Mechanics 1

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Mathematics

Modelling Assumptions

You need to be able to recognise when the following modelling assumptions are appropriate and, if they are, what the consequences of using these assumptions are.

- A **particle** is an object whose dimensions are small compared with the other lengths involved. When an object is modelled as a particle its mass can be considered to be at a single point, we can ignore any rotational effects (such as moments) of any forces acting upon it, and we can ignore the effect of air resistance.
- An object is **light** if its mass is very small compared with the other masses involved. When an object is modelled as light we can treat it as having zero mass.
- A body is **uniform** if its mass is evenly distributed over its entire volume. When a body is modelled as uniform its mass can be considered to be concentrated at a single point (the **centre of mass**), at the geometrical centre of the object.
- A string is **inextensible** if it does not stretch under a load. If two objects are connected by an inextensible taut string then they will have the same velocity and the same acceleration.
- A surface is **smooth** if there is no friction between an object which is moving, or is tending to move, along it.
- A surface is **rough** if it is not smooth. For a rough surface, we need to consider the effect of friction on an object which is moving, or tending to move, along it.
- A pulley is **smooth** if there is no friction at the bearing of the pulley or between the pulley and the string. If the pulley is smooth then the tension in the string is the same on either side of the pulley.
- A **rod** is an object which has one dimension that is small in comparison with another. When an object is modelled as a rod it has no thickness, the mass can be considered to be distributed along a straight line, and it is rigid, i.e., it does not bend or buckle.

Forces

A **force** is any external influence that causes an object to undergo a certain change. As far as Mechanics 1 is concerned, a force may change an object's movement or direction but will not cause the object to deform.

Friction

$$F_{\text{max}} = \mu R,$$

where F_{max} is the maximum or limiting value of the frictional force (measured in newtons, N), μ is the coefficient of friction (this is a pure number), and R is the normal reaction between the two surfaces (measured in newtons, N).

- Friction always acts in the opposite direction to the direction of motion or intended motion.
- Friction is a 'lazy' force, i.e., it will do all that is required to oppose motion up to its maximum possible value of F_{max}.
- Thus, in practice, $F \leq \mu R$, and you may need to be able to work with the inequality in an examination problem.

Newton's First Law

An object will either stay at rest or will move in a straight line at constant speed unless it is acted upon by an unbalanced force.

Newton's Second Law

When an object is acted on by an unbalanced force, it will accelerate and its acceleration is proportional to its mass. This in encapsulated in the equation

$$F = ma$$

where F is the unbalanced force (measured in newtons, N), m is the mass of the object (measured in kilograms, kg), and a is the acceleration of the object (measured in metres per second per second, ms^{-2}).

Newton's Third Law

For every action there is an equal and opposite reaction.

Kinematic equations

$$\begin{aligned} s &= \tfrac{1}{2}(u+v)t \\ v &= u + at \\ s &= ut + \tfrac{1}{2}at^2 \\ s &= vt - \tfrac{1}{2}at^2 \\ v^2 &= u^2 + 2as \end{aligned}$$

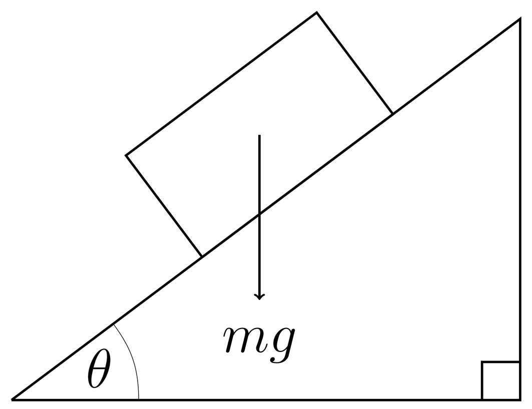
where

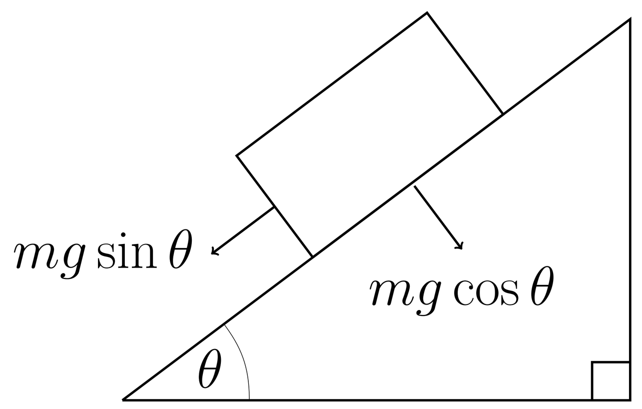
- a is the constant acceleration,
- s is displacement,
- t is time,
- u is the initial velocity,
- v is the final velocity.

You are advised to make a clear statement of which of these variables you know in order to assist you in choosing the best equation to use.

Resolving Forces

Always resolve any forces so that you have the components of the force parallel and perpendicular to the direction of motion (or the direction of intended motion). For example, if you have an object on an inclined plane:





Statics

In statics problems, there is no net force acting on the object. Thus, however you resolve the forces (horizontally and vertically, perpendicular or parallel to a plane, and so on) there will be no resultant force.

Dynamics

Once all of the forces have been resolved appropriately then you should use Newton's Second Law ($F = ma$) in the direction of motion and balance the forces that act in any other direction.

Vectors

Many of the kinematic and force equations have vector equivalents, e.g.,

$$\begin{aligned} \mathbf{s} &= \tfrac{1}{2}(\mathbf{u}+\mathbf{v})t \\ \mathbf{v} &= \mathbf{u} + \mathbf{a}t \\ \mathbf{s} &= \mathbf{u}t + \tfrac{1}{2}\mathbf{a}t^2 \\ \mathbf{s} &= \mathbf{v}t - \tfrac{1}{2}\mathbf{a}t^2 \\ \mathbf{F} &= m\mathbf{a} \end{aligned}$$

Moments

The **moment** of a force about a point is given by

$$F \times d$$

where F is the magnitude of the force (measured in newtons, N) and d is the perpendicular distance of the line of action of the force from the point (measured in metres, m).

- Moments are a vector quantity and their units are newton-metres (N m).
- The total moment of several forces acting on a body is the sum of the individual moments; you may find it helpful to gather together the clockwise moments and then the anticlockwise moments separately.
- When a rigid body is equilibrium then the resultant force in any direction is zero and the sum of the moments about any point is zero.
- In examination problems, particularly those with objects placed on a beam, it is usual to model the beam as a rod (it may or may not be uniform) and each of the masses as a particle.
- If the rod is not uniform then the mass can be modelled as acting at its centre of mass.
- If a beam is balanced on two supports and it is about to rotate about one of them then the normal reaction at the other support is zero.