Dr Oliver Mathematics
OCR FMSQ Additional Mathematics
2009 Paper
2 hours

The total number of marks available is 100.
You must write down all the stages in your working.
You are permitted to use a scientific or graphical calculator in this paper.
Final answers should be given correct to three significant figures where appropriate.

Section A

1. The angle θ is greater than 90° and less than 360° and $\cos\theta = \frac{2}{3}$. (3)

   Find the exact value of $\tan\theta$.

2. Find the equation of the normal to the curve (5)

$$y = x^3 + 5x - 7$$

   at the point $(1, -1)$.

3. A is the point $(1, 5)$ and C is the point $(3, p)$.

   (a) Find the equation of the line through A which is parallel to the line (2)

$$2x + 5y = 7.$$

   This line also passes through the point C.

   (b) Find the value of p. (2)

4. AB is a diameter of a circle, where A is $(1, 1)$ and B is $(5, 3)$.

   Find

   (a) the exact length of AB, (2)

   (b) the coordinates of the midpoint of AB, (1)

   (c) the equation of the circle. (3)

5. Parcels slide down a ramp. Due to resistance, the deceleration is 0.25 ms^{-2}.

   One parcel is given an initial velocity of 2 ms^{-1}.

(a) Find the distance travelled before the parcel comes to rest. (3)

A second parcel is given an initial velocity of 3 ms^{-1} and takes 4 seconds to reach the bottom of the ramp.

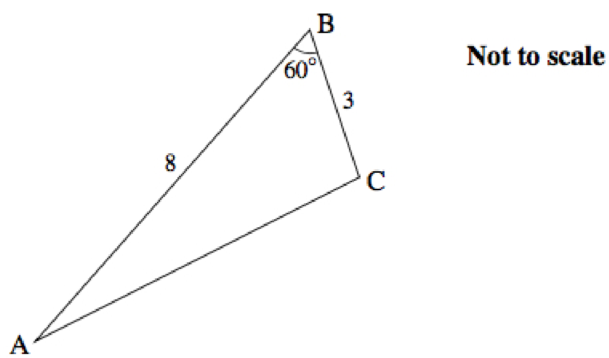
(b) Find the length of the ramp. (3)

6. The gradient function of a curve is given by (4)

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 1 - 4x + 3x^2.$$

Find the equation of the curve given that it passes through the point $(2, 6)$.

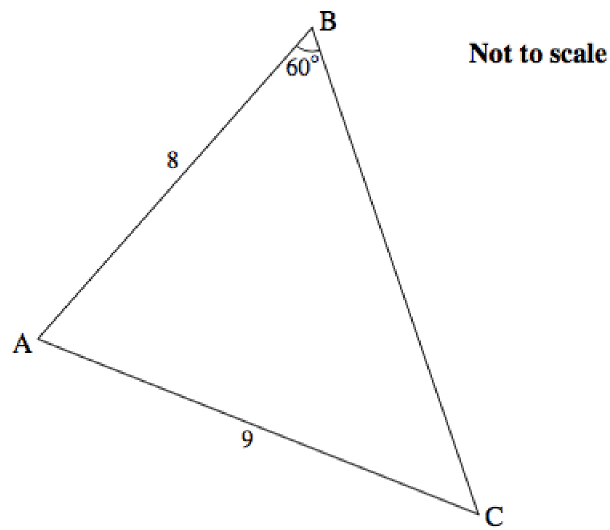
7. The course of a cross-country race is in the shape of a triangle ABC.



$AB = 8$ km, $BC = 3$ km, and angle $ABC = 60°$.

(a) Calculate the distance AC and hence the total length of the course. (4)

The organisers extend the course so that $AC = 9$ km.



(b) Calculate the angle BCA. (3)

8. Calculate the x-coordinates of the points of intersection of the line (5)

$$y = 2x + 11$$

and the curve

$$y = x^2 - x + 5.$$

Give your answers correct to 2 decimal places.

9. A car accelerates from rest. At time t seconds, its acceleration is given by

$$a = (4 - 0.2t) \text{ ms}^{-2}$$

until $t = 20$.

(a) Find the velocity after 5 seconds. (3)

(b) What is happening to the velocity at $t = 20$? (1)

(c) Find the distance travelled in the first 20 seconds. (3)

10. (a) Illustrate on one graph the following three inequalities. (4)

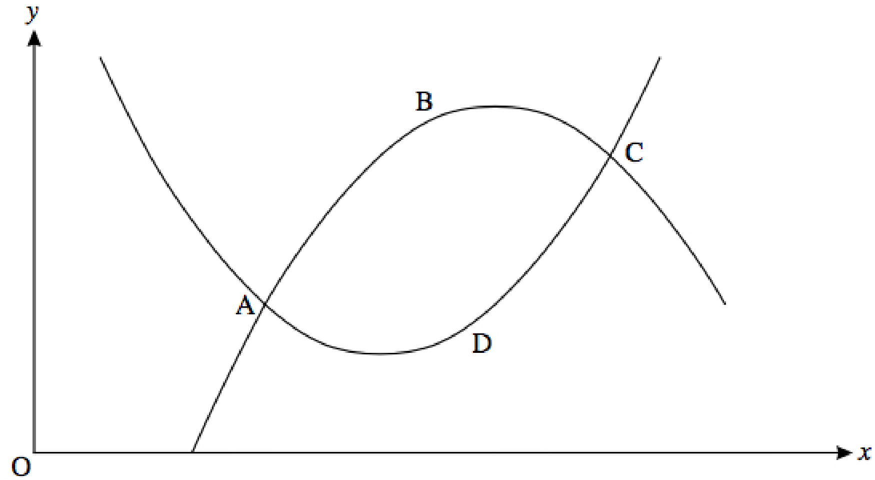
$$y \geqslant x - 1$$
$$x \geqslant 2$$
$$2x + y \geqslant 8.$$

Draw suitable boundaries and shade areas that are **excluded**.

(b) Write down the minimum value of y in this region. (1)

Section B

11. The shape $ABCD$ below represents a leaf.
The curve ABC has equation
$$y = -x^2 + 8x - 9.$$
The curve ADC has equation
$$y = x^2 - 6x + 11.$$

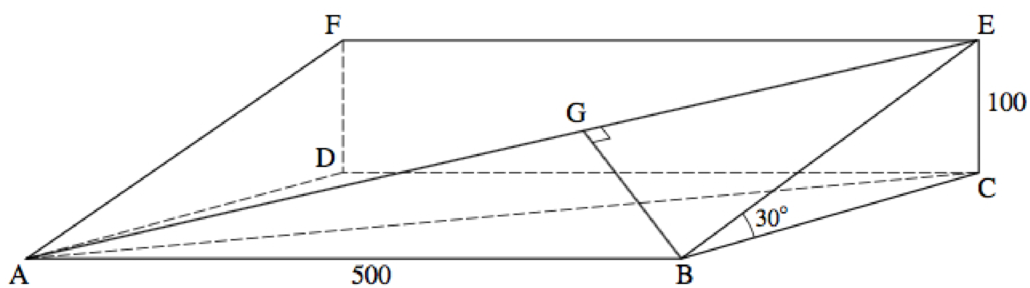


(a) Find algebraically the coordinates of A and C, the points where the curves intersect. (5)

(b) Find the area of the leaf. (7)

12. The diagram shows a rectangle $ABEF$ on a plane hillside which slopes at an angle of 30° to the horizontal. $ABCD$ is a horizontal rectangle. E and F are 100 m vertically above C and D respectively. $AB = DC = FE = 500$ m.

AE is a straight path.
From B there is a straight path which runs at right angles to AE, meeting it at G.



(a) Find the distance BE. (3)

(b) Find the angle that the path AE makes with the horizontal. (4)

(c) Find the area of the triangle ABE and hence find the length BG. (5)

13. In a supermarket chain there are a large number of employees, of whom 40% are male.

One employee is chosen to undergo training.

(a) What assumption is made if 0.4 is taken to be the probability that this employee is male? (1)

6 employees are chosen at random to undergo training.

(b) (i) Show that (2)

$$\text{P(all 6 chosen are female)} = 0.0467,$$

correct to 4 decimal places.

Find the probability that

(ii) 3 are male and 3 are female, (4)

(iii) there are more females than males chosen. (5)

14. (a) (i) On the same graph, draw sketches of the curve (2)

$$y = x^3 \text{ and the line } y = 3 - 2x.$$

(ii) Use your sketch to explain why the equation (1)

$$x^3 + 2x - 3 = 0$$

has only one root.

(b) (i) Show by differentiation that there are no stationary points on the curve (3)

$$y = x^3 + 3x - 4.$$

(ii) Hence explain why the equation (1)

$$x^3 + 3x - 4 = 0$$

has only one root.

(c) (i) Use the factor theorem to find an integer root of the equation (1)

$$x^3 + x - 10 = 0.$$

(ii) Write the equation (2)

$$x^3 + x - 10 = 0$$

in the form

$$(x - a)(x^2 + px + q) = 0$$

where a, p, and q are values to be determined.

(iii) By considering the quadratic equation (1)

$$x^2 + px + q = 0$$

found in part (ii), show that the cubic equation

$$x^3 + x - 10 = 0$$

has only one root.

You are given that r and s are positive numbers.

(d) What do the results in parts (a), (b) and (c) suggest about the equation (1)

$$x^3 + rx - s = 0?$$