Dr Oliver Mathematics
Mathematics: National Qualifications N5
2023 Paper 2: Calculator
1 hour 30 minutes

The total number of marks available is 50.
You must write down all the stages in your working.

1. A caravan was bought for £20 000. (3)

   It depreciated by 11% in the first year.

   It then depreciated by a further 6% each year over the next two years.

   Calculate the value of the caravan three years after it was bought.

Solution

$$\begin{aligned}\text{Value} &= 20\,000 \times (1 - 0.11) \times (1 - 0.06)^2 \\ &= 20\,000 \times 0.89 \times (0.94)^2 \\ &= \underline{\underline{£15\,728.08}}.\end{aligned}$$

2. The mass of a helium atom is 6.64×10^{-24} grams. (3)

   A flask contains 300 grams of helium.

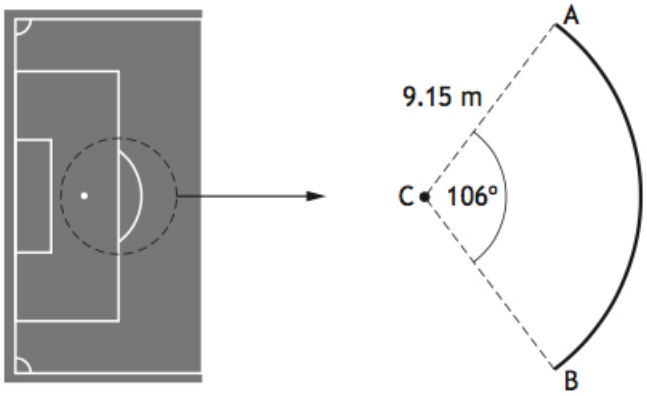
   Calculate the number of helium atoms in this flask.

   Give your answer in scientific notation, correct to 3 significant figures.

Solution

$$\begin{aligned}\text{Number of helium atoms} &= \frac{300}{6.64 \times 10^{-24}} \\ &= 4.518\,072\,289 \times 10^{25} \text{ (FCD)} \\ &= \underline{\underline{4.52 \times 10^{25} \text{ (3 sf)}}}.\end{aligned}$$

3. The diagram shows part of a football pitch. (3)



The penalty spot is marked at point C.

AB is an arc of a circle, centre C, radius 9.15 metres.

Calculate the length of the arc AB.

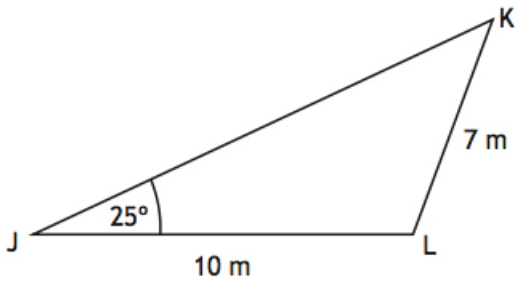
Solution

Well,

$$\begin{aligned}\text{the length of arc } AB &= \tfrac{106}{360} \times (2 \times \pi \times 9.15) \\ &= 16.927\,948\,42 \text{ (FCD)} \\ &= \underline{\underline{17.0 \text{ m (3 sf)}}}.\end{aligned}$$

4. The diagram shows triangle JKL. (3)

- Angle $KJL = 25°$.
- $JL = 10$ metres.
- $KL = 7$ metres.



Calculate the size of angle JKL.

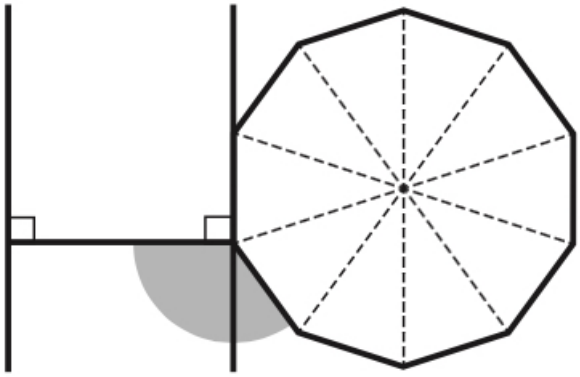
Solution

Sine rule:

$$\frac{\sin JKL}{JL} = \frac{\sin KJL}{KL} \Rightarrow \frac{\sin JKL}{10} = \frac{\sin 25^\circ}{7}$$
$$\Rightarrow \sin JKL = \frac{10 \sin 25^\circ}{7}$$
$$\Rightarrow \angle JKL = 37.138\,254\,51 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{\angle JKL = 37.1^\circ \text{ (3 sf)}}}.$$

5. A logo consists of an H-shape and a regular decagon. (2)

The diagram represents the logo.



Calculate the size of the shaded angle.

Solution

$$\begin{aligned}\text{Shaded angle} &= \text{right-angle} + \text{external angle}\\ &= 90 + \frac{360}{10}\\ &= 90 + 36\\ &= \underline{\underline{126^\circ}}.\end{aligned}$$

6. Nadim bought a flat last year. (3)

   The value of the flat has increased by 8% and it is now worth £94 500.

   Calculate how much Nadim paid for the flat.

Solution

Well,

$$\begin{aligned}\text{new price} = 1.08 \times \text{old price} &\Rightarrow 94\,500 = 1.08 \times \text{old price}\\ &\Rightarrow \text{old price} = \frac{94\,500}{1.08}\\ &\Rightarrow \text{old price} = \underline{\underline{£87\,500}}.\end{aligned}$$

7. Change the subject of the formula (3)

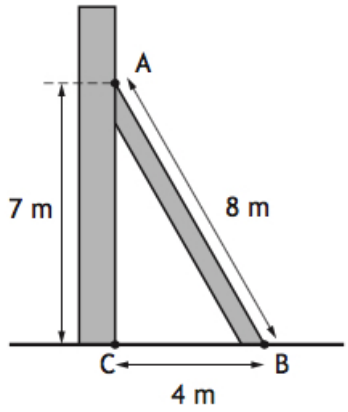
$$P = \tfrac{1}{3}mn - r$$

   to m.

Solution

$$\begin{aligned}P = \tfrac{1}{3}mn - r &\Rightarrow \tfrac{1}{3}mn = P + r\\ &\Rightarrow mn = 3(P + r)\\ &\Rightarrow \underline{\underline{m = \frac{3(P + r)}{n}}}.\end{aligned}$$

8. A wooden beam is used to support a wall built on horizontal ground as shown in the diagram. (4)



- The edge of the beam, AB, is 8 metres long.
- C is at the foot of the wall.
- A is 7 metres from C.
- B is 4 metres from C.

Determine whether the wall is perpendicular to the ground.
Justify your answer.

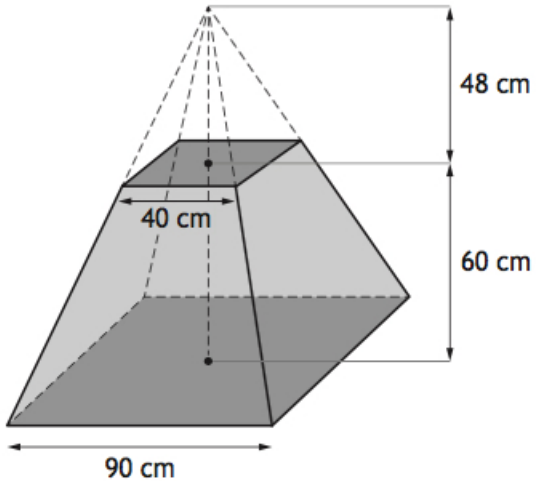
Solution

Well,

$$\begin{aligned} AC^2 + BC^2 &= 7^2 + 4^2 \\ &= 49 + 16 \\ &= 65 \\ &> 64 \\ &= AC^2; \end{aligned}$$

the angle ACB is larger than a right-angle so it is not perpendicular to the ground

9. A concrete block is in the shape of a large pyramid with a small pyramid removed. (4)



The large pyramid has a square base of length 90 centimetres.

The small pyramid has a square base of length 40 centimetres and a height of 48 centimetres.

The block has height 60 centimetres.

Calculate the volume of the block.

Solution

Well,

$$\begin{aligned}\text{volume of the block} &= \text{big pyramid} - \text{small pyramid}\\ &= \left(\tfrac{1}{3} \times [48+60] \times 90^2\right) - \left(\tfrac{1}{3} \times 48 \times 40^2\right)\\ &= 291\,600 - 25\,600\\ &= \underline{\underline{266\,000\ \text{cm}^3}}.\end{aligned}$$

10. Express (3)

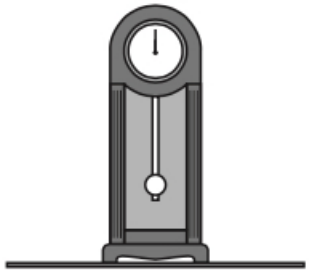
$$\frac{7}{x-3} - \frac{2}{x},\ x \neq 3,\ x \neq 0,$$

as a single fraction in its simplest form.

Solution

$$\begin{aligned}\frac{7}{x-3}-\frac{2}{x} &= \frac{7x-2(x-3)}{x(x-3)}\\ &= \frac{7x-2x+6}{x(x-3)}\\ &= \underline{\underline{\frac{5x+6}{x(x-3)}}}.\end{aligned}$$

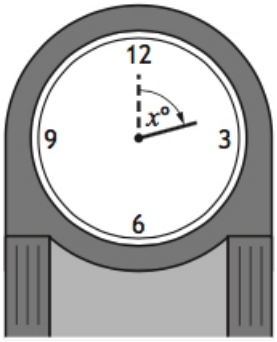
11. Anna has a grandfather clock in her house. (4)



The height of the tip of the hour hand above the floor, in centimetres, is given by

$$h = 20\cos x^\circ + 147,$$

where x° is the angle the hour hand has rotated through since 12 o'clock.



Calculate the first two values of x for which the tip of the hour hand is 150 centimetres above the floor.

Solution

$$\begin{aligned} h = 150 &\Rightarrow 20\cos x^\circ + 147 = 150 \\ &\Rightarrow 20\cos x^\circ = 3 \\ &\Rightarrow \cos x^\circ = \tfrac{3}{20} \\ &\Rightarrow x = 81.373\,073\,44,\ 278.626\,926\,6 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{x = 81.4,\ 279 \text{ (3 sf)}}}. \end{aligned}$$

12. Simplify (3)

$$\frac{x^2 - 16}{x^2 + x - 20}.$$

Solution

Difference of two squares:

$$x^2 - 16 = (x - 4)(x + 4).$$

$$\left.\begin{array}{ll} \text{add to:} & +1 \\ \text{multiply to:} & -20 \end{array}\right\} +5,\ -4$$

$$x^2 + x - 20 = (x + 5)(x - 4).$$

Finally,

$$\begin{aligned} \frac{x^2 - 16}{x^2 + x - 20} &= \frac{(x - 4)(x + 4)}{(x + 5)(x - 4)} \\ &= \underline{\underline{\frac{x + 4}{x + 5}}}. \end{aligned}$$

13. Simplify (2)

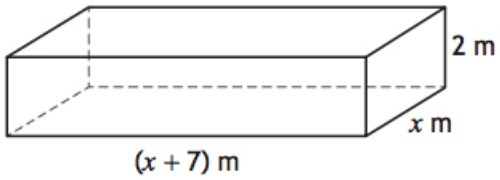
$$2\sin^2 x^\circ + 2\cos^2 x^\circ.$$

Show your working.

Solution

$$\begin{aligned} 2\sin^2 x^\circ + 2\cos^2 x^\circ &= 2(\sin^2 x^\circ + \cos^2 x^\circ) \\ &= 2(1) \\ &= \underline{\underline{2}}. \end{aligned}$$

14. A storage unit, built in the shape of a cuboid, is shown.



It has length $(x + 7)$ metres, breadth x metres, and height 2 metres.
The volume of this unit is 45 cubic metres.

(a) Show that (2)

$$2x^2 + 14x - 45 = 0.$$

Solution

$$\begin{aligned} 2 \times x \times (x + 7) = 45 &\Rightarrow 2x(x + 7) = 45 \\ &\Rightarrow 2x^2x + 14x = 45 \\ &\Rightarrow \underline{\underline{2x^2x + 14x - 45 = 0}}, \end{aligned}$$

as required.

(b) Calculate x, the breadth of the storage unit. (4)
Give your answer correct to 1 decimal place.

Solution

Quadratic formula: $a = 2$, $b = 14$, and $c = -45$:

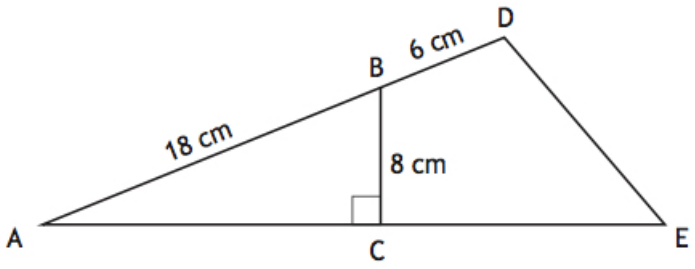
$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-14 \pm \sqrt{14^2 - 4(2)(-45)}}{2(2)} \\ &= \frac{-14 \pm \sqrt{556}}{4} \\ &= -9.394\,913\,061,\ 2.394\,913\,061 \text{ (FCD)}; \end{aligned}$$

now, $x > 0$ (why?) so

$$\underline{\underline{x = 2.4 \text{ cm}}}.$$

15. In the diagram: (4)

- AC is perpendicular to BC,
- $AB = 18$ centimetres,
- $BD = 6$ centimetres, and
- $BC = 8$ centimetres.



The **area** of triangle ADE is 160 square centimetres.

Calculate the length of AE.

Solution

We mark the point on AE which is directly below D as F.

Similar triangles:

$$\frac{DF}{BC} = \frac{AC}{AB} \Rightarrow \frac{DF}{8} = \frac{18+6}{18}$$
$$\Rightarrow \frac{DF}{8} = \frac{24}{18}$$
$$\Rightarrow DF = \tfrac{32}{3}.$$

Finally,

$$\text{area} = \tfrac{1}{2}bh \Rightarrow 160 = \tfrac{1}{2} \times DE \times AE$$
$$\Rightarrow 160 = \tfrac{1}{2} \times \tfrac{32}{3} \times AE$$
$$\Rightarrow AE = \frac{160}{\frac{1}{2} \times \frac{32}{3}}$$
$$\Rightarrow AE = \frac{160}{\frac{32}{6}}$$
$$\Rightarrow \underline{\underline{AE = 30 \text{ cm}}}.$$