Dr Oliver Mathematics
Applied Mathematics: Mechanics or Statistics
Section B
2013 Paper
1 hour

The total number of marks available is 32.
You must write down all the stages in your working.

1. Given that (2)

$$y = \sin(e^{5x}),$$

find $\dfrac{dy}{dx}$.

2. Matrices are given as

$$\mathbf{A} = \begin{pmatrix} 4 & x \\ 0 & 2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 5 & 1 \\ 0 & 1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} y & 3 \\ -1 & 2 \end{pmatrix}.$$

(a) Write (2)

$$\mathbf{A}^2 - 3\mathbf{B}$$

as a single matrix.

(b) (i) Given that $\mathbf{C}$ is non-singular, find $\mathbf{C}^{-1}$, the inverse of $\mathbf{C}$. (2)

(ii) For what value of y would matrix $\mathbf{C}$ be singular? (1)

3. Use integration by parts to obtain (4)

$$\int \frac{\ln x}{x^3}\, dx,$$

where $x > 0$.

4. (a) State (2)

$$\sum_{r=1}^{n} r \text{ and } \sum_{r=1}^{n} r^3$$

in terms of n.

(b) Hence show that (2)

$$\sum_{r=1}^{n} (r^3 - 3r) = \frac{n(n+1)(n-2)(n+3)}{4}.$$

(c) Use the above result to evaluate (2)

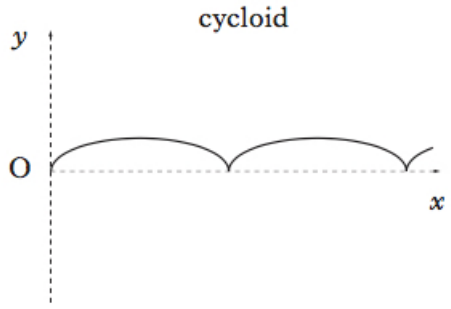
$$\sum_{r=5}^{15}(r^3 - 3r).$$

5. Find the general solution of the differential equation (6)

$$\frac{1}{x}\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = 6,\ x \neq 0.$$

6. The cycloid curve below is defined by the parametric equations

$$x = t - \sin t,\ y = 1 - \cos t.$$



(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t. (2)

(b) Show that the value of $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ is always negative, in the case where $0 < t < 2\pi$. (5)

A particle follows the path of the cycloid where t is the time elapsed since the particle's motion commenced.

(c) Calculate the speed of the particle when $t = \frac{1}{3}\pi$. (2)