# Dr Oliver Mathematics
# GCSE Mathematics
# 2010 June Paper 3H: Non-Calculator
# 1 hour 45 minutes

The total number of marks available is 100.
You must write down all the stages in your working.

1. Simplify $6x + 9y + 2x - 3y$. (2)

**Solution**

$$6x + 9y + 2x - 3y = \underline{\underline{8x + 6y}}.$$

2. Here are the weights, in grams, of 16 eggs. (3)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 47 | 45 | 50 | 53 | 43 | 61 | 53 | 62 |
| 58 | 56 | 57 | 47 | 55 | 62 | 58 | 58 |

Draw an ordered stem and leaf diagram to show this information.
You must include a key.

**Solution**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 6 | 1 | 2 | 2 | | | | | | |
| 5 | 0 | 3 | 3 | 5 | 6 | 7 | 8 | 8 | 8 |
| 4 | 3 | 5 | 7 | 7 | | | | | |

Key: 4|3 means 43 grams.

3. $PQR$ is a straight line. (4)

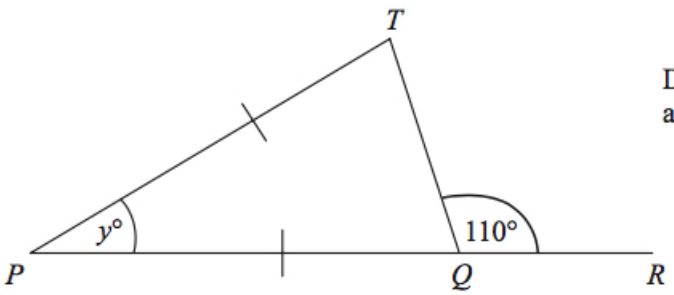



Diagram **NOT** accurately drawn

$PT = PQ$.

(a) Work out the value of $y$.

**Solution**

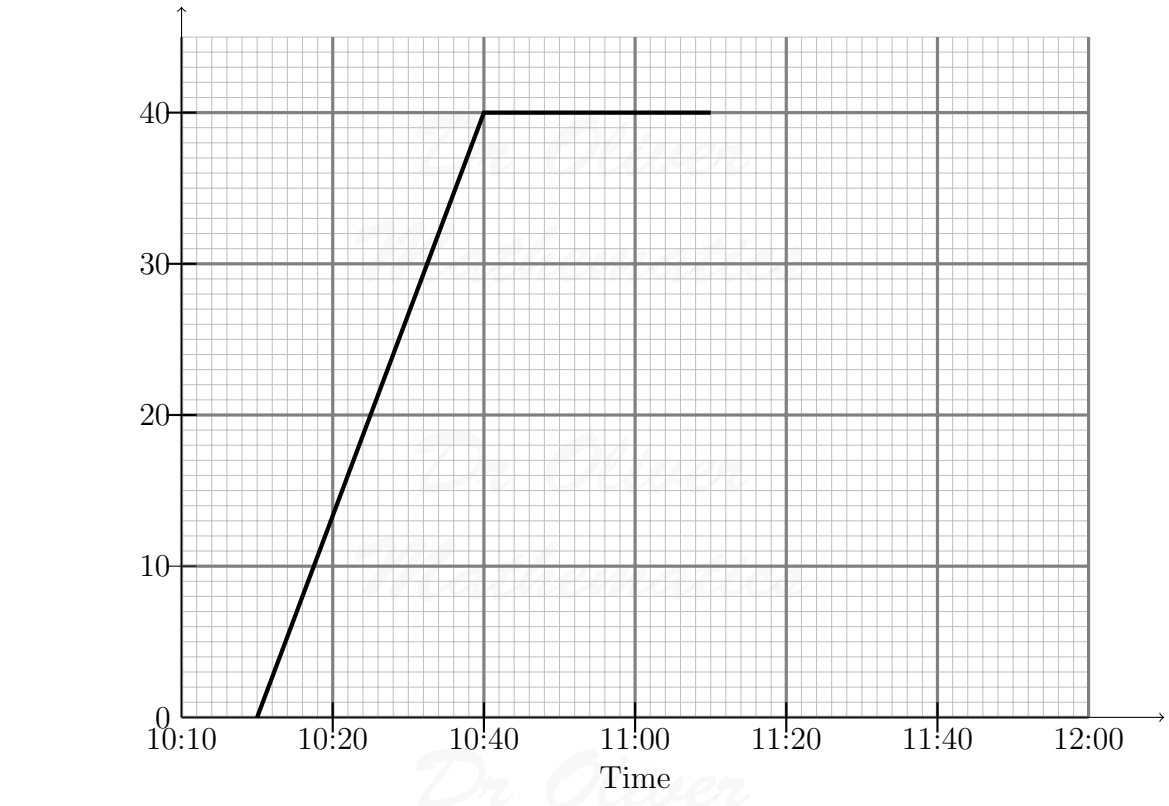
$\angle QPT = \underline{\underline{40^\circ}}$.

(b) Give reasons for your answer.

**Solution**

$\angle PQT = 180 - 110 = 70^\circ$ (angles on a straight line),
$\angle PTQ = 70^\circ$ (base angle in an isosceles triangle), and
$\angle QPT = 180 - 70 - 70 = 40^\circ$.

4. Nigel travelled from his home to his friend?s house 40 km away.
He stayed at his friend's house for 30 minutes.
Nigel then travelled home.
Here is part of the distance-time graph for Nigel's journey.




(a) At what time did Nigel leave home? (1)

**Solution**

10:10.

(b) How far was Nigel from home at 10:20? (1)

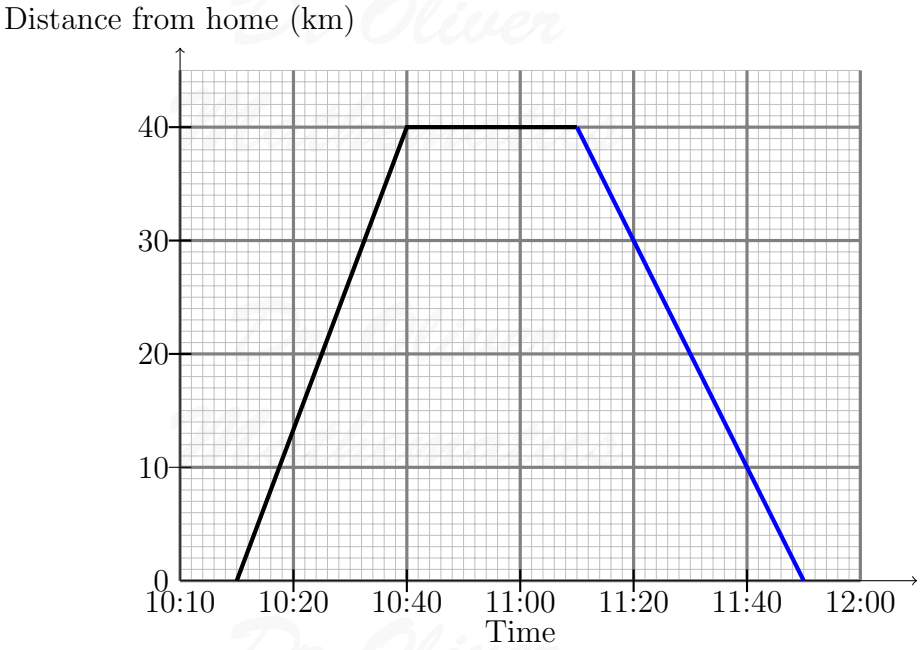
**Solution**

13 miles.

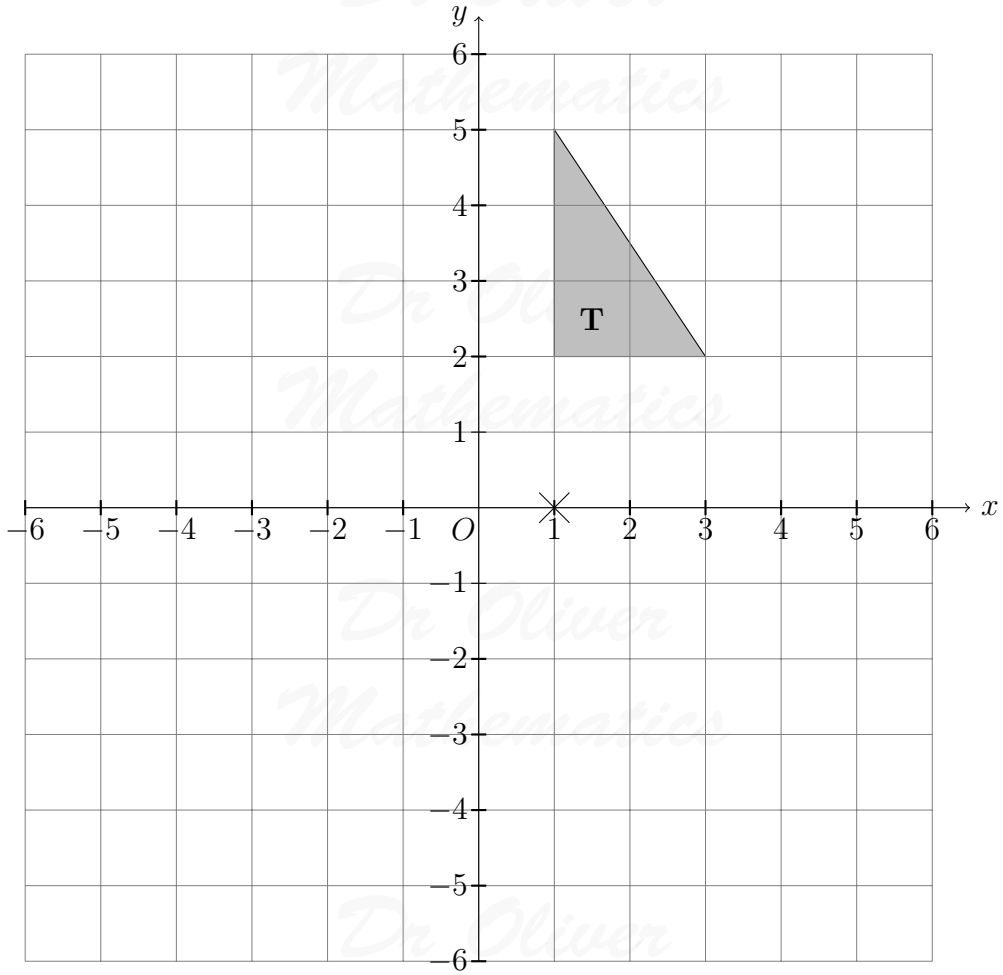
Nigel arrived home at 11:50.

(c) Complete the distance-time graph. (1)

**Solution**

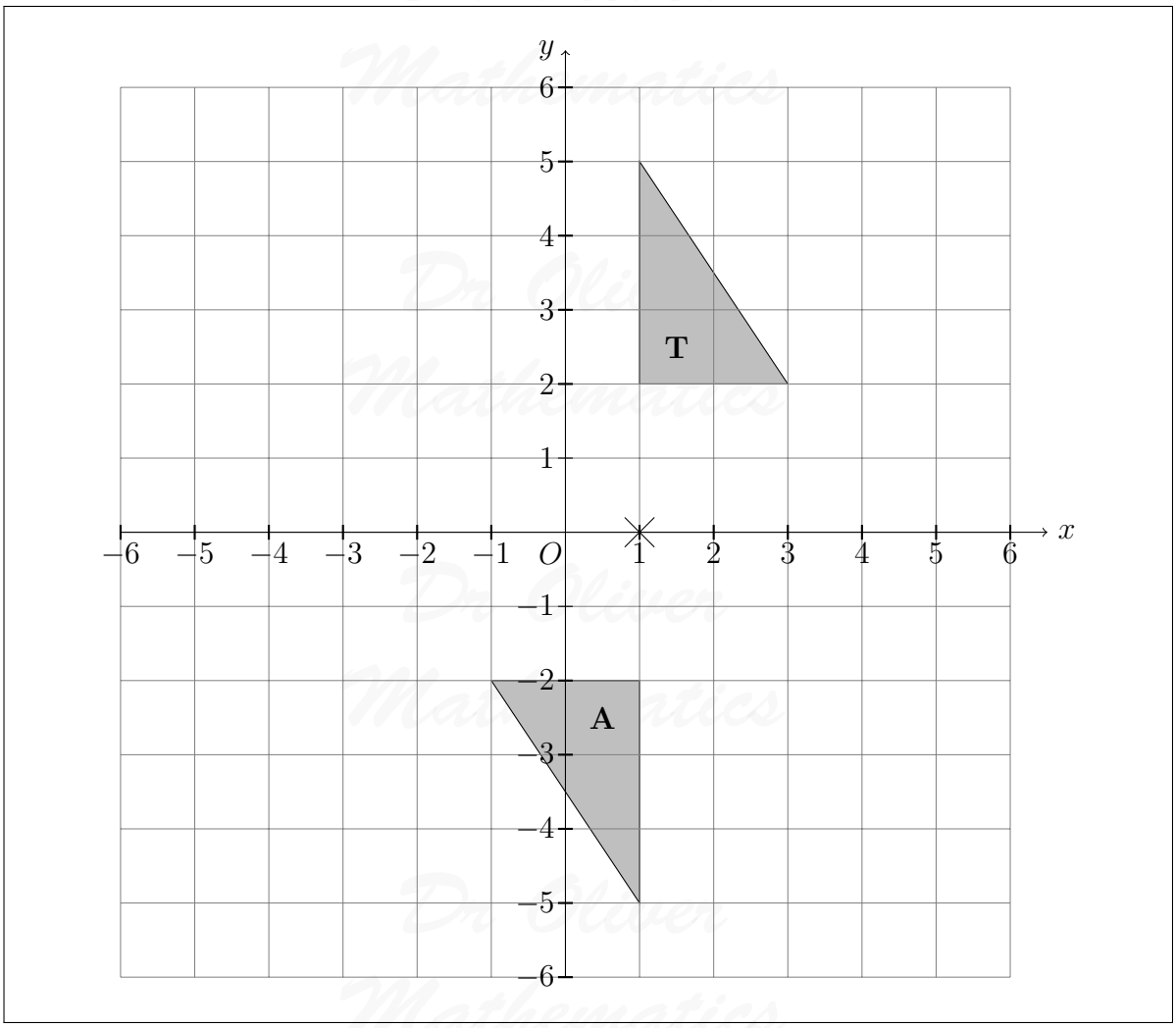



5. Triangle **T** has been drawn on the grid. (5)

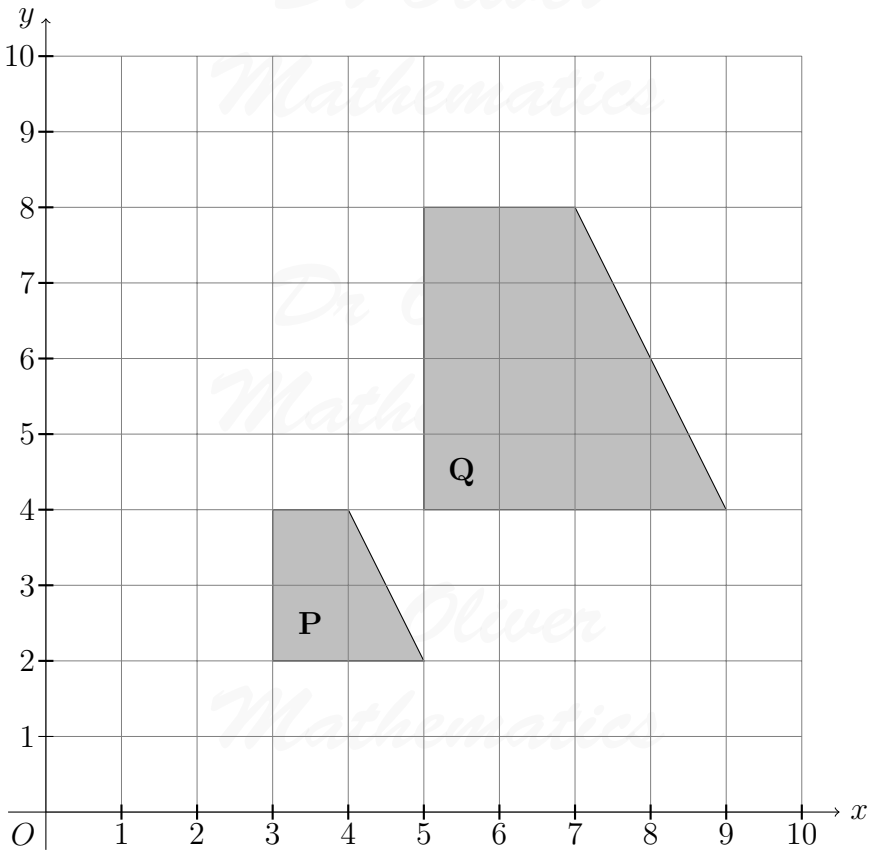



Rotate triangle **T** 180° about the point $(1, 0)$.
Label the new triangle **A**.

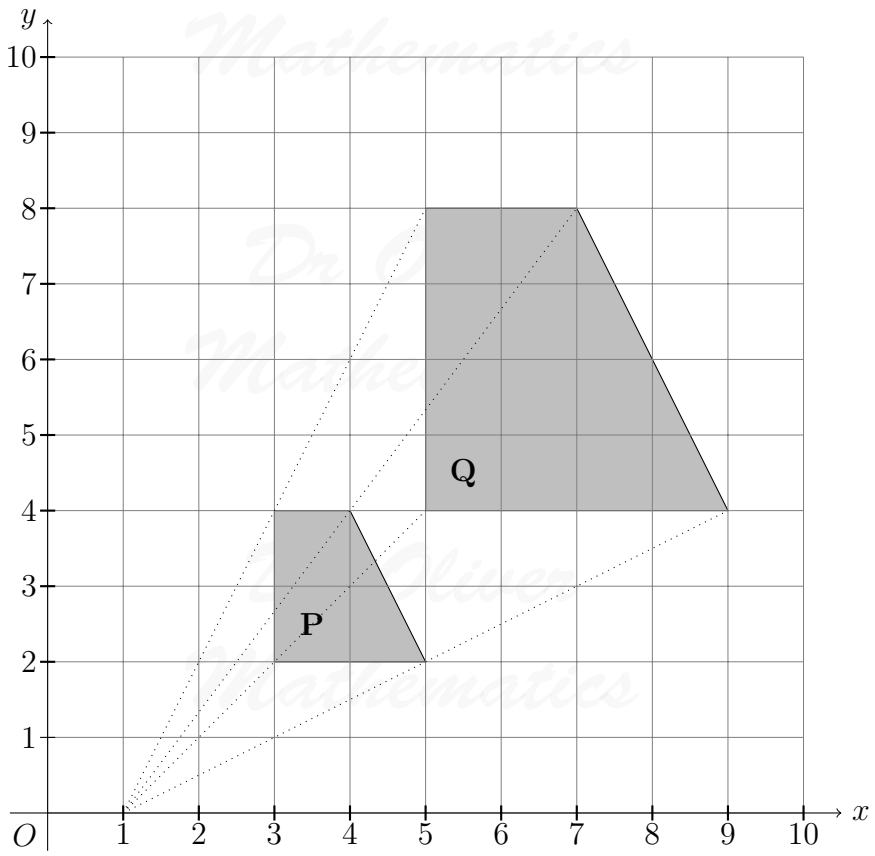
**Solution**




6. Describe fully the single transformation which maps shape **P** onto shape **Q**.




**Solution**




It is an enlargement, scale factor 2, about $(1, 0)$.

7. Anna and Bill share £40 in the ratio 2 : 3. (3)
Work out how much each person gets.

**Solution**

Well,

$$2 + 3 = 5$$

so Anna gets

$$\tfrac{2}{5} \times 40 = 2 \times 8 = £16$$

whilst Bill gets

$$\tfrac{3}{5} \times 40 = 3 \times 8 = £24.$$

8. Sasha carried out a survey of 60 students.
She asked them how many CDs they each have.
This table shows information about the numbers of CDs these students have.

| Number of CDs | $0-4$ | $5-9$ | $10-14$ | $15-19$ | $20-24$ |
|---|---|---|---|---|---|
| Frequency | 8 | 11 | 9 | 14 | 18 |

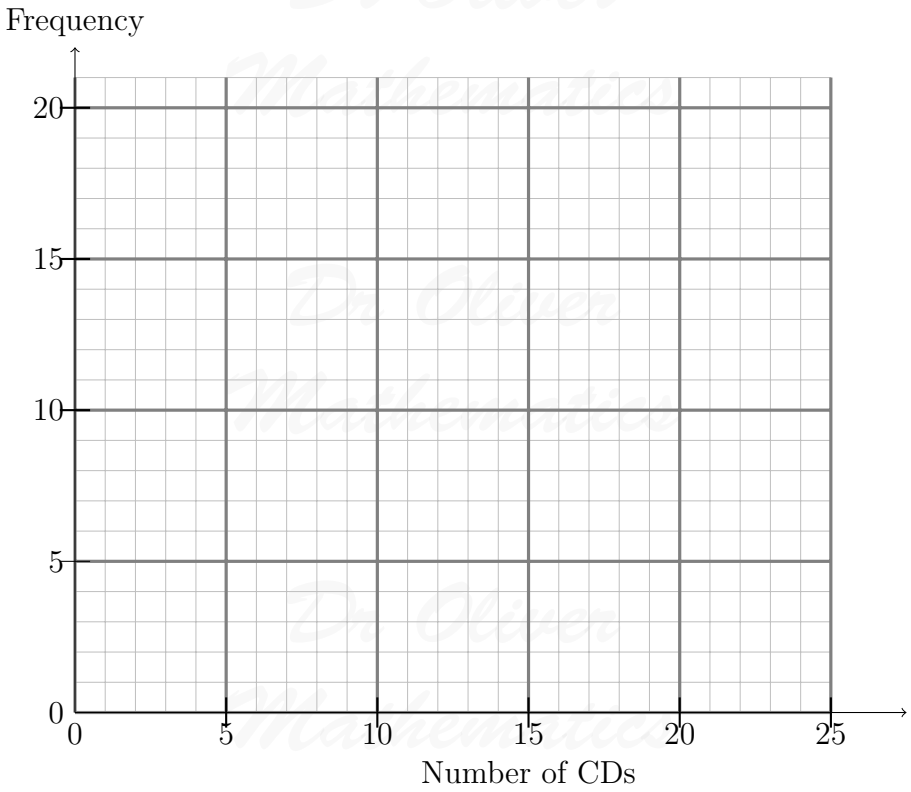
(a) Write down the class interval containing the median. (1)

**Solution**

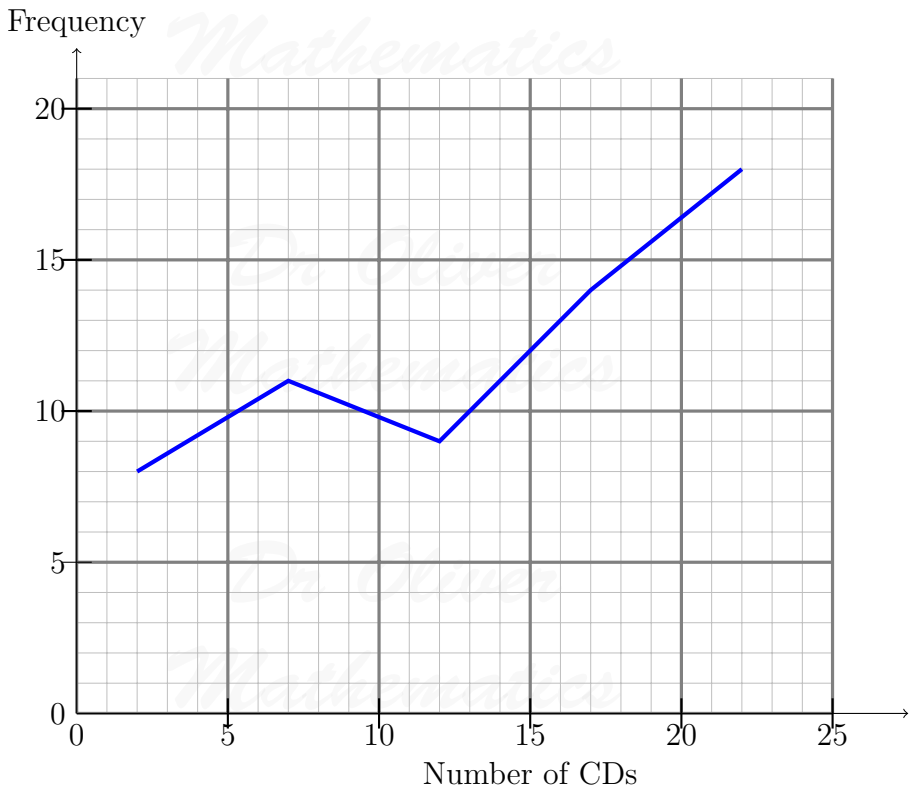
| Number of CDs | Running Total |
|---|---|
| $0-4$ | 8 |
| $0-9$ | $8+11=19$ |
| $0-14$ | $19+9=28$ |
| $0-19$ | $28+14=42$ |
| $0-24$ | $42+18=60$ |

There are 60 students and the class interval containing the median is $\underline{\underline{15-19}}$.

(b) On the grid, draw a frequency polygon to show the information given in the table. (2)




**Solution**




There is no need to do a frequency diagram: just mark in the points ($0-4$ has a midpoint of 2 so plot $(2,8)$), etc.

9. Work out the volume of the triangular prism. (2)

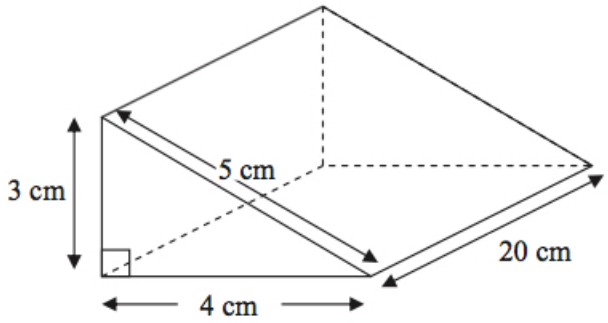



Diagram **NOT** accurately drawn

**Solution**

$$\begin{aligned}\text{Volume} &= \tfrac{1}{2} \times 3 \times 4 \times 20 \\ &= 6 \times 20 \\ &= \underline{\underline{120 \text{ cm}^3}}.\end{aligned}$$

10. Work out $4.52 \times 36$. (3)

**Solution**

| $\times$ | 30 | 6 |
|---|---|---|
| 4 | 120 | 24 |
| 0.5 | 15 | 3 |
| 0.02 | 0.6 | 0.12 |

$$\begin{aligned}4.52 \times 36 &= 120 + 24 + 15 + 3 + 0.6 + 0.12 \\ &= 144 + 18 + 0.72 \\ &= \underline{\underline{162.72}}.\end{aligned}$$

11. There are 300 people in the cinema. (4)
$\frac{1}{6}$ of the 300 people are boys.
$\frac{3}{10}$ of the 300 people are girls.
The rest of the people are adults.
Work out how many people are adults.

**Solution**

The number of boys is

$$\tfrac{1}{6} \times 300 = 50$$

and the number of girls is

$$\tfrac{3}{10} \times 300 = 90.$$

Finally, the number of adults is

$$300 - 50 - 90 = \underline{\underline{160}}.$$

12. Work out the size of an exterior angle of a regular pentagon. (2)

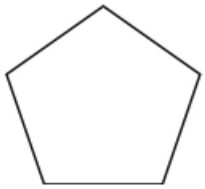


Diagram **NOT** accurately drawn

**Solution**

The exterior angle of a regular pentagon is

$$\frac{360}{5} = \underline{\underline{72^\circ}}.$$

13. Anil wants to find out how many DVDs people buy. (2)
He uses this question on a questionnaire.

How many DVDs do you buy?

☐ $1-5$ ☐ $5-10$ ☐ $10-15$ ☐ $15-20$

Write down **two** different things wrong with this question.

**Solution**

E.g., there is no space to write down 0, there is no space to write down 21 or more, 5 goes into two answer boxes, 10 goes into two answer boxes, 15 goes into two answer boxes, there is no time frame ("last month"), etc.

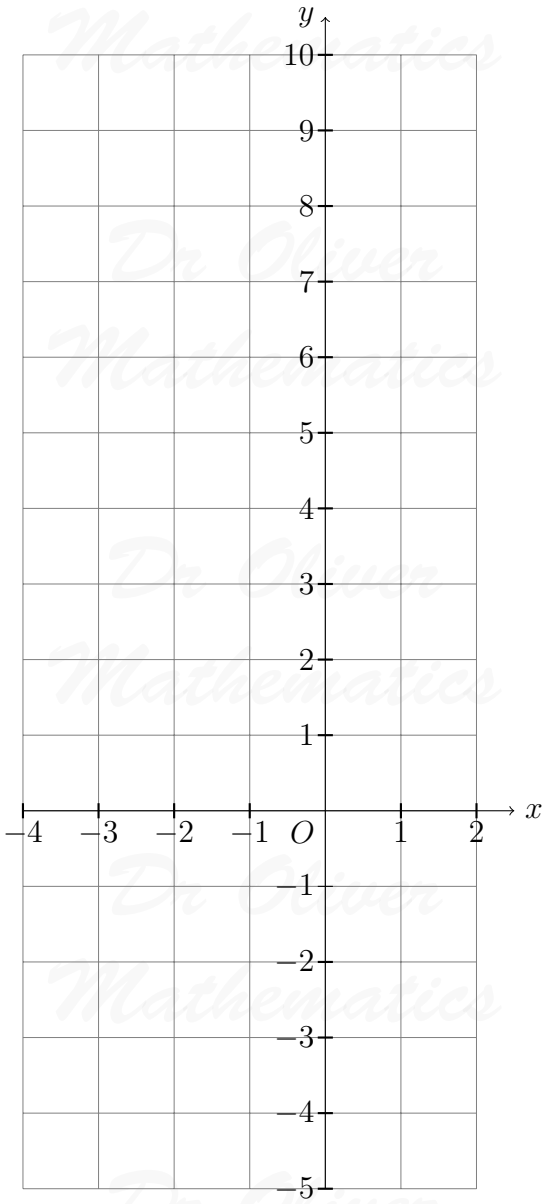
14. (a) Complete the table of values for $y = x^2 + x - 3$. (2)

| $x$ | $-4$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|---|---|
| $y$ | $9$ | | $-1$ | $-3$ | | | $3$ |

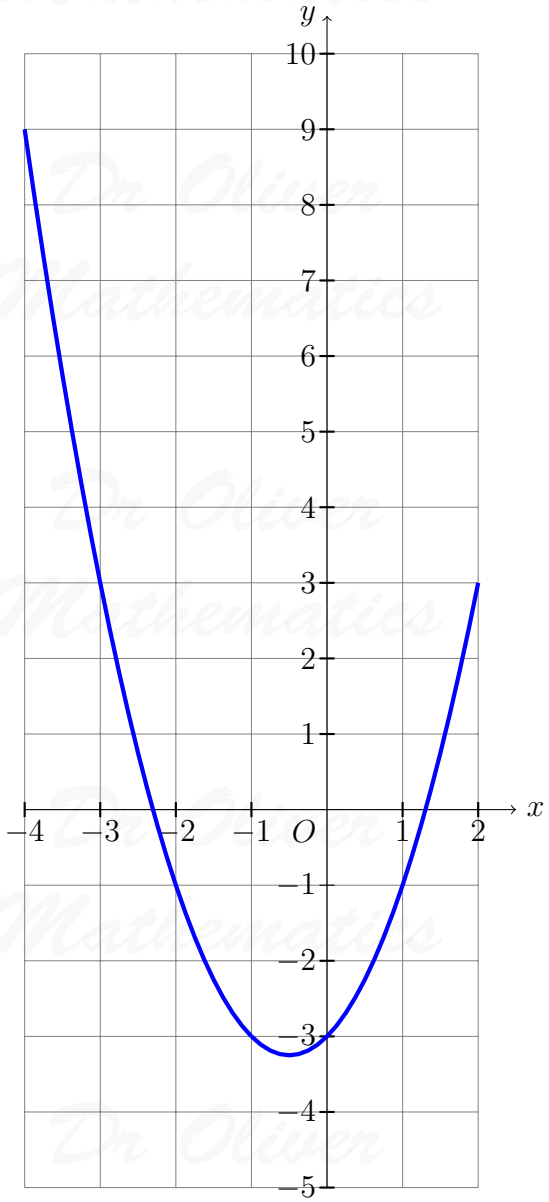
**Solution**

| $x$ | $-4$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|---|---|
| $y$ | $9$ | $\underline{\underline{3}}$ | $-1$ | $-3$ | $\underline{\underline{-3}}$ | $\underline{\underline{-1}}$ | $3$ |

(b) On the grid below, draw the graph of $y = x^2 + x - 3$ for values of $x$ from $-4$ to $2$. (2)



**Solution**




(c) Use your graph to find estimates for the solutions of $y = x^2 + x - 3$. (1)

**Solution**

Correct read-off: approximately $\underline{\underline{x = -2.3}}$ and $\underline{\underline{x = 1.3}}$.

15. Express 180 as a product of its prime factors. (3)

**Solution**

| | |
|---|---|
| | 180 |
| 2 | 90 |
| 2 | 45 |
| 3 | 15 |
| 3 | 5 |
| 5 | 1 |

and so

$$\begin{aligned} 180 &= 2 \times 2 \times 3 \times 3 \times 5 \\ &= \underline{\underline{2^2 \times 3^2 \times 5}}. \end{aligned}$$

16. Work out (3)

$$3\tfrac{1}{4} \times 2\tfrac{2}{3}.$$

Give your answer in its simplest form.

**Solution**

$$\begin{aligned} 3\tfrac{1}{4} \times 2\tfrac{2}{3} &= \frac{13}{4} \times \frac{8}{3} \\ &= \frac{13}{1} \times \frac{2}{3} \\ &= \frac{26}{3} \\ &= \underline{\underline{8\tfrac{2}{3}}}. \end{aligned}$$

17. (a) Factorise $3x + 12$. (1)

**Solution**

$$3x + 12 = \underline{\underline{3(x + 4)}}.$$

(b) Solve $4(2x - 3) = 5x + 7$. (3)

**Solution**

$$\begin{aligned} 4(2x-3) = 5x+7 &\Rightarrow 8x - 12 = 5x + 7 \\ &\Rightarrow 3x = 19 \\ &\Rightarrow \underline{\underline{x = \tfrac{19}{3} \text{ or } 6\tfrac{1}{3}}}. \end{aligned}$$

(c) Expand and simplify $(y+4)(y+5)$. (2)

**Solution**

| $\times$ | $y$ | $+4$ |
|---|---|---|
| $y$ | $y^2$ | $+4$ |
| $+5$ | $+5y$ | $+20$ |

Hence

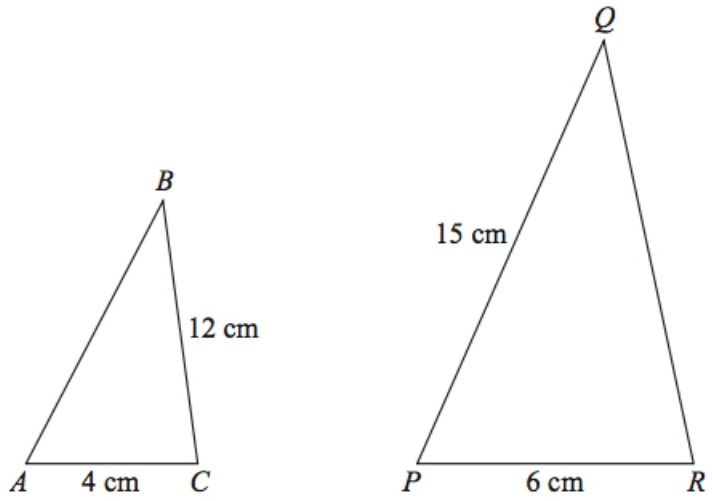
$$(y+4)(y+5) = \underline{\underline{y^2 + 9y + 20}}.$$

(d) Factorise fully $8x^2 + 12xy$. (2)

**Solution**

$$8x^2 + 12xy = \underline{\underline{4x(2x+3y)}}.$$

18. Triangles $ABC$ and $PQR$ are mathematically similar.




Diagrams **NOT** accurately drawn

Angle $A$ = angle $P$.
Angle $B$ = angle $Q$.
Angle $C$ = angle $R$.
$AC = 4$ cm.
$BC = 12$ cm.
$PR = 6$ cm.
$PQ = 15$ cm.

(a) Work out the length of $QR$. (2)

**Solution**

Length scale ratio is $\frac{6}{4} = \frac{3}{2}$ and so

$$QR = \tfrac{3}{2} \times 12 = \underline{\underline{18 \text{ cm}}}.$$

(b) Work out the length of $AB$. (2)

**Solution**

$$AB = \frac{15}{\frac{3}{2}} = 15 \times \tfrac{2}{3} = \underline{\underline{10 \text{ cm}}}.$$

19. Arwen buys a car for £4 000. (3)
The value of the car depreciates by 10% each year.
Work out the value of the car after two years.

**Solution**

$100 - 10 = 90$ and so the value of the car after two years is

$$4\,000 \times 0.9^2 = 4\,000 \times 0.81$$
$$= \underline{\underline{£3\,240}}.$$

20. (a) Here are some expressions. (2)

| $a^3$ | $a^2(c+b)$ | $4abc$ | $ab+c^3$ | $4\pi c^2$ |
|---|---|---|---|---|
| | | | | |

The letters $a$, $b$, and $c$ represent lengths.
$\pi$ and 4 are numbers that have no dimension.
Two of the expressions could represent volumes.
Tick the boxes underneath these two expressions.

**Solution**

| $a^3b$ | $a^2(c+b)$ | $4abc$ | $ab+c^3$ | $4\pi c^2$ |
|---|---|---|---|---|
| | ✓ | ✓ | | |

The volume of this cube is 8 $m^3$.

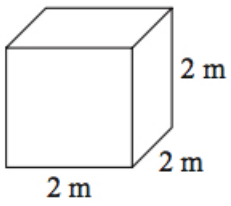



Diagram **NOT** accurately drawn

(b) Change 8 $m^3$ into $cm^3$. (2)

**Solution**

$$\begin{aligned} 8\ \text{m}^3 &= 8 \times 1\ \text{m} \times 1\ \text{m} \times 1\ \text{m} \\ &= 8 \times 100\ \text{cm} \times 100\ \text{cm} \times 100\ \text{cm} \\ &= 8 \times 1\,000\,000\ \text{cm}^3 \\ &= \underline{\underline{8\,000\,000\ \text{cm}^3}}. \end{aligned}$$

21. Solve the simultaneous equations (4)

$$\begin{aligned} 3x + 2y &= 8 \\ 2x + 5y &= -2. \end{aligned}$$

**Solution**

E.g.,

$$\begin{aligned} 3x + 2y &= 8 \quad (1) \\ 2x + 5y &= -2 \quad (2) \end{aligned}$$

Now,

$$2 \times (1) : 6x + 4y = 16 \quad (3)$$

and

$$3 \times (2) : 6x + 15y = -6 \quad (4).$$

Next, do (4) − (3):

$$\begin{aligned} 15y - 4y = -6 - 16 &\Rightarrow 11y = -22 \\ &\Rightarrow y = -2. \end{aligned}$$

Finally, substitute $y = -2$ into equation (1):

$$\begin{aligned} 3x + 2(-2) = 8 &\Rightarrow 3x - 4 = 8 \\ &\Rightarrow 3x = 12 \\ &\Rightarrow x = 4. \end{aligned}$$

Hence,

$$\underline{\underline{x = 4, y = -2}}.$$

22. The table gives some information about the delays, in minutes, of 80 flights.

| Delay ($n$ minutes) | Frequency |
|---|---|
| $0 < n \leqslant 20$ | 16 |
| $20 < n \leqslant 30$ | 26 |
| $30 < n \leqslant 40$ | 23 |
| $40 < n \leqslant 50$ | 10 |
| $50 < n \leqslant 60$ | 5 |

(a) Write down the modal class interval. (1)

**Solution**

$\underline{\underline{20 < n \leqslant 30}}$.

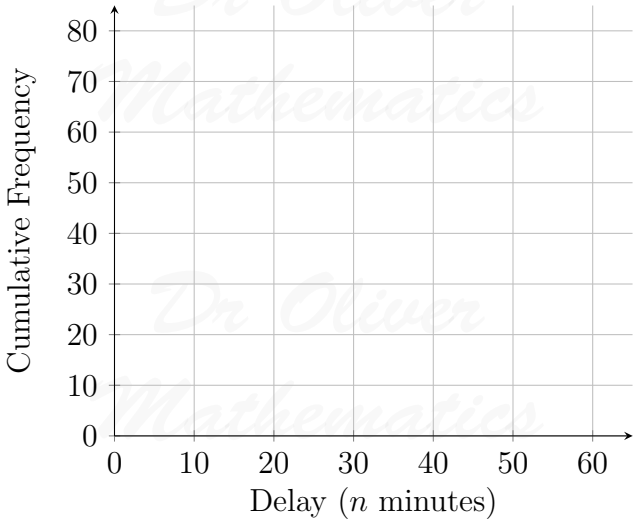
(b) Complete the cumulative frequency table. (1)

| Delay ($n$ minutes) | Cumulative Frequency |
|---|---|
| $0 < n \leqslant 20$ | 16 |
| $0 < n \leqslant 30$ | |
| $0 < n \leqslant 40$ | |
| $0 < n \leqslant 50$ | |
| $0 < n \leqslant 60$ | |

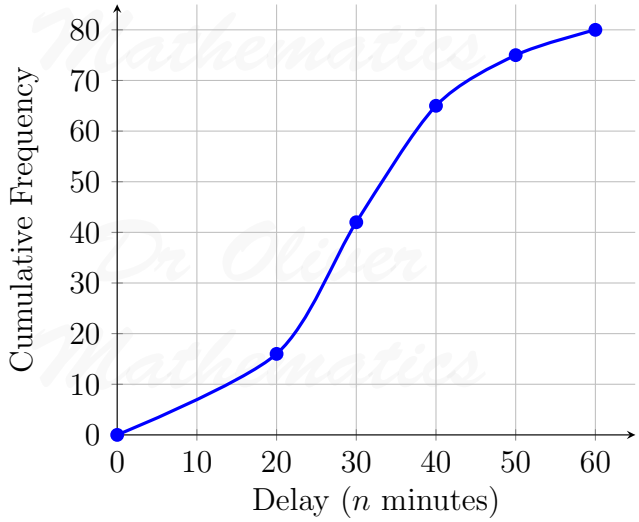
**Solution**

| Delay ($n$ minutes) | Cumulative Frequency |
|---|---|
| $0 < n \leqslant 20$ | $\underline{\underline{16}}$ |
| $0 < n \leqslant 30$ | $16 + 26 = \underline{\underline{42}}$ |
| $0 < n \leqslant 40$ | $42 + 23 = \underline{\underline{65}}$ |
| $0 < n \leqslant 50$ | $65 + 10 = \underline{\underline{75}}$ |
| $0 < n \leqslant 60$ | $75 + 5 = \underline{\underline{80}}$ |

(c) Draw a cumulative frequency graph for your table. (2)



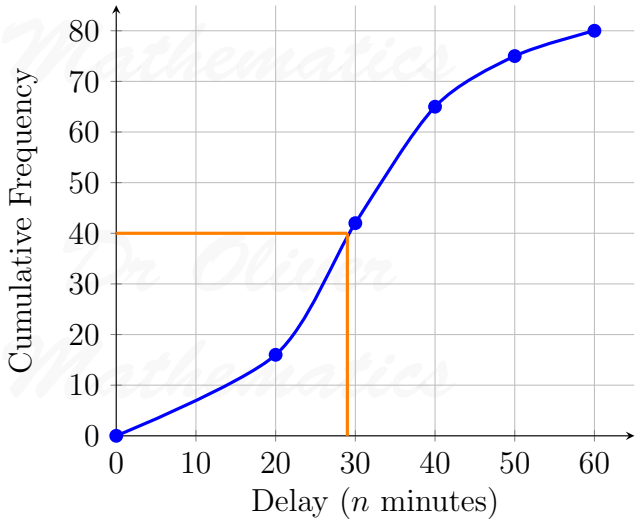

**Solution**




(d) Use your graph to find an estimate for (3)

(i) the median delay,

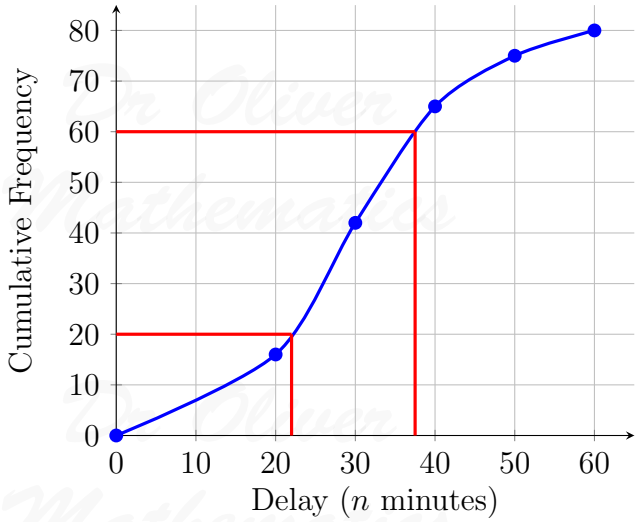
**Solution**




The median delay is 29 minutes.

(ii) the interquartile range of the delays.

**Solution**




The interquartile range of the delays is

$$37.5 - 22 = 15.5 \text{ minutes}.$$

23. A straight line passes through $(0, -2)$ and $(3, 10)$. (3)

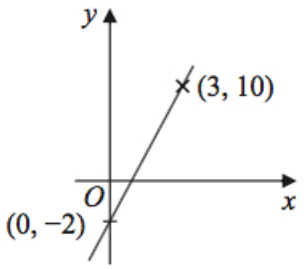



Diagram **NOT** accurately drawn

Find the equation of the straight line.

**Solution**

$$\text{Gradient} = \frac{10-(2)}{3-0} = \frac{12}{3} = 4$$

and the equation of the straight line is

$$\underline{\underline{y = 4x - 2}}.$$

24. Find the value of (4)

(a) $6^0$,

**Solution**

$$6^0 = \underline{\underline{1}}.$$

(b) $64^{\frac{1}{2}}$,

**Solution**

$$64^{\frac{1}{2}} = \sqrt{64} = \underline{\underline{8}}.$$

(c) $\left(\frac{27}{8}\right)^{-\frac{2}{3}}$.

**Solution**

$$\left(\frac{27}{8}\right)^{-\frac{2}{3}} = \left(\frac{8}{27}\right)^{\frac{2}{3}}$$
$$= \left(\frac{2}{3}\right)^{2}$$
$$= \underline{\underline{\frac{4}{9}}}.$$

25. $ABC$ is a right-angled triangle.
All the measurements are in centimetres.

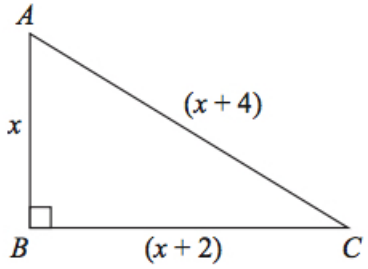



Diagram **NOT** accurately drawn

$AB = x$.
$BC = (x + 2)$.
$AC = (x + 4)$.

(a) Show that $x^2 - 4x - 12 = 0$. (3)

**Solution**

$$AC^2 = AB^2 + BC^2 \Rightarrow (x+4)^2 = (x+2)^2 + x^2$$
$$\Rightarrow x^2 + 8x + 16 = (x^2 + 4x + 4) + x^2$$
$$\Rightarrow \underline{\underline{x^2 - 4x - 12 = 0}},$$

as required.

(b) (i) Solve $x^2 - 4x - 12 = 0$. (4)

**Solution**

$$x^2 - 4x - 12 = 0 \Rightarrow (x-6)(x+2) = 0$$
$$\Rightarrow \underline{\underline{x = -2 \text{ or } x = 6}}.$$

(ii) Hence, write down the length of $AC$.

**Solution**

$$AC = 6 + 4 = \underline{\underline{10 \text{ cm}}}.$$

26. There are 3 orange sweets, 2 red sweets, and 5 yellow sweets in a bag. (4)
Sarah takes a sweet at random.
She eats the sweet.
She then takes another sweet at random.
Work out the probability that both the sweets are the same colour.

**Solution**

$$\begin{aligned} \text{P(same colour)} &= \text{P}(OO) + \text{P}(RR) + \text{P}(YY) \\ &= \left(\tfrac{3}{10} \times \tfrac{2}{9}\right) + \left(\tfrac{2}{10} \times \tfrac{1}{9}\right) + \left(\tfrac{5}{10} \times \tfrac{4}{9}\right) \\ &= \tfrac{6}{90} + \tfrac{2}{90} + \tfrac{20}{90} \\ &= \underline{\underline{\tfrac{28}{90}}}. \end{aligned}$$

27. $P$, $Q$, and $T$ are points on the circumference of a circle, centre $O$. (5)

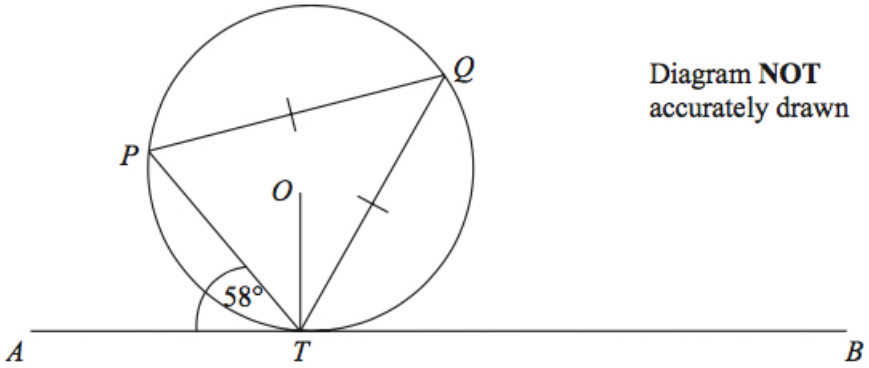



Diagram **NOT** accurately drawn

The line $ATB$ is the tangent at $T$ to the circle.
$PQ = TQ$.

Angle $ATP = 58^\circ$.
Calculate the size of angle $OTQ$.
Give a reason for each stage in your working.

**Solution**

Angle $PQT = 58^\circ$ (alternate segment theroem).
Angle $QTP = \frac{1}{2}(180 - 58) = 61^\circ$ (base angles in a triangle).
Angle $OTP = 90 - 58 = 32^\circ$ (complementary angles).
Angle $OTQ = 61 - 32 = \underline{\underline{29^\circ}}$ (angle between tangent and radius).