Dr Oliver Mathematics
AQA Further Maths Level 2
June 2015 Paper 1
1 hour 30 minutes

The total number of marks available is 70.
You must write down all the stages in your working.
You are **not** permitted to use a scientific or graphical calculator in this paper.

1. GH is a straight line. (2)

   The coordinates of G are $(-2, 8)$.
   The midpoint of GH is $(5, -3)$.

   Work out the coordinates of H.

2. A straight line with equation $y = mx + c$ has gradient m and y-intercept c.
   Here are the equations of four straight lines: P, Q, R, and S.

$$\begin{aligned} P: &\quad 2y - 4x = 5 \\ Q: &\quad 5y = 2x - 4 \\ R: &\quad 2y - 4 = 5x \\ S: &\quad 4y = 5 - 2x. \end{aligned}$$

   (a) Circle the line that passes through $(7, 2)$. (1)

   $P \quad Q \quad R \quad S$

   (b) Circle the line with gradient $2\frac{1}{2}$. (1)

   $P \quad Q \quad R \quad S$

   (c) Circle the line with y-intercept $2\frac{1}{2}$. (1)

   $P \quad Q \quad R \quad S$

   (d) Circle the line with a negative gradient. (1)

   $P \quad Q \quad R \quad S$

   (e) Circle a pair of perpendicular lines. (1)

   $P \quad Q \quad R \quad S$

3. Solve (2)

$$2(3x + 1) > 3 - 4x.$$

4. The equation of a curve is

$$y = x^2 - 5x.$$

(a) Work out $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. (2)

P is a point on the curve.
The tangent to the curve at P has gradient 1.

(b) Work out the coordinates of P. (2)

5. In the expansion of

$$(x + 2)(x^2 + kx - 3)$$

the coefficient of x^2 is zero.

(a) Work out the value of k. (1)

(b) Work out the value of x. (2)

6. A bag contains $5x$ red balls and $2x$ blue balls. (4)

The number of red balls is **decreased** by 20%.
The number of blue balls is **increased** by 30%.
There are now 35 more red balls than blue balls in the bag.

Work out the value of x.

7. (3)

$$3x^3 - 2x^2 - 147x + 98 \equiv (ax - c)(bx + d)(bx - d)$$

where a, b, c, and d are positive integers.

Work out the values of a, b, c, and d.

8. Simplify fully (4)

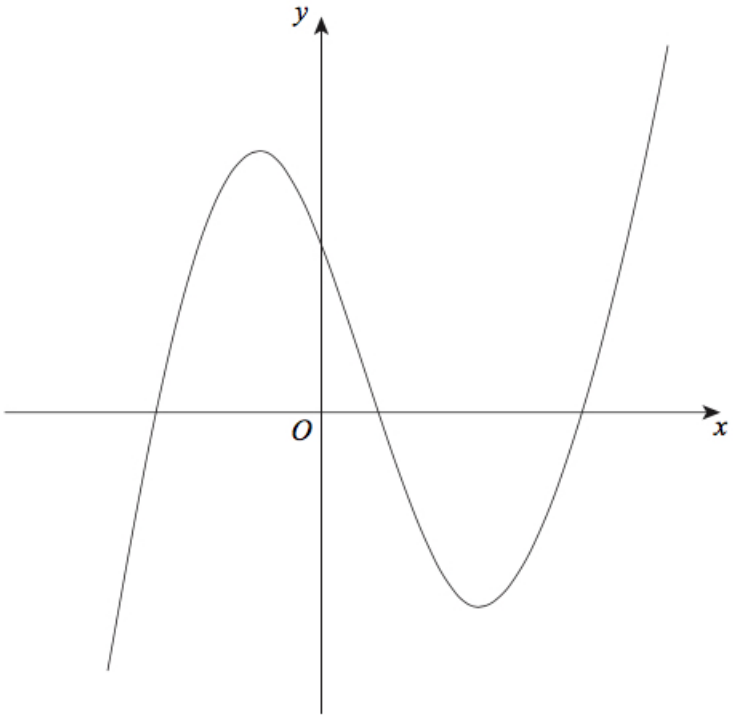
$$\frac{5x}{(x + 4)(x - 6)} - \frac{3}{x - 6}.$$

9. Given that (5)

$$\begin{pmatrix} 3 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} b \\ a + 1 \end{pmatrix},$$

work out the values of a and b.

10. This is a sketch of the curve $y = \mathrm{f}(x)$.



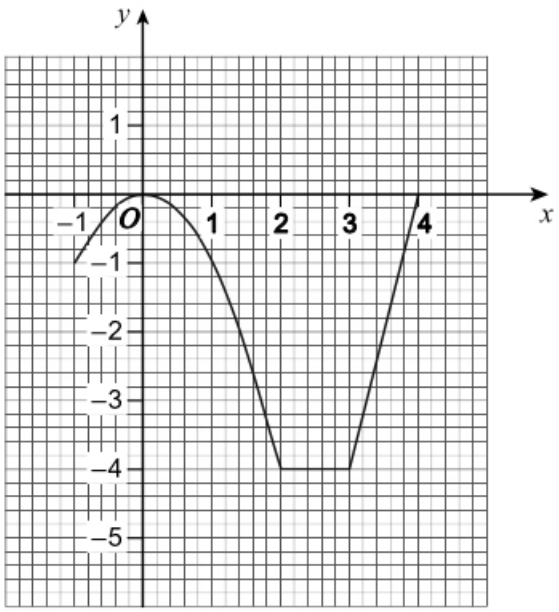
For this curve,

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 4x - 4.$$

(a) Work out the range of values of x for which $\mathrm{f}(x)$ is a decreasing function. (4)
Write your answer as an inequality.

(b) Work out the equation of the normal to the curve at the point $(1, -2)$. (5)
Give your answer in the form $y = mx + c$.

11. Here is the graph of $y = \mathrm{f}(x)$. (4)
It consists of a quadratic curve and two straight lines.



Define f(x), stating clearly the domain for each part.

12. Make y the subject for (4)

$$\sqrt{\frac{3xy}{x+y}} = 4.$$

13. (3)

$$x^2 + 2ax + b \equiv (x-5)^2 - a.$$

Work out the values of a and b.

14. Write (5)

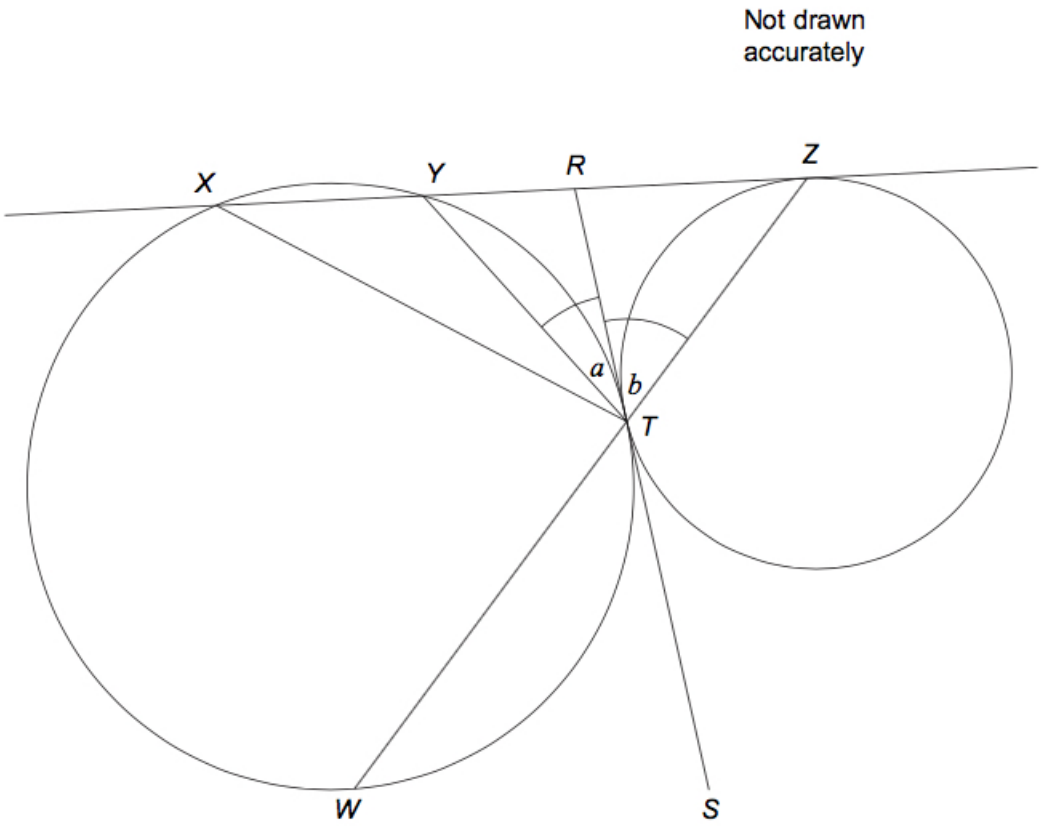
$$\frac{5\sqrt{2}}{3\sqrt{6}-7}$$

in the form $\sqrt{w} + \sqrt{k}$ where w and k are integers.

15. The diagram shows two circles touching externally at T.
Points X, Y, and W lie on the larger circle.

- RTS is a tangent to both circles.
- $XYRZ$ is a tangent to the smaller circle at Z.
- ZTW is a straight line.

Angle $YTR = a$ and angle $ZTR = b$.



(a) Give reasons why angle $RZT = b$. (2)

Angle $RZT = b$.

(b) Prove that (3)

$$\text{angle } XTW = \text{angle } YTZ.$$

16. By factorising fully, simplify (5)

$$\frac{x^4 - x^3 - 2x^2}{x^4 - 5x^2 + 4}.$$

17. Prove that (3)

$$2\tan^2\theta + 1 \equiv \frac{1 + \sin^2\theta}{1 - \sin^2\theta},$$

where $\sin^2\theta \neq 1$.