Dr Oliver Mathematics
GCSE Mathematics
2011 November Paper 4H: Calculator
1 hour 45 minutes

The total number of marks available is 100.
You must write down all the stages in your working.

1. (a) Use your calculator to work out (2)

$$\frac{\sqrt{21.5}}{5.8 - 2.36}.$$

Write down all the figures on your calculator display.

Solution

$$\frac{\sqrt{21.5}}{5.8 - 2.36} = \frac{\sqrt{21.5}}{3.44}$$
$$= \underline{\underline{1.347\,909\,665 \text{ (FCD)}}}.$$

(b) Write down your answer to part (a) correct to 2 decimal places. (1)

Solution

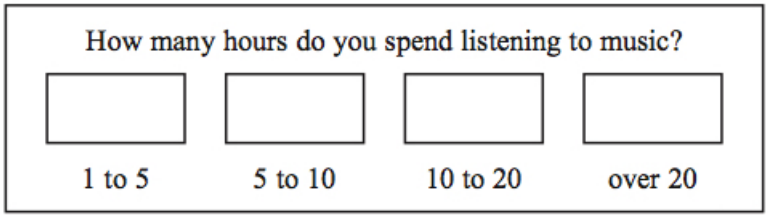
$\underline{\underline{1.35 \text{ (2 dp)}}}$.

2. Ishmal invested £3 500 for 3 years at 2.5% per annum **simple interest**. (3)
Work out the total amount of interest Ishmal earned.

Solution

$$3 \times 3\,500 \times 0.025 = \underline{\underline{£262.50}}.$$

3. Gary wants to find out how much time teenagers spend listening to music.



He uses this question on a questionnaire.

(a) Write down **two** things wrong with this question. (2)

Solution

E.g., there is no place to write down 0.5, the '5' appears in multiple boxes, the '10' appears in multiple boxes,

(b) Design a better question for Gary's questionnaire to find out how much time teenagers spend listening to music. (2)

Solution

A suitable question with a time frame, e.g., "How many hours do you spend listening to music today/last week/last month? Tick the appropriate box."
At least three exhaustive and non-overlapping tick boxes (best defined using inequality notation): for example, Never listen to music, $0 < t$ hours < 1, $1 < t$ hours < 3, t hours $\geqslant 3$.

4. (a) Find the highest common factor (HCF) of 24 and 30. (1)

Solution

$$\begin{array}{r|l} & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline 2 & 3 \\ \hline 3 & 1 \\ \hline \end{array}$$

So

$$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3.$$

$$\begin{array}{r|l} & 30 \\ \hline 2 & 15 \\ \hline 3 & 5 \\ \hline 5 & 1 \\ \hline \end{array}$$

So

$$30 = 2 \times 3 \times 5.$$

Hence,

$$\text{HCF}(24, 30) = 2 \times 3 = \underline{\underline{6}}.$$

(b) Find the lowest common multiple (LCM) of 4, 5, and 6. (2)

Solution

Well, $4 = 2^2$ and $6 = 2 \times 3$ and so the

$$\text{LCM}(4, 5, 6) = 2^2 \times 3 \times 5 = \underline{\underline{60}}.$$

5. Melissa is 13 years old. (4)
Becky is 12 years old.
Daniel is 10 years old. Melissa, Becky, and Daniel share £28 in the ratio of their ages.
Becky gives a third of her share to her mother.
How much should Becky now have?

Solution

Well,

$$13 + 12 + 10 = 35$$

and Becky has

$$\frac{12}{35} \times 28 = £9.60.$$

She retains

$$\frac{2}{3} \times 9.60 = \underline{\underline{£6.40}}.$$

6. All angles are measured in degrees.

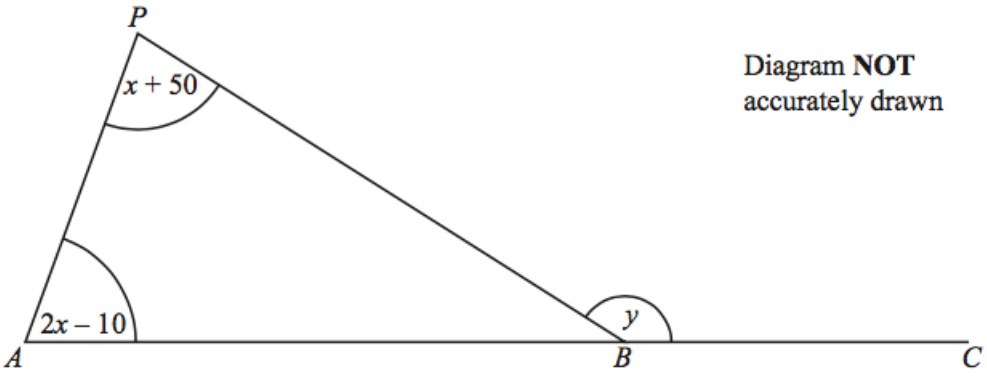


Diagram **NOT** accurately drawn

ABC is a straight line.
Angle $APB = x + 50$.

Angle $PAB = 2x - 10$.
Angle $PBC = y$.

(a) Show that (3)

$$y = 3x + 40.$$

Give reasons for each stage of your working.

Solution

$$\begin{aligned} y &= (2x - 10) + (x + 50) \\ &= \underline{\underline{3x + 40}} \end{aligned}$$

as the exterior angles equals the sum of the two interior opposite angles.

(b) Given that $y = 145$, (4)

(i) work out the value of x,

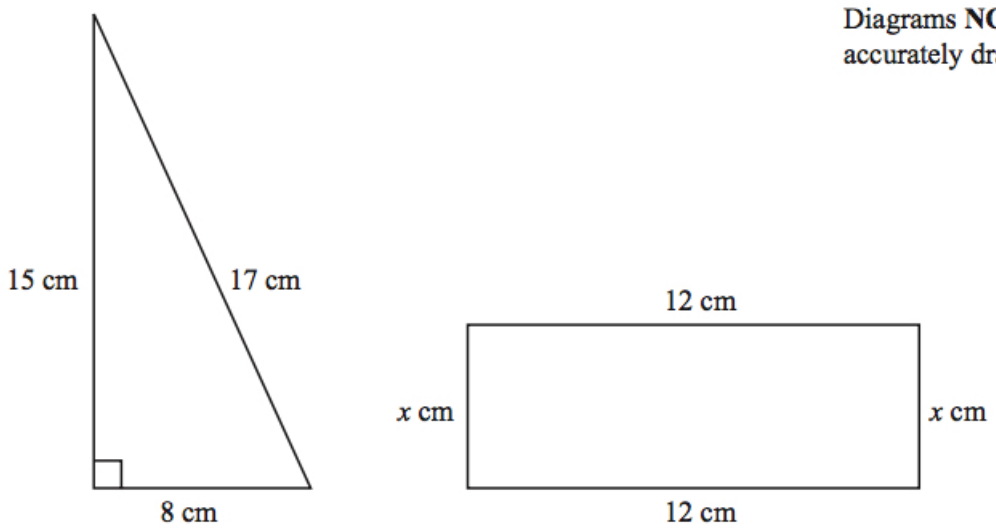
Solution

$$\begin{aligned} 145 = 3x + 40 \Rightarrow 3x &= 105 \\ \Rightarrow \underline{\underline{x = 35}}&. \end{aligned}$$

(ii) work out the size of the largest angle in triangle ABP.

Solution
$\angle PAC = 2 \times 35 - 10 = 60$.
$\angle APC = 35 + 50 = 85$.
$\angle PBA = 180 - 145 = 35$.
Hence, it is $\underline{\underline{\angle APC = 85}}$.

7. The diagrams show a right-angled triangle and a rectangle. (4)



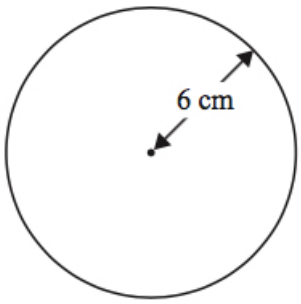
The area of the right-angled triangle is equal to the area of the rectangle.
Find the value of x.

Solution

The areas areas are equal:

$$12 \times x = \tfrac{1}{2} \times 8 \times 15 \Rightarrow 12x = 60$$
$$\Rightarrow \underline{\underline{x = 5}}.$$

8. The diagram shows a CD.



The CD is a circle of radius 6 cm.

(a) Work out the circumference of the CD. (2)

Solution

$$2 \times \pi \times 6 = \underline{\underline{37.699\,111\,84 \text{ cm (FCD)}}}.$$

CDs of this size are cut from rectangular sheets of plastic.
Each sheet is 1 metre long and 50 cm wide.

(b) Work out the greatest number of CDs that can be cut from one rectangular sheet. (2)

Solution

$$\frac{100}{2 \times 6} = 8\tfrac{1}{3}$$

and

$$\frac{50}{2 \times 6} = 4\tfrac{1}{6}.$$

Hence, the greatest number of CDs that can be cut from one rectangular sheet is

$$4 \times 8 = \underline{\underline{32}}.$$

9. The exchange rate in London is £1 = €1.14. (3)
The exchange rate in Paris is €1 = £0.86.
Elaine wants to change some pounds into euros.
In which of these cities would Elaine get the most euros?
You must show all of your working.

Solution

$$\frac{1}{0.86} = 1.162\ldots$$

so Elaine get the most euros in Paris.

10. The temperature (T°C) at noon at a seaside resort was recorded for a period of 60 days. (4)
The table shows some of this information.

Temperature (T°C)	Number of days
$10 < T \leqslant 14$	2
$14 < T \leqslant 18$	8
$18 < T \leqslant 22$	14
$22 < T \leqslant 26$	23
$26 < T \leqslant 30$	9
$30 < T \leqslant 34$	4

Calculate an estimate for the mean temperature at noon during these 60 days.
Give your answer correct to 3 significant figures.

Solution

Temperature (T°C)	Number of days	Midpoint	Frequency×Midpoint
$10 < T \leqslant 14$	2	12	$2 \times 12 = 24$
$14 < T \leqslant 18$	8	16	$8 \times 16 = 128$
$18 < T \leqslant 22$	14	20	$14 \times 20 = 280$
$22 < T \leqslant 26$	23	24	$23 \times 24 = 552$
$26 < T \leqslant 30$	9	28	$9 \times 28 = 252$
$30 < T \leqslant 34$	4	32	$4 \times 32 = 128$
Total	60		1 364

$$\begin{aligned}\text{Mean} &\approx \frac{1\,364}{60}\\ &= 22.7\dot{3} \text{ (FCD)}\\ &= \underline{\underline{22.7^\circ\text{C (3 sf)}}}.\end{aligned}$$

11. (a) Simplify (1)

$$m^3 \times m^6.$$

Solution

$$m^3 \times m^6 = \underline{\underline{m^9}}.$$

(b) Simplify (1)

$$\frac{p^8}{p^2}.$$

Solution

$$\frac{p^8}{p^2} = \underline{\underline{p^6}}.$$

(c) Simplify (2)

$$(2n^3)^4.$$

Solution

$$(2n^3)^4 = \underline{\underline{16n^{12}}}.$$

12. $-2 \leqslant n < 5$.
n is an integer.

(a) Write down all the possible values of n. (2)

Solution

$\underline{\underline{-2, -1, 0, 1, 2, 3, 4}}$.

(b) Solve the inequality (2)

$$4x + 1 > 11.$$

Solution

$$4x + 1 > 11 \Rightarrow 4x > 10$$
$$\Rightarrow \underline{\underline{x > 2\tfrac{1}{2}}}.$$

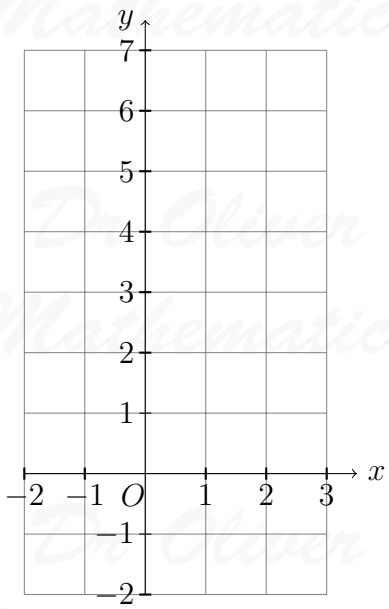
13. (a) Complete the table of values for $3x + 2y = 6$. (2)

x	-2	-1	0	1	2	3
y		4.5	3			-1.5

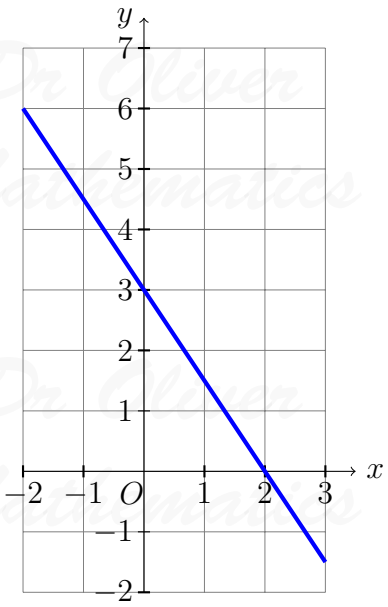
Solution

x	-2	-1	0	1	2	3
y	$\underline{\underline{6}}$	4.5	3	$\underline{\underline{1.5}}$	$\underline{\underline{0}}$	-1.5

(b) On the grid, draw the graph of $3x + 2y = 6$. (2)



Solution



(c) Find the gradient of the graph of $3x + 2y = 6$. (2)

Solution

$$3x + 2y = 6 \Rightarrow 2y = -3x + 6$$
$$\Rightarrow y = -1.5x + 2;$$

hence, the gradient is $\underline{\underline{-1.5}}$.

14. (a) Factorise $6x + 4$. (1)

Solution

$$6x + 4 = \underline{\underline{2(3x + 2)}}.$$

(b) Factorise fully (2)

$$9x^2y - 15xy.$$

Solution

$$9x^2y - 15xy = \underline{\underline{3xy(3x - 5)}}.$$

15. A garage sells used cars.
The table shows the number of used cars it sold from July to December.

July	August	September	October	November	December
28	25	34	46	28	40

(a) Work out the 3-point moving averages for the information in the table. (2)
The first two have been worked out for you: 29 and 35.

Solution

Sept - Nov:

$$\frac{34 + 46 + 28}{3} = \frac{108}{3} = \underline{\underline{36}}.$$

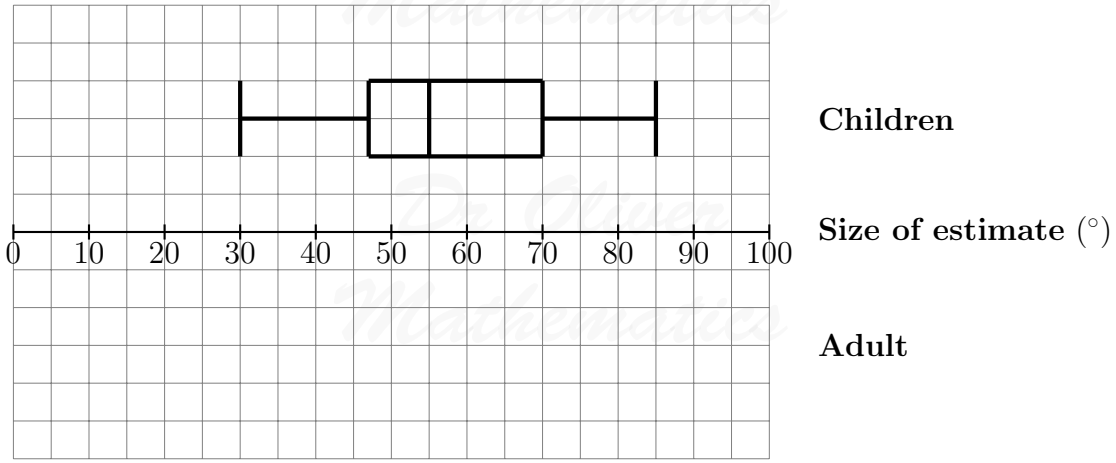
Oct - Dec:

$$\frac{46 + 28 + 40}{3} = \frac{114}{3} = \underline{\underline{38}}.$$

(b) Comment on the trend shown by the 3-point moving averages (1)

Solution
They are increasing.

16. Barry drew an angle of 60°.
He asked some children to estimate the size of the angle he had drawn.
He recorded their estimates.
The box plot gives some information about these estimates.



(a) Write down the median of the children's estimates. (1)

Solution
55°.

(b) Find the interquartile range of the children's estimates. (2)

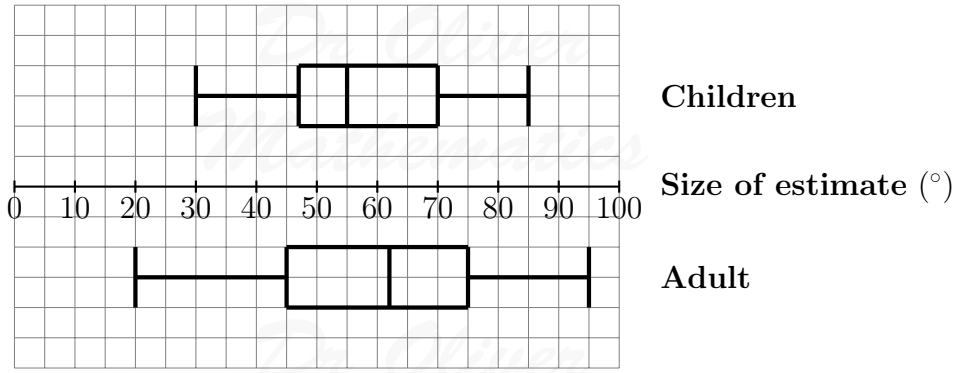
Solution
IQR = 70 − 47 = 23°.

Barry then asked some adults to estimate the size of the angle he had drawn.
The table gives some information about the adults' estimates.

	Angle
Lowest estimate	20°
Lower quartile	45°
Median	62°
Upper quartile	75°
Highest estimate	95°

(c) On the grid above, draw a box plot to show this information. (2)

Solution



(d) Use the two box plots, to compare the distribution of the children's estimates with the distribution of the adults' estimates. (2)

Solution

Average

Since the median for the adults is is more accurate than the median for the children, the adults were more accurate on average.

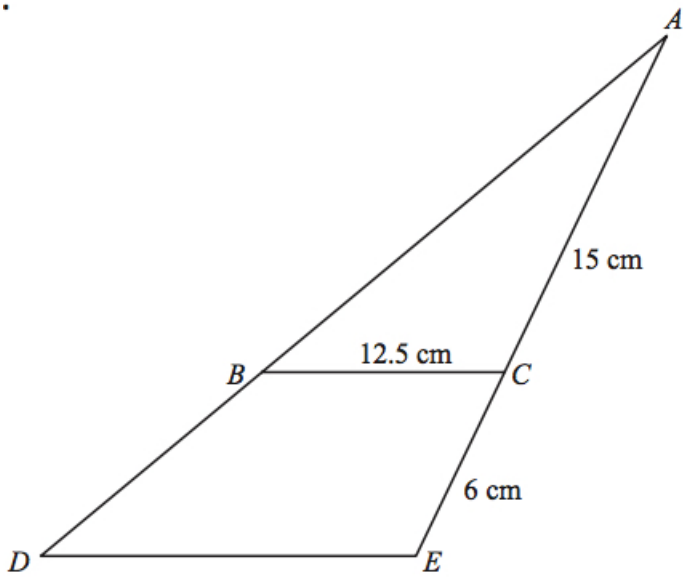
Spread

Since the range for the children $(85 - 30 = 55)$ is smaller than the range for the adults $(95 - 20 = 65)$, the times were more consistent in the children.

OR

Since the IQR for the children (23) is smaller than the range for the adults $(75 - 45 = 30)$, the times were more consistent in in the children.

17. Triangle ABC is similar to triangle ADE. (3)



$AC = 15$ cm.
$CE = 6$ cm.
$BC = 12.5$ cm.
Work out the length of DE.

Solution

$$DE = \left(\frac{15+6}{15}\right) \times 12.5$$
$$= \frac{21}{15} \times 12.5$$
$$= \underline{\underline{17.5 \text{ cm}}}.$$

18. Change 9 cm^2 to mm^2. (2)

Solution

$$\begin{aligned} 9\text{ cm}^2 &= 9 \times 1\text{ cm}^2 \\ &= 9 \times 1\text{ cm} \times 1\text{ cm} \\ &= 9 \times 10\text{ mm} \times 10\text{ mm} \\ &= 9 \times 100\text{ mm}^2 \\ &= \underline{\underline{900\text{ mm}^2}}. \end{aligned}$$

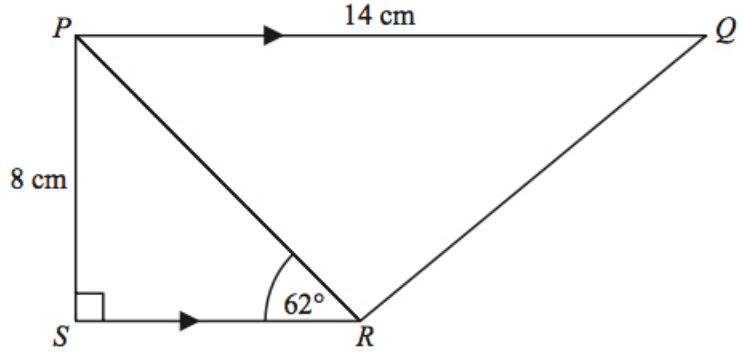
19. Find the exact solutions of (3)

$$x + \frac{3}{x} = 7.$$

Solution

$$\begin{aligned} x + \frac{3}{x} = 7 &\Rightarrow x^2 + 3 = 7x \\ &\Rightarrow x^2 - 7x = -3 \\ &\Rightarrow x^2 - 7x + \left(\tfrac{7}{2}\right)^2 = -3 + \left(\tfrac{7}{2}\right)^2 \\ &\Rightarrow \left(x - \tfrac{7}{2}\right)^2 = \tfrac{37}{4} \\ &\Rightarrow x - \tfrac{7}{2} = \pm\sqrt{\tfrac{37}{4}} \\ &\Rightarrow x - \tfrac{7}{2} = \pm\tfrac{\sqrt{37}}{2} \\ &\Rightarrow \underline{\underline{x = \tfrac{7 \pm \sqrt{37}}{2}}}. \end{aligned}$$

20. $PQRS$ is a trapezium.



PQ is parallel to SR.
Angle $PSR = 90°$.
Angle $PRS = 62°$.
$PQ = 14$ cm.
$PS = 8$ cm.

(a) Work out the length of PR. (3)
Give your answer correct to 3 significant figures.

Solution

$$\text{hyp} = \frac{\text{opp}}{\sin} \Rightarrow PR = \frac{8}{\sin 62°}$$
$$\Rightarrow PR = 9.060\,560\,406 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{PR = 9.06 \text{ cm (3 sf)}}}.$$

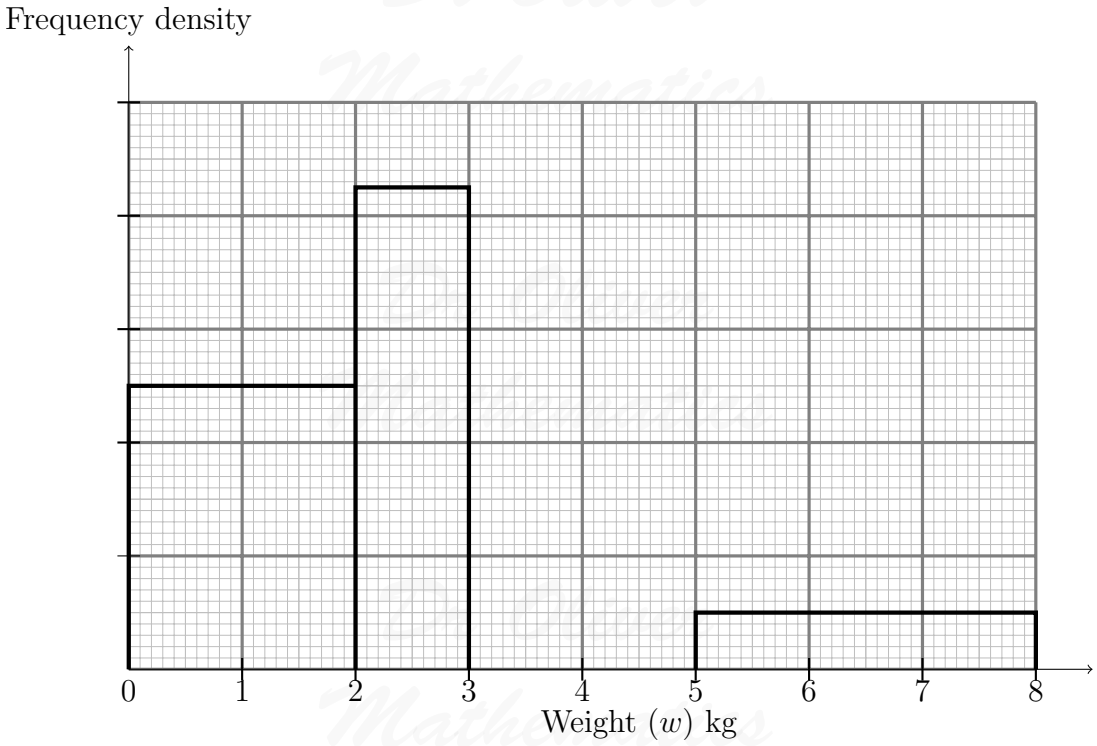
(b) Work out the length of QR. (4)
Give your answer correct to 3 significant figures.

Solution

Now, $\angle QPR = 62°$ (alternate angles) and

$$QR = \sqrt{14^2 + 9.060\ldots^2 - 2 \times 14 \times 9.060\ldots \times \cos 62°}$$
$$= 12.609\,157\,08 \text{ (FCD)}$$
$$= \underline{\underline{12.6 \text{ cm (3 sf)}}}.$$

21. The table and histogram give some information about the weights of parcels received at a post office during one day.



(a) Use the histogram to complete the frequency table. (2)

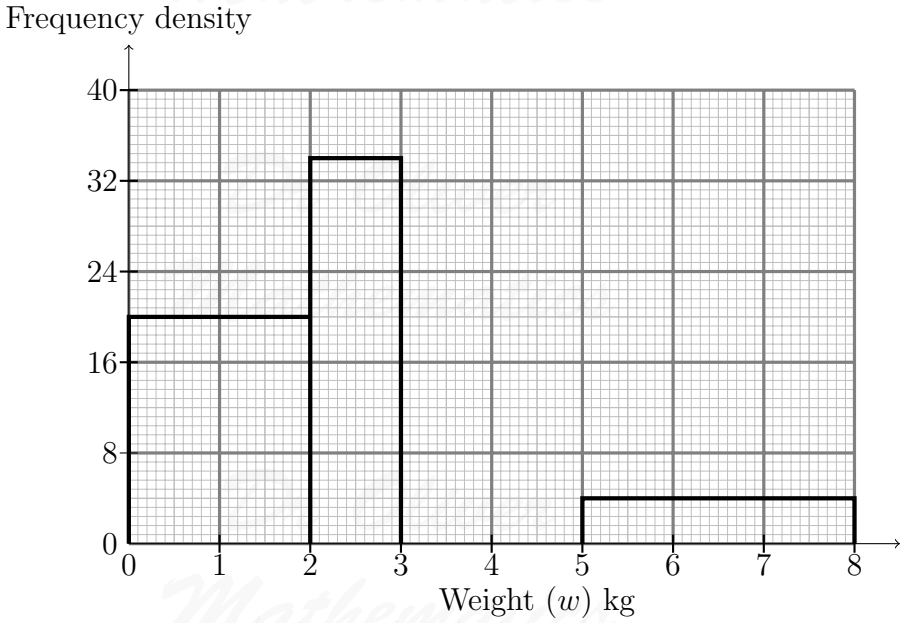
Weight (w) kg	Frequency
$2 < w \leqslant 4$	40
$2 < w \leqslant 3$	
$3 < w \leqslant 4$	24
$4 < w \leqslant 5$	18
$5 < w \leqslant 8$	

Solution

First, we write down the width of the five intervals and we complete, as far we can, the frequency densities.

Weight (w) kg	Frequency	Width	Frequency Density
$2 < w \leqslant 4$	40	2	$\frac{40}{2} = 20$
$2 < w \leqslant 3$		1	
$3 < w \leqslant 4$	24	1	$\frac{24}{1} = 24$
$4 < w \leqslant 5$	18	1	$\frac{18}{1} = 18$
$5 < w \leqslant 8$		3	

This gives us some information about the frequency densities.

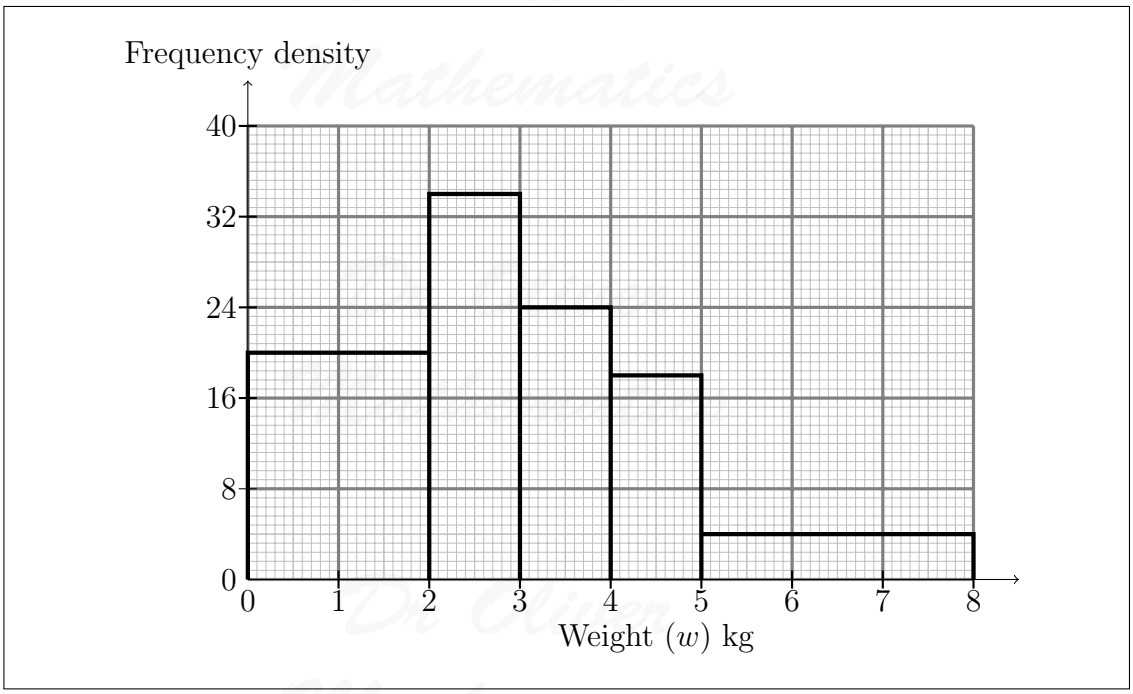


Finally, we complete the frequency table.

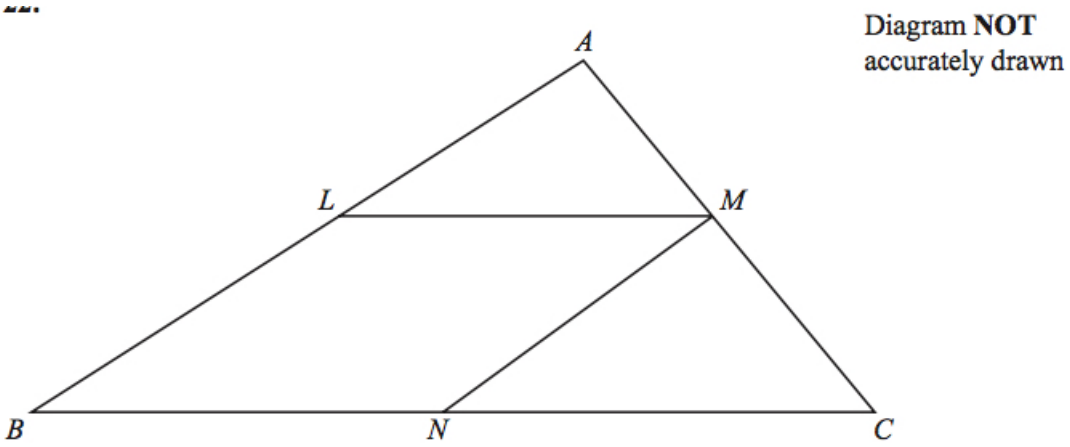
Weight (w) kg	Frequency	Width	Frequency Density
$2 < w \leqslant 4$	40	2	$\frac{40}{2} = 20$
$2 < w \leqslant 3$	$\underline{\underline{34}}$	1	$\frac{34}{1} = 34$
$3 < w \leqslant 4$	24	1	$\frac{24}{1} = 24$
$4 < w \leqslant 5$	18	1	$\frac{18}{1} = 18$
$5 < w \leqslant 8$	$\underline{\underline{12}}$	3	$\frac{12}{3} = 4$

(b) Use the table to complete the histogram. (2)

Solution



22. The diagram shows a triangle ABC. (3)



$LMNB$ is a parallelogram where L is the midpoint of AB.
M is the midpoint of AC.
N is the midpoint of BC.
Prove that triangle ALM and triangle MNC are congruent.
You must give reasons for each stage of your proof.

Solution

$AM = MC$ (M is midpoint)

$AL = LB$ (L is midpoint)
$LB = MN$ (opposite sides of a parallolgram)
So $AL = MN$.

$BN = NC$ (N is midpoint)
$BN = LM$ (opposite sides of a parallolgram)
So $LM = NC$.

Hence, triangle ALM and triangle MNC are congruent (SSS).

23. (a) Factorise (2)

$$x^2 + px + qx + pq.$$

Solution

$$\left.\begin{array}{ll} \text{add to:} & +p+q \\ \text{multiply to:} & +pq \end{array}\right\} +p,\ +q$$

$$\begin{aligned} x^2 + px + qx + pq &= x^2 + (p+q)x + pq \\ &= \underline{\underline{(x+p)(x+q)}}. \end{aligned}$$

(b) Factorise (1)

$$m^2 - 4.$$

Solution

$$\left.\begin{array}{ll} \text{add to:} & 0 \\ \text{multiply to:} & -4 \end{array}\right\} -2,\ +2$$

$$m^2 - 4 = \underline{\underline{(m-2)(m+2)}}.$$

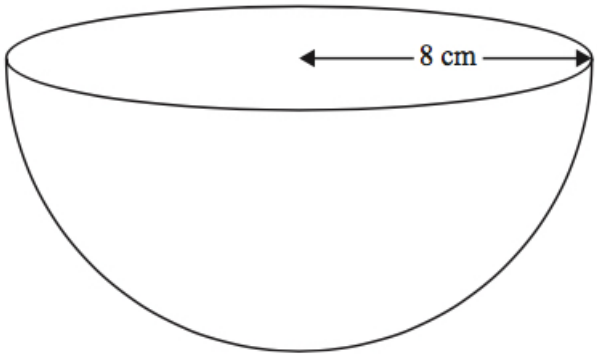
(c) Write as a single fraction in its simplest form (3)

$$\frac{2}{x-4} - \frac{1}{x+3}.$$

Solution

$$\begin{aligned}\frac{2}{x-4}-\frac{1}{x+3} &= \frac{2(x+3)}{(x-4)(x+3)}-\frac{x-4}{(x-4)(x+3)}\\ &= \frac{2(x+3)-(x-4)}{(x-4)(x+3)}\\ &= \frac{2x+6-x+4}{(x-4)(x+3)}\\ &= \underline{\underline{\frac{x+10}{(x-4)(x+3)}}}.\end{aligned}$$

24. The diagram shows a solid hemisphere of radius 8 cm. (3)



Work out the total surface area of the hemisphere.
Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned}\text{Total surface area} &= (2\times\pi\times 8^2)+(\pi\times 8^2)\\ &= 192\pi\\ &= 603.185\,789\,5 \text{ (FCD)}\\ &= \underline{\underline{603 \text{ cm}^2 \text{ (3 sf)}}}.\end{aligned}$$

25. Steve measured the length and the width of a rectangle. (3)
He measured the length to be 645 mm, correct to the nearest 5 mm.
He measured the width to be 400 mm, correct to the nearest 5 mm.
Calculate the lower bound for the area of this rectangle.
Give your answer correct to 3 significant figures.

Solution

$$642.5 \leqslant \text{length} < 647.5$$

and

$$397.5 \leqslant \text{width} < 402.5.$$

Finally, the lower bound for the area of this rectangle is

$$\begin{aligned} 642.5 \times 397.5 &= 255\,393.75 \\ &= \underline{\underline{255\,000 \text{ (3 sf)}}}. \end{aligned}$$