Dr Oliver Mathematics
Cambridge O Level Additional Mathematics
2007 June Paper 2: Calculator
2 hours

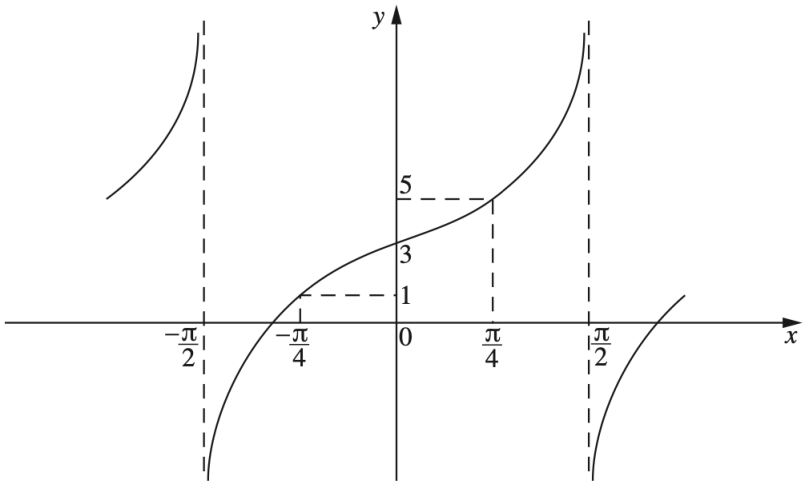
The total number of marks available is 80.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You must write down all the stages in your working.

1. A triangle has a base of length $(13 - 2x)$ m and a perpendicular height of x m. (3)

   Calculate the range of values of x for which the area of the triangle is greater than 3 m^2.

2. The diagram shows part of the graph of

$$y = a\tan(bx) + c.$$



   Find the value of

   (a) c, (1)

   (b) b, (1)

   (c) a. (1)

3. The roots of the equation (5)

$$x^2 - \sqrt{28}x + 2 = 0$$

are p and q, where $p > q$.

Without using a calculator, express $\dfrac{p}{q}$ in the form $m + \sqrt{n}$, where m and n are integers.

4. An artist has 6 watercolour paintings and 4 oil paintings.

She wishes to select 4 of these 10 paintings for an exhibition.

(a) Find the number of different selections she can make. (2)

(b) In how many of these selections will there be more watercolour paintings than oil paintings? (3)

5. (a) Express (1)

$$\frac{1}{\sqrt{32}}$$

as a power of 2.

(b) Express (1)

$$(64)^{\frac{1}{x}}$$

as a power of 2.

(c) Hence solve the equation (3)

$$\frac{(64)^{\frac{1}{x}}}{2^x} = \frac{1}{\sqrt{32}}.$$

6. (a) Differentiate (2)

$$x^2 \ln x$$

with respect to x.

(b) Use your result to show that (4)

$$\int_1^{\mathrm{e}} 4x \ln x \, \mathrm{d}x = \mathrm{e}^2 + 1.$$

7. Given that

$$\mathbf{A} = \begin{pmatrix} 2 & 3 \\ -2 & -1 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 8 & 10 \\ -4 & 2 \end{pmatrix},$$

find the matrices $\mathbf{X}$ and $\mathbf{Y}$ such that

(a) $\mathbf{X} = \mathbf{A}^2 + 2\mathbf{B}$, (3)

(b) $\mathbf{YA} = \mathbf{B}$. (4)

8. The equation of the curve C is

$$2y = x^2 + 4.$$

The equation of the line L is

$$y = 3x - k,$$

where k is an integer.

(a) Find the largest value of the integer k for which L intersects C. (4)

(b) In the case where $k = -2$, show that the line joining the points of intersection of L and C is bisected by the line $y = 2x + 5$. (4)

9. The position vectors, relative to an origin O, of three points P, Q, and R are $\mathbf{i} + 3\mathbf{j}$, $5\mathbf{i} + 11\mathbf{j}$, and $9\mathbf{i} + 9\mathbf{j}$ respectively.

(a) By finding the magnitude of the vectors $\overrightarrow{PR}$, $\overrightarrow{RQ}$, and $\overrightarrow{QP}$, show that angle PQR is 90°. (4)

(b) Find the unit vector parallel to $\overrightarrow{PR}$. (2)

(c) Given that (3)

$$\overrightarrow{OQ} = m\overrightarrow{OP} + n\overrightarrow{OR},$$

where m and n are constants, find the value of m and of n.

10. The functions f and g are defined, for $x \in \mathbb{R}$, by

$$\text{f} : x \mapsto 3x - 2,$$
$$\text{g} : x \mapsto \frac{7x - a}{x + 1}, \text{ where } x \neq -1 \text{ and } a \text{ is a positive constant.}$$

(a) Obtain expressions for f^{-1} and g^{-1}. (3)

(b) Determine the value of a for which (3)

$$\text{f}^{-1}\,\text{g}(4) = 2.$$

(c) If $a = 9$, show that there is only one value of x for which (3)

$$\text{g}(x) = \text{g}^{-1}(x).$$

11. A particle, moving in a straight line, passes through a fixed point O with velocity $14\ \text{ms}^{-1}$.

The acceleration, $a\ \text{ms}^{-2}$ of the particle, t seconds after passing through O, is given by particle by

$$a = 2t - 9.$$

The particle subsequently comes to instantaneous rest, firstly at A and later at B.

Find

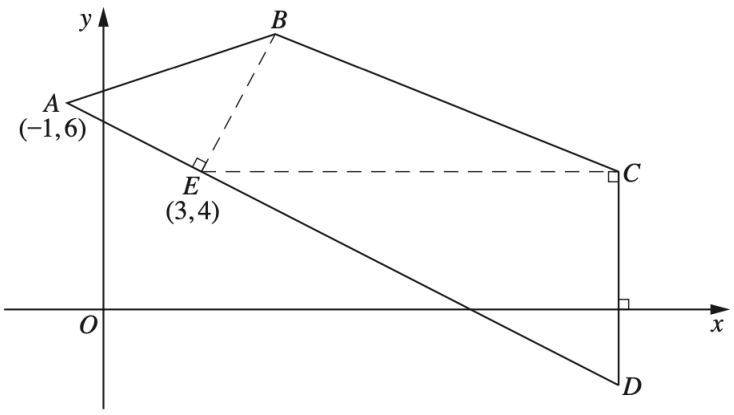
(a) the acceleration of the particle at A and at B, (4)

(b) the greatest speed of the particle as it travels from A to B, (2)

(c) the distance AB. (4)

EITHER

12. The diagram shows a quadrilateral $ABCD$.



- The point E lies on AD such that angle $AEB = 90°$.
- The line EC is parallel to the x-axis and the line CD is parallel to the y-axis.
- The points A and E are $(-1, 6)$ and $(3, 4)$ respectively.

Given that the gradient of AB is $\frac{1}{3}$,

(a) find the coordinates of B. (5)

Given also that the area of triangle EBC is 24 units^2,

(b) find the coordinates of C, (3)

(c) find the coordinates of D. (2)

OR

13. (a) The expression

$$\mathrm{f}(x) = x^3 + ax^2 + bx + c$$

leaves the same remainder, R, when it is divided by $(x + 2)$ and when it is divided by $(x - 2)$.

(i) Evaluate b. (2)

$\mathrm{f}(x)$ also leaves the same remainder, R, when divided by $(x - 1)$.

(ii) Evaluate a. (2)

$\mathrm{f}(x)$ leaves a remainder of 4 when divided by $(x - 3)$.

(iii) Evaluate c. (1)

(b) Solve the equation (5)

$$x^3 + 3x^2 = 2,$$

giving your answers to 2 decimal places where necessary.