

Dr Oliver Mathematics
GCSE Mathematics
2013 November Paper 2H: Calculator
1 hour 45 minutes

The total number of marks available is 100.

You must write down all the stages in your working.

1. (a) Use your calculator to work out (2)

$$\frac{\sqrt{7056}}{0.35 \times 12.8}$$

Write down all the figures on your calculator display.

You must give your answer as a decimal.

Solution

$$\begin{aligned} \frac{\sqrt{7056}}{0.35 \times 12.8} &= \frac{84}{4.48} \\ &= \underline{\underline{18.75}}. \end{aligned}$$

- (b) Write your answer to part (a) correct to 1 significant figure. (1)

Solution

20.

2. Pavel and Katie share some sweets in the ratio 3 : 8.

Katie gets 32 sweets.

- (a) How many sweets does Pavel get? (2)

Solution

$$3 : 8 = 3 \times 4 : 8 \times 4 = 12 : 32$$

and Pavel gets 12 sweets.

Katie also has a tin of chocolates.

There are 80 chocolates in the tin.

45% of the chocolates have toffee in the middle.

- (b) Work out the number of chocolates that have toffee in the middle. (2)

Solution

$$0.45 \times 80 = \underline{\underline{36}}.$$

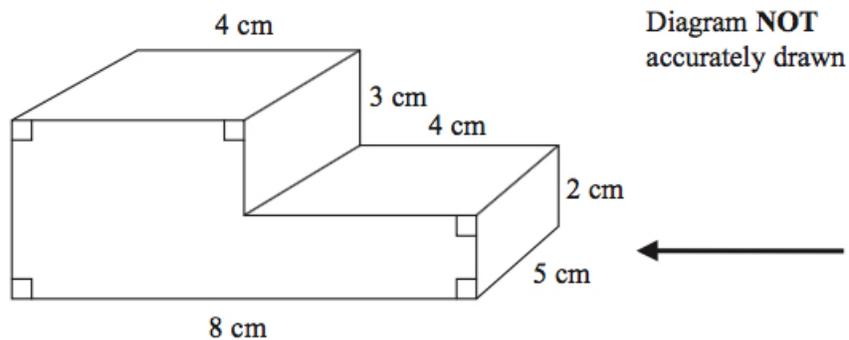
3. Bill has some counters in a bag. (2)
3 of the counters are red.
7 of the counters are blue.
The rest of the counters are yellow.
Bill takes at random a counter from the bag. The probability that he takes a yellow counter is $\frac{2}{7}$.
How many yellow counters are in the bag before Bill takes a counter?

Solution

Let x be the number of yellow counters. Now,

$$\begin{aligned} \frac{2}{7} &= \frac{x}{3 + 7 + x} \Rightarrow \frac{2}{7} = \frac{x}{10 + x} \\ &\Rightarrow 2(10 + x) = 7x \\ &\Rightarrow 20 + 2x = 7x \\ &\Rightarrow 20 = 5x \\ &\Rightarrow \underline{\underline{x = 4}}. \end{aligned}$$

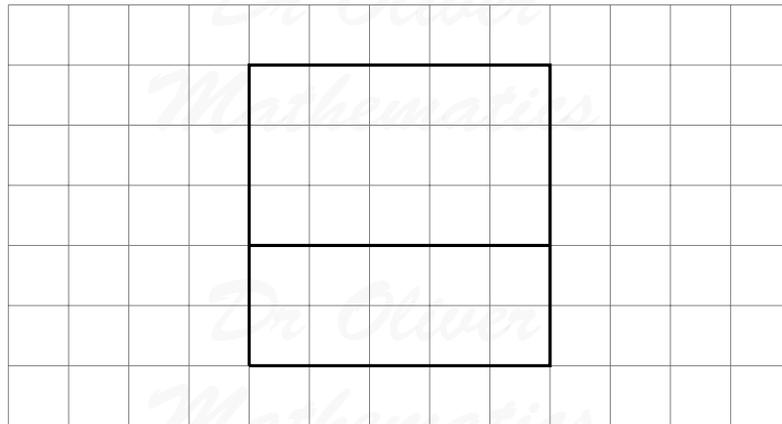
4. The diagram shows a solid prism. (2)



On the centimetre square grid, draw the side elevation of the solid prism from the direction shown by the arrow.



Solution



5. Ben goes on holiday to Hong Kong. (3)
In Hong Kong, Ben sees a camera costing HK\$3 179.55.
In London, an identical camera costs £285.
The exchange rate is £1 = HK\$12.30.
Ben buys the camera in Hong Kong.
How much cheaper is the camera in Hong Kong than in London?

Solution

$$12.30 \times 285 = \text{HK\$}3\,505.50$$

and it is cheaper by

$$3\,505.50 - 3\,179.55 = \underline{\underline{\text{HK\$}325.95}}$$

or

$$\frac{325.95}{12.30} = \underline{\underline{\pounds 26.50.}}$$

6. There are 130 adults at a language school. (4)
Each adult studies one of French or Spanish or German.
96 of the adults are women.
12 of the women study French.
73 of the adults study Spanish.
55 of the women study Spanish.
9 of the men study German.
How many of the adults study French?

Solution

There are

$$130 - 96 = 34 \text{ men.}$$

Now,

$$96 - 12 - 55 = 29 \text{ women}$$

study German. Finally, the adults studying French are

$$130 - 73 - 9 - 29 = \underline{\underline{19.}}$$

7. Plants are sold in three different sizes of tray. (4)
A small tray of 30 plants costs £6.50.
A medium tray of 40 plants costs £8.95.
A large tray of 50 plants costs £10.99.
Kaz wants to buy the tray of plants that is the best value for money.
Which size tray of plants should she buy?
You must show all your working.

Solution

Small:

$$\frac{6.50}{30} = 0.21\bar{6}.$$

Medium:

$$\frac{8.95}{40} = 0.22375.$$

Large:

$$\frac{10.99}{50} = 0.2198.$$

Kaz should buy the small tray as it is better value for money.

8. Here are the first four terms of an arithmetic sequence.

$$3 \quad 10 \quad 17 \quad 24$$

(a) Find, in terms of n , an expression for the n th term of this arithmetic sequence. (2)

Solution

Let the

$$nth \text{ term} = an + b.$$

$$\begin{array}{cccc} 3 & 10 & 17 & 24 \\ & 7 & 7 & 7 \\ a + b & 2a + b & 3a + b & 4a + b \\ & a & a & a \end{array}$$

We compare terms:

$$a = 7$$

and

$$\begin{aligned} a + b = 3 &\Rightarrow 7 + b = 3 \\ &\Rightarrow b = -4. \end{aligned}$$

Hence,

$$nth \text{ term} = \underline{\underline{7n - 4}}.$$

(b) Is 150 a term of this sequence? (2)

You must explain how you get your answer.

Solution

$$\begin{aligned} 7n - 4 = 150 &\Rightarrow 7n = 154 \\ &\Rightarrow n = 22; \end{aligned}$$

hence, yes: it is the 22nd term.

9. The diagram shows a pattern using four identical rhombuses.

(4)

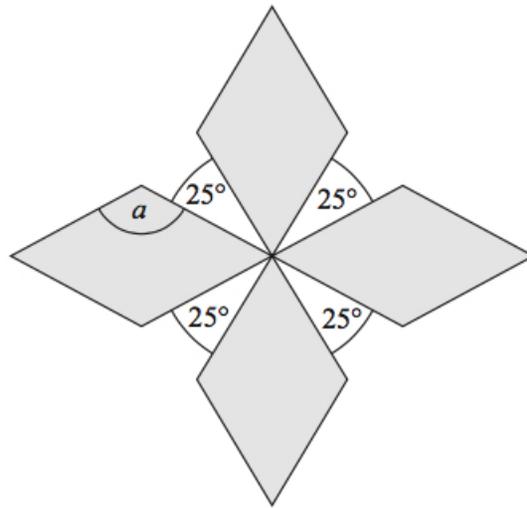


Diagram **NOT**
accurately drawn

Work out the size of the angle marked a .
You must show your working.

Solution

Let b be the light grey angles at the centre of the circle. Now,

$$\begin{aligned}4(b + 25) &= 360 \text{ (sum of the angles)} \Rightarrow b + 25 = 90 \\ &\Rightarrow b = 65 \\ &\Rightarrow a + 65 = 180 \text{ (interior angles)} \\ &\Rightarrow \underline{\underline{a = 115}}.\end{aligned}$$

10. Sasha takes a music exam.

(3)

The table shows the result that Sasha can get for different percentages in her music exam.

Percentage	Result
50% - 69%	Pass
70% - 84%	Merit
85% - 100%	Distinction

Sasha gets 62 out of 80 in her music exam.
What result does Sasha get?
You must show your working.

Solution

$$\frac{62}{80} \times 100\% = 77\frac{1}{2}\%$$

so Sasha got a merit.

11. (a) Simplify

$$x^7 \times x^3.$$

(1)

Solution

$$x^7 \times x^3 = \underline{x^{10}}.$$

(b) Simplify

$$(m^4)^3.$$

(1)

Solution

$$(m^4)^3 = \underline{m^{12}}.$$

(c) Simplify

$$\frac{36af^8}{12a^5f^2}.$$

(2)

Solution

$$\frac{36af^8}{12a^5f^2} = \underline{\frac{3f^6}{a^4}}.$$

12. A circle has a diameter of 140 cm.

Work out the circumference of the circle.

Give your answer correct to 3 significant figures.

(2)

Solution

The radius is

$$\frac{140}{2} = 70 \text{ cm}$$

and the circumference of the circle is

$$\begin{aligned} 2 \times \pi \times 70 &= 439.8229715 \text{ (FCD)} \\ &= \underline{\underline{440 \text{ cm (3 sf)}}}. \end{aligned}$$

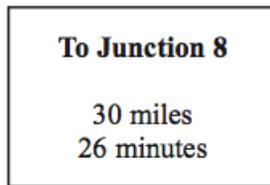
13. Axel and Lethna are driving along a motorway.

(3)

They see a road sign.

The road sign shows the distance to Junction 8.

It also shows the average time drivers will take to get to Junction 8.



The speed limit on the motorway is 70 mph.

Lethna says, 'We will have to drive faster than the speed limit to go 30 miles in 26 minutes.'

Is Lethna right?

You must show how you got your answer.

Solution

$$\begin{aligned} \text{Time} &= \frac{30 \text{ miles}}{26 \text{ minutes}} \\ &= \frac{30 \text{ miles}}{\frac{26}{60} \text{ hours}} \\ &= 69\frac{3}{13} \text{ mph}; \end{aligned}$$

hence, Lethna is not correct.

14. The table gives information about the temperature, $T^{\circ}\text{C}$, at noon in a town for 50 days.

Temperature $T^{\circ}\text{C}$	Frequency
$8 < T \leq 12$	6
$12 < T \leq 16$	8
$16 < T \leq 20$	13
$20 < T \leq 24$	21
$24 < T \leq 28$	2

- (a) Write down the modal class interval. (1)

Solution

$20 < T \leq 24$.

- (b) Calculate an estimate for the mean temperature. (4)

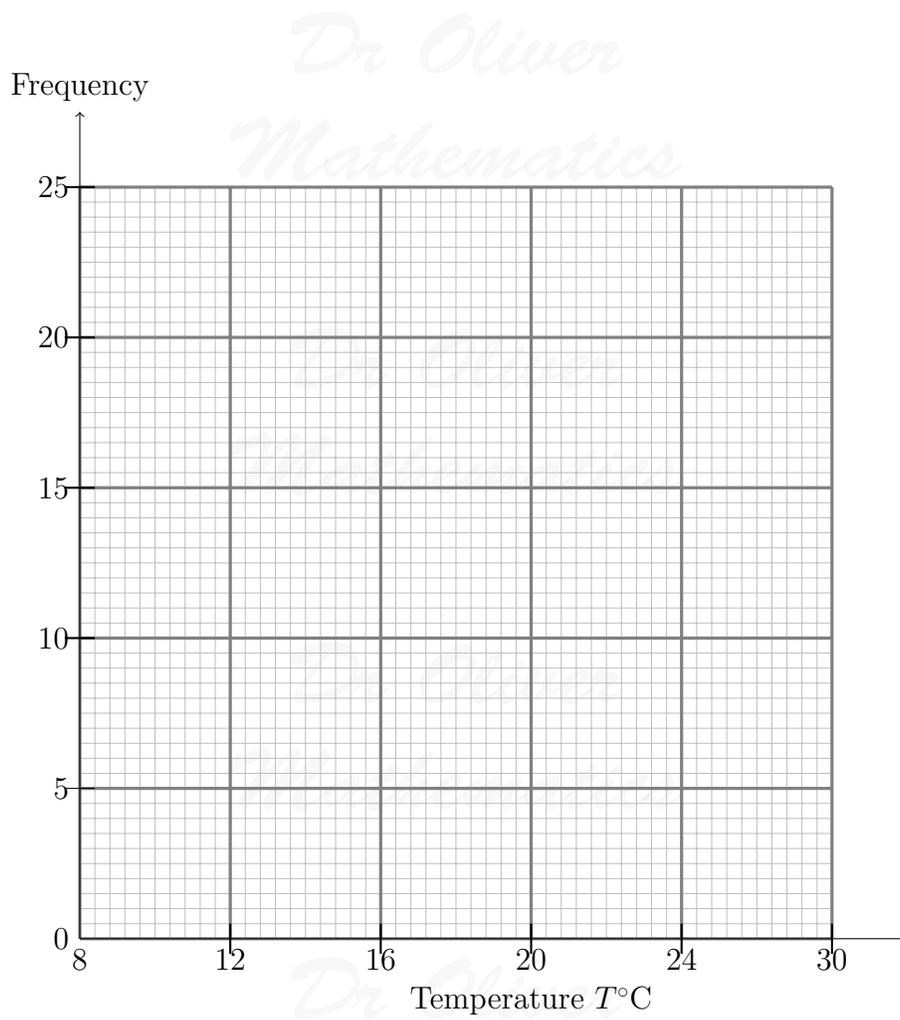
Solution

Temperature $T^{\circ}\text{C}$	Frequency	Midpoint	Frequency \times Midpoint
$8 < T \leq 12$	6	10	$6 \times 10 = 60$
$12 < T \leq 16$	8	14	$8 \times 14 = 112$
$16 < T \leq 20$	13	18	$13 \times 18 = 234$
$20 < T \leq 24$	21	22	$21 \times 22 = 462$
$24 < T \leq 28$	2	26	$2 \times 26 = 52$
Total	50		920

An estimate for the mean temperature is

$$\frac{\sum fx}{\sum f} \approx \frac{920}{50} = \underline{18.4^{\circ}\text{C}}.$$

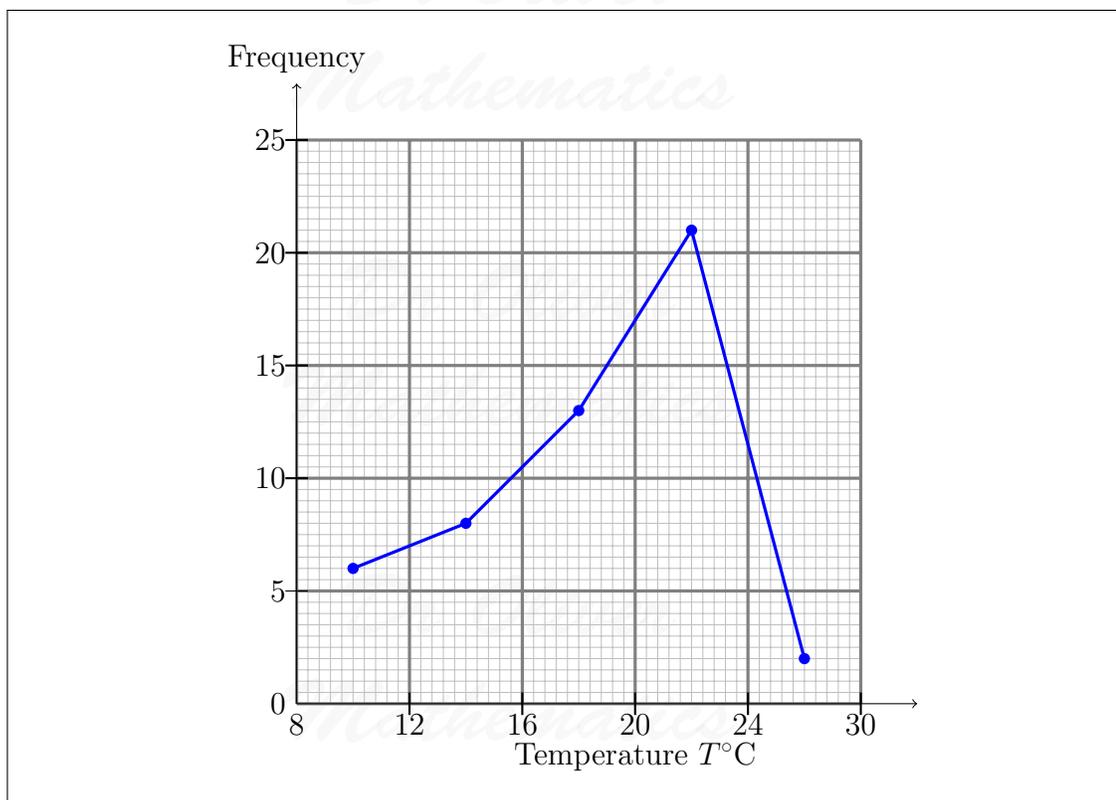
- (c) Draw a frequency polygon for the information in the table. (2)



Solution

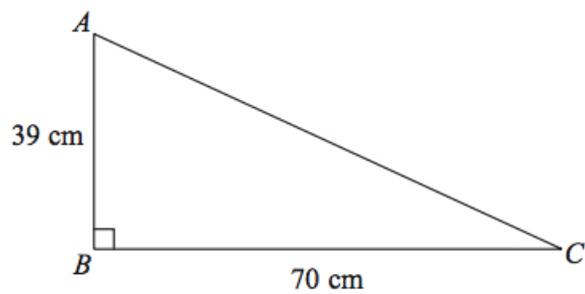
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15. Here is a right-angled triangle.

(3)



Work out the length of AC .
Give your answer correct to 1 decimal place.

Solution

$$\begin{aligned}
 AC &= \sqrt{AB^2 + BC^2} \\
 &= \sqrt{39^2 + 70^2} \\
 &= 80.131\ 142\ 51 \text{ (FCD)} \\
 &= \underline{\underline{80.1 \text{ cm (3 sf)}}}.
 \end{aligned}$$

16. (a) Solve

$$5(f - 3) = f + 10.$$

(3)

Solution

$$\begin{aligned}
 5(f - 3) &= f + 10 \Rightarrow 5f - 15 = f + 10 \\
 &\Rightarrow 4f = 25 \\
 &\Rightarrow \underline{\underline{f = 6\frac{1}{4}}}.
 \end{aligned}$$

(b) Solve

$$\frac{h + 7}{3} + \frac{2h - 1}{2} = \frac{5}{6}.$$

(4)

Solution

Multiply all the terms by 6:

$$\begin{aligned}
 \frac{h + 7}{3} + \frac{2h - 1}{2} &= \frac{5}{6} \\
 \Rightarrow 2(h + 7) + 3(2h - 1) &= 5 \\
 \Rightarrow 2h + 14 + 6h - 3 &= 5 \\
 \Rightarrow 8h &= -6 \\
 \Rightarrow \underline{\underline{h = -\frac{3}{4}}}.
 \end{aligned}$$

17. (a) Complete the table of values for $y = x^3 - 4x$.

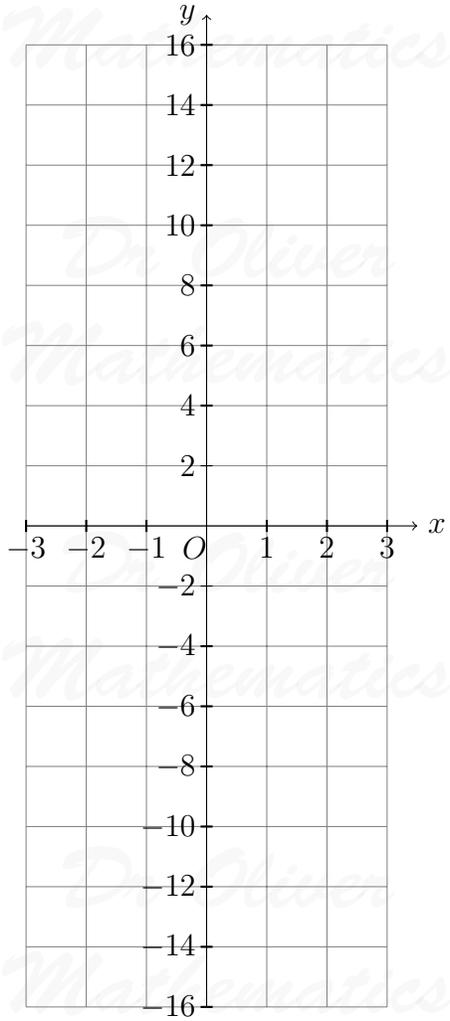
(2)

x	-3	-2	-1	0	1	2	3
y			3	0			15

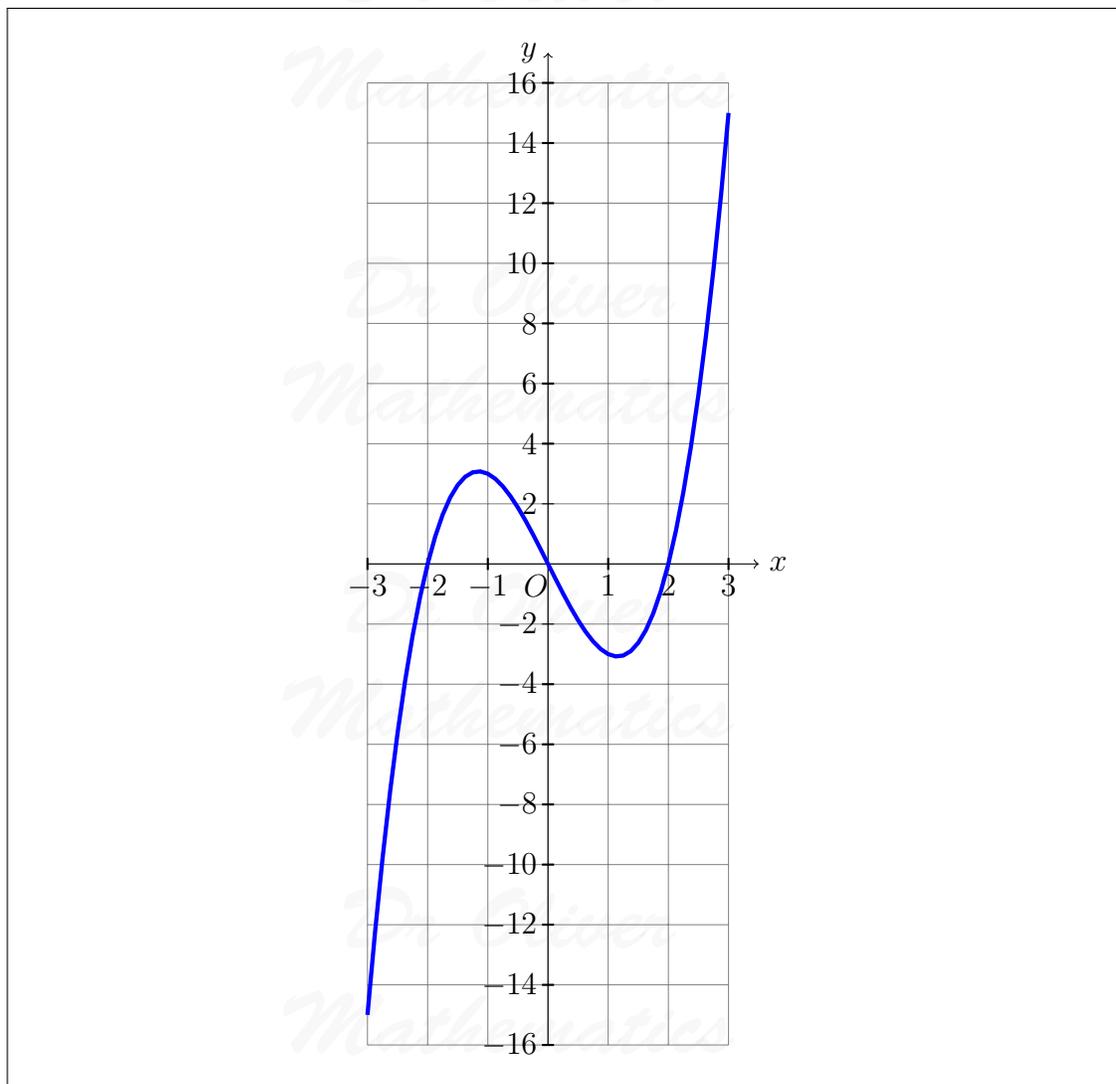
Solution

x	-3	-2	-1	0	1	2	3
y	<u>-15</u>	<u>0</u>	3	0	<u>-3</u>	<u>0</u>	15

(b) On the grid, draw the graph of $y = x^3 - 4x$ from $x = -3$ to $x = 3$. (2)

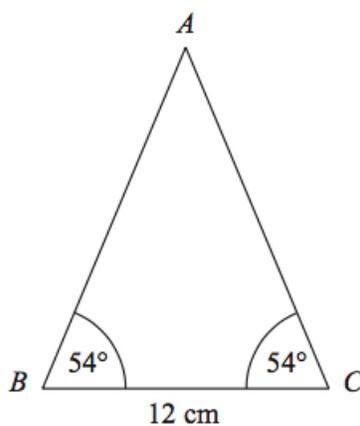


Solution



18. ABC is an isosceles triangle.

(4)



Work out the area of the triangle.
Give your answer correct to 3 significant figures.

Solution

First,

$$\angle BAC = 180 - 54 - 54 = 72^\circ$$

and, second,

$$\frac{AB}{\sin 54^\circ} = \frac{12}{\sin 72^\circ} \Rightarrow AB = \frac{12 \sin 54^\circ}{\sin 72^\circ}$$
$$\Rightarrow AB = 10.207\ 809\ 7 \text{ cm (FCD).}$$

Finally,

$$\text{area} = \frac{1}{2} \times 10.207 \dots \times 10.207 \dots \times \sin 72^\circ$$
$$= 49.549\ 749\ 14 \text{ (FCD)}$$
$$= \underline{\underline{49.6 \text{ cm}^2}} \text{ (3 sf).}$$

19. (a) Write

$$7.8 \times 10^{-4}$$

(1)

as an ordinary number

Solution

$$7.8 \times 10^{-4} = \underline{\underline{0.000\ 78}}.$$

(b) Write

$$95\ 600\ 000$$

(1)

as a number in standard form.

Solution

$$95\ 600\ 000 = \underline{\underline{9.56 \times 10^7}}.$$

20. In a sale normal prices are reduced by 20%.

A washing machine has a sale price of £464.

By how much money is the normal price of the washing machine reduced?

(3)

Solution

$$\begin{aligned}\text{Normal price} &= \frac{464}{1 - 0.2} \\ &= \frac{464}{0.8} \\ &= \underline{\underline{\pounds 580}}\end{aligned}$$

and it is reduced by

$$580 - 464 = \underline{\underline{\pounds 116}}.$$

21. (a) Factorise

$$4x^2 - 9.$$

(1)

Solution

$$\begin{array}{l} \text{add to:} \quad 0 \\ \text{multiply to: } (+4) \times (-9) = -36 \end{array} \left. \vphantom{\begin{array}{l} \text{add to:} \\ \text{multiply to:} \end{array}} \right\} -6, +6$$

$$\begin{aligned}4x^2 - 9 &= 4x^2 - 6x + 6x - 9 \\ &= 2x(2x - 3) + 3(2x - 3) \\ &= \underline{\underline{(2x + 3)(2x - 3)}}.\end{aligned}$$

(b) Make m the subject of

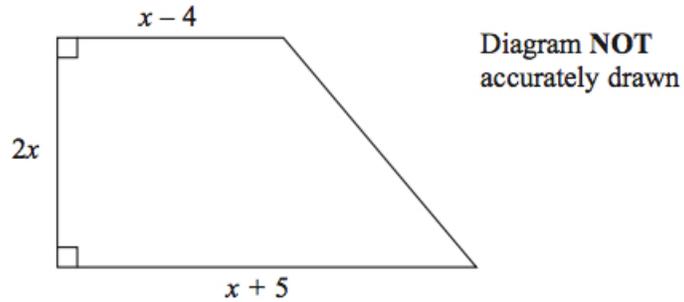
$$g - 3m = am + 5.$$

(3)

Solution

$$\begin{aligned}g - 3m &= am + 5 \Rightarrow g - 5 = am + 3m \\ &\Rightarrow g - 5 = m(a + 3) \\ &\Rightarrow m = \underline{\underline{\frac{g - 5}{a + 3}}}.\end{aligned}$$

22. The diagram shows a trapezium.



All the measurements are in centimetres.
The area of the trapezium is 351 cm^2 .

(a) Show that

$$2x^2 + x - 351 = 0. \quad (2)$$

Solution

$$\begin{aligned} \frac{1}{2} \times [(x - 4) + (x + 5)] \times 2x &= 351 \Rightarrow x(2x + 1) = 351 \\ &\Rightarrow 2x^2 + x = 351 \\ &\Rightarrow \underline{\underline{2x^2 + x - 351 = 0}}, \end{aligned}$$

as required.

(b) Work out the value of x .

Solution

$$\left. \begin{array}{l} \text{add to:} \\ \text{multiply to: } (+2) \times (-351) = -702 \end{array} \right\} -26, +27$$

$$\begin{aligned} 2x^2 + x - 351 = 0 &\Rightarrow 2x^2 + 27x - 26x - 351 = 0 \\ &\Rightarrow x(2x + 27) - 13(2x + 27) = 0 \\ &\Rightarrow (x - 13)(2x + 27) = 0 \\ &\Rightarrow x - 13 = 0 \text{ or } 2x + 27 = 0 \\ &\Rightarrow x = 13 \text{ or } x = -13\frac{1}{2}. \end{aligned}$$

Hence, as $x - 4 > 0$, $\underline{\underline{x = 13}}$.

23. The table shows information about 1 065 students.

(2)

	Male	Female
Year 7	126	109
Year 8	112	134
Year 9	121	114
Year 10	87	94
Year 11	88	80

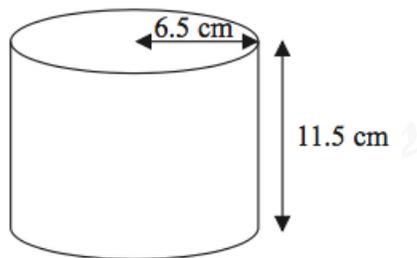
Elena takes a stratified sample of 120 students by year group and by gender.
Work out the number of Year 8 female students in her sample.

Solution

$$\begin{aligned}
 \text{Year 8 female students} &= \frac{134}{1065} \times 120 \\
 &= 15.098\dots \\
 &= \underline{\underline{15}}.
 \end{aligned}$$

24. The diagram shows a large tin of pet food in the shape of a cylinder.

(3)



The large tin has a radius of 6.5 cm and a height of 11.5 cm.
A pet food company wants to make a new size of tin.
The new tin will have a radius of 5.8 cm.
It will have the same volume as the large tin.
Calculate the height of the new tin.
Give your answer correct to one decimal place.

Solution

Let h cm be the new tin's height. Now,

$$\begin{aligned}\pi \times 5.8^2 \times h &= \pi \times 6.5^2 \times 11.5 \\ \Rightarrow 33.64h &= 485.875 \\ \Rightarrow h &= 14.443\,370\,99 \text{ (FCD)} \\ \Rightarrow h &= \underline{\underline{14.4 \text{ cm (1 dp)}}}.\end{aligned}$$

25. **A** and **B** are straight lines.

Line **A** has equation $2y = 3x + 8$.

Line **B** goes through the points $(-1, 2)$ and $(2, 8)$.

Do lines **A** and **B** intersect?

You must show all your working.

(3)

Solution

Line **A** has equation

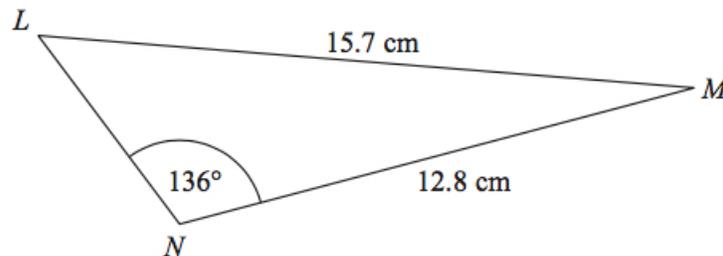
$$2y = 3x + 8 \Rightarrow y = \frac{3}{2}x + 4.$$

Line **B**:

$$\begin{aligned}\text{gradient} &= \frac{8 - 2}{2 - (-1)} \\ &= \frac{6}{3} \\ &= 2.\end{aligned}$$

Hence, they must cross as the gradients are not the same.

26. The diagram shows triangle LMN .



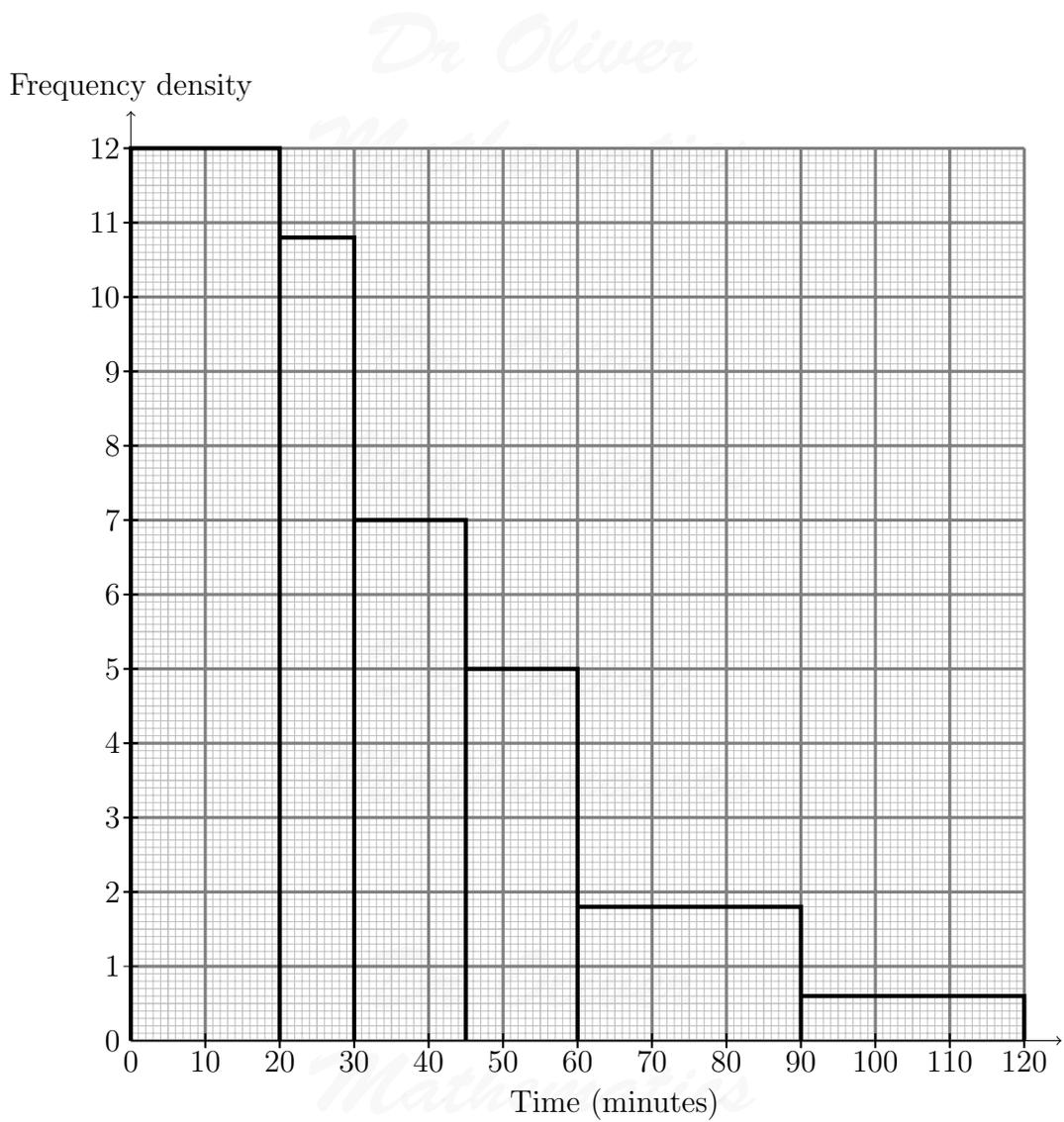
(5)

Calculate the length of LN .
Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned}\frac{\sin MLN}{12.8} &= \frac{\sin 136^\circ}{15.7} \Rightarrow \sin MLN = \frac{12.8 \sin 136^\circ}{15.7} \\ &\Rightarrow \angle MLN = 34.49578985 \text{ (FCD)} \\ &\Rightarrow \angle LMN = 180 - 136 - 34.495\dots \text{ (FCD)} \\ &\Rightarrow \angle LMN = 9.504210148 \text{ (FCD)} \\ &\Rightarrow LN = \sqrt{15.7^2 + 12.8^2 - 2 \times 15.7 \times 12.8 \times \cos 9.504\dots^\circ} \\ &\Rightarrow LN = 3.731885117 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{LN = 3.73 \text{ cm (3 sf)}}}.\end{aligned}$$

27. The histogram shows information about the times, in minutes, that some passengers had to wait at an airport. (3)



Work out the percentage of the passengers who had to wait for more than one hour.

Solution

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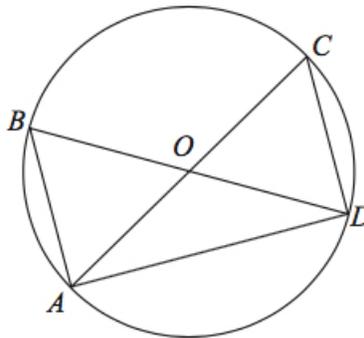
Time	Frequency Density	Width	Frequency
$0 < t \leq 20$	12	20	$12 \times 20 = 240$
$20 < t \leq 30$	10.8	10	$11.8 \times 10 = 108$
$30 < t \leq 45$	7	15	$7 \times 15 = 105$
$45 < t \leq 60$	5	15	$5 \times 15 = 75$
$60 < t \leq 90$	1.8	30	$1.8 \times 30 = 54$
$90 < t \leq 120$	0.6	30	$0.6 \times 30 = 18$
Total			600

The percentage of the passengers who had to wait for more than one hour is

$$\frac{(54 + 18)}{600} \times 100\% = \underline{\underline{12\%}}.$$

28. AOC and BOD are diameters of a circle, centre O .

(3)



Prove that triangle ABD and triangle DCA are congruent.

Solution

AD is common.

$BD = AC$ (diameters)

$\angle BAD = \angle ADC$ (they are both right-angles)

So, triangle ABD and triangle DCA are congruent (RHS).