of B , and of C .

(b) Hence, using the substitution (4)

$$y = x^2,$$

solve, for x , the equation

$$(3 + x)^5 + (3 - x)^5 = 1086.$$

7. (a) Show that (3)

$$\frac{(4 - \sqrt{x})^2}{\sqrt{x}}$$

can be written in the form

$$px^{-\frac{1}{2}} + q + rx^{\frac{1}{2}},$$

where p , q , and r are integers to be found.

(b) A curve is such that (5)

$$\frac{dy}{dx} = \frac{(4 - \sqrt{x})^2}{\sqrt{x}} \text{ for } x > 0.$$

Given that the curve passes through the point (9, 30), find the equation of the curve.

8. The line CD is the perpendicular bisector of the line joining the point $A(-1, -5)$ and the point $B(5, 3)$.

(a) Find the equation of the line CD . (4)

(b) Given that (3)

- M is the midpoint of AB ,
- that $2CM = MD$, and that
- the x -coordinate of C is -2 ,

find the coordinates of D .

(c) Find the area of the triangle CAD . (2)

9. (a) Given that (3)

$$y = x \sin 4x,$$

find $\frac{dy}{dx}$.

(b) Hence find

$$\int x \cos 4x \, dx$$

and evaluate

$$\int_0^{\frac{1}{8}\pi} x \cos 4x \, dx.$$

(6)

10. (a) Solve

$$2 \sec^2 x = 5 \tan x + 5, \text{ for } 0^\circ < x < 360^\circ.$$

(5)

(b) Solve

$$\sqrt{2} \sin\left(\frac{1}{2}y + \frac{1}{3}\pi\right) = 1 \text{ for } 0 < y < 4\pi \text{ radians.}$$

(5)

EITHER

11. A curve has equation

$$y = e^{-x}(A \cos 2x + B \sin 2x).$$

At the point $(0, 4)$ on the curve, the gradient of the tangent is 6.

(a) Find the value of A .

(1)

(b) Show that $B = 5$.

(5)

(c) Find the value of x , where $0 < x < \frac{1}{2}\pi$ radians, for which y has a stationary value.

(5)

OR

12. A curve has equation

$$y = \frac{\ln(x^2 - 1)}{x^2 - 1} \text{ for } x > 1.$$

(a) Show that

$$\frac{dy}{dx} = \frac{kx [1 - \ln(x^2 - 1)]}{(x^2 - 1)^2},$$

(4)

where k is a constant to be found.

(b) Hence find the approximate change in y when x increases from $\sqrt{5}$ to $(\sqrt{5} + p)$, where p is small.

(2)

(c) Find, in terms of e , the coordinates of the stationary point on the curve.

(5)