Dr Oliver Mathematics
Mathematics: National Qualifications N5
2019 Paper 1: Non-Calculator
1 hour 15 minutes

The total number of marks available is 50.
You must write down all the stages in your working.

1. Given that (2)

$$f(x) = 5x^3,$$

evaluate $f(-2)$.

Solution

$$\begin{aligned} f(-2) &= 5 \cdot (-2)^3 \\ &= 5 \cdot (-8) \\ &= \underline{\underline{-40}}. \end{aligned}$$

2. Evaluate (2)

$$\tfrac{3}{8} \times 1\tfrac{5}{7}.$$

Give your answer in its simplest form.

Solution

$$\begin{aligned} \tfrac{3}{8} \times 1\tfrac{5}{7} &= \tfrac{3}{8} \times \tfrac{12}{7} \\ &= \tfrac{3}{2} \times \tfrac{3}{7} \\ &= \underline{\underline{\tfrac{9}{14}}}. \end{aligned}$$

3. Expand and simplify (3)

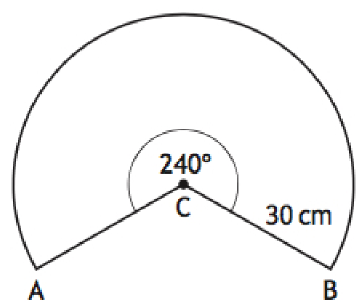
$$(x + 5)(2x^2 - 7x - 3).$$

Solution

$\times$	$2x^2$	$-7x$	-3
x	$2x^3$	$-7x^2$	$-3x$
$+5$	$+10x^2$	$-35x$	-15

$$(x+5)(2x^2-7x-3) = \underline{\underline{2x^3+3x^2-38x-15}}.$$

4. The diagram below shows a sector of a circle, centre C. (3)



The radius of the circle is 30 centimetres.
Calculate the length of the major arc AB.
Take $pi = 3.14$.

Solution

$$\begin{aligned}\text{Major arc} &= \frac{240}{360} \times 2 \times \pi \times 30 \\ &= \tfrac{4}{3} \times 3.14 \times 30 \\ &= 40 \times 3.14 \\ &= \underline{\underline{125.6 \text{ cm}}}.\end{aligned}$$

5. The midday temperatures in Grantford were recorded over a nine-day period.
The temperatures, in °C, were

$$4 \quad 7 \quad 4 \quad 3 \quad 6 \quad 10 \quad 9 \quad 5 \quad 3.$$

(a) Calculate the median and semi-interquartile range for these temperatures. (3)

Solution

First, we sort them into order:

$$3 \quad 3 \quad 4 \quad 4 \quad 5 \quad 6 \quad 7 \quad 9 \quad 10.$$

$$\begin{aligned}\text{Median} &= \frac{(9+1)}{2}\text{th number} \\ &= 5\text{th number} \\ &= \underline{\underline{5^\circ\text{C}}}\end{aligned}$$

and

$$\begin{aligned}\text{semi-interquartile range} &= \tfrac{1}{2}(7\tfrac{1}{2}\text{th number} - 2\tfrac{1}{2}\text{th number}) \\ &= \tfrac{1}{2}\left(\frac{7+9}{2} - \frac{3+4}{2}\right) \\ &= \tfrac{1}{2}(8 - 3.5) \\ &= \tfrac{1}{2}(4.5) \\ &= \underline{\underline{2.25^\circ\text{C}}}.\end{aligned}$$

Over the same nine day period the midday temperatures in Endoch were also recorded. The median temperature was 8°C, and the semi-interquartile range was 1.5°C.

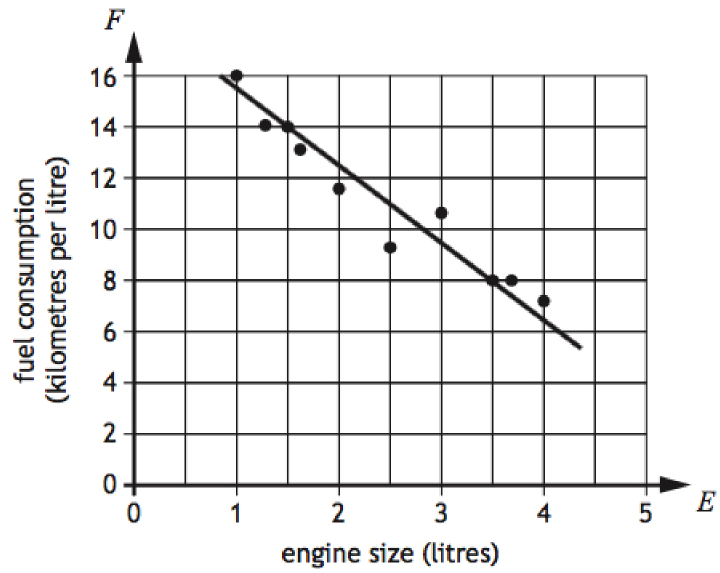
(b) Make two valid comments comparing the midday temperatures of Grantford and Endoch during this period. (2)

Solution

The average temperature was warmer in Endoch and the semi-interquartile range was more consistent in Endoch.

6. The fuel consumption of a group of cars is recorded.

The scattergraph shows the relationship between the fuel consumption, F kilometres per litre, and the engine size, E litres, of the cars.



A line of best fit has been drawn.

(a) Find the equation of the line of best fit in terms of F and E. Give the equation in its simplest form. (3)

Solution

The line of best fit goes between $(1.5, 14)$ and $(3.5, 8)$. Now,

$$\text{gradient} = \frac{14-8}{1.5-3.5} = \frac{6}{-2} = -3$$

and the equation is

$$F - 14 = -3(E - 1.5) \Rightarrow F - 14 = -3E + 4.5 \Rightarrow \underline{\underline{F = 18.5 - 3E}}.$$

Amaar's car has an engine size of 1.1 litres.

(b) Use your equation from part (a) to estimate how many kilometres per litre he should expect to get. (1)

Solution

$$\begin{aligned} E = 1.1 \Rightarrow F &= 18.5 - 3(1.1) \\ \Rightarrow F &= 18.5 - 3.3 \\ \Rightarrow \underline{\underline{F = 15.2 \text{ kilometres per litre}}}. \end{aligned}$$

7. The area of a trapezium is given by the formula (3)

$$A = \tfrac{1}{2}h(x + y).$$

Make x the subject of the formula.

Solution

$$\begin{aligned} A = \tfrac{1}{2}h(x+y) &\Rightarrow 2A = h(x+y) \\ &\Rightarrow x + y = \frac{2A}{h} \\ &\Rightarrow \underline{\underline{x = \frac{2A}{h} - y}}. \end{aligned}$$

8. John bought 7 bags of cement and 3 bags of gravel.
The total weight of these bags was 215 kilograms.

(a) Write down an equation to illustrate this information. (1)

Solution

(Sigh: we're using 'weight' and 'mass' interchangeably? Go and speak to your Physics teacher ...) Let c kg and g kg be the weight of cement and the weight of gravel respectively. Then

$$\underline{\underline{7c + 3g = 215}} \quad (1)$$

Shona bought 5 bags of cement and 4 bags of gravel.
The total weight of her bags was 200 kilograms.

(b) Write down an equation to illustrate this information. (1)

Solution

$$\underline{\underline{5c + 4g = 200}} \quad (2)$$

(c) Calculate the weight of one bag of cement and the weight of one bag of gravel. (4)

Solution

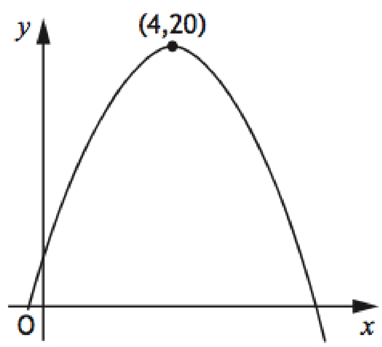
$$\begin{aligned} 4 \times (1): &\quad 28c + 12g = 860 \quad (3) \\ 3 \times (2): &\quad 15c + 12g = 600 \quad (4). \end{aligned}$$

Now, (3) − (4):

$$\begin{aligned} 13c = 260 \Rightarrow c &= 20 \\ &\Rightarrow 140 + 3g = 215 \\ &\Rightarrow 3g = 75 \\ &\Rightarrow g = 25; \end{aligned}$$

hence, the weight of one bag of cement is $\underline{\underline{20 \text{ kg}}}$ and the weight of one bag of gravel is $\underline{\underline{25 \text{ kg}}}$.

9. The graph shows a parabola.



The maximum turning point has coordinates $(4, 20)$ as shown in the diagram.

(a) Write down the equation of the axis of symmetry of the graph. (1)

Solution

$\underline{\underline{x = 4}}$.

The equation of the parabola is of the form

$$y = b - (x + a)^2.$$

(b) State the values of

(i) a, (1)

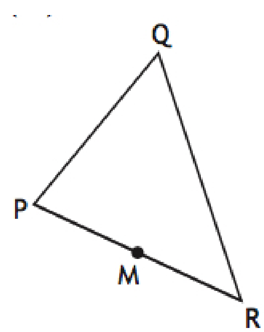
> **Solution**
> $\underline{\underline{a = -4}}$.

(ii) b. (1)

> **Solution**
> $\underline{\underline{b = 20}}$.

10. In triangle PQR,

$$\overrightarrow{PR} = \begin{pmatrix} 6 \\ -4 \end{pmatrix} \text{ and } \overrightarrow{RQ} = \begin{pmatrix} -1 \\ 8 \end{pmatrix}.$$



(a) Express $\overrightarrow{PQ}$ in component form. (1)

> **Solution**
>
> $$\begin{aligned} \overrightarrow{PQ} &= \overrightarrow{PR} + \overrightarrow{RQ} \\ &= \begin{pmatrix} 6 \\ -4 \end{pmatrix} + \overrightarrow{RQ} = \begin{pmatrix} -1 \\ 8 \end{pmatrix} \\ &= \underline{\underline{\begin{pmatrix} 5 \\ 4 \end{pmatrix}}}. \end{aligned}$$

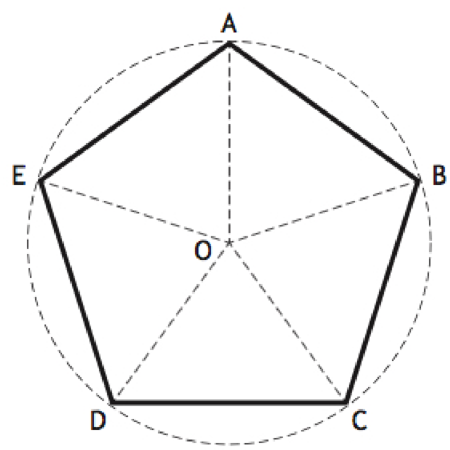
M is the midpoint of PR.

(b) Express $\overrightarrow{MQ}$ in component form. (2)

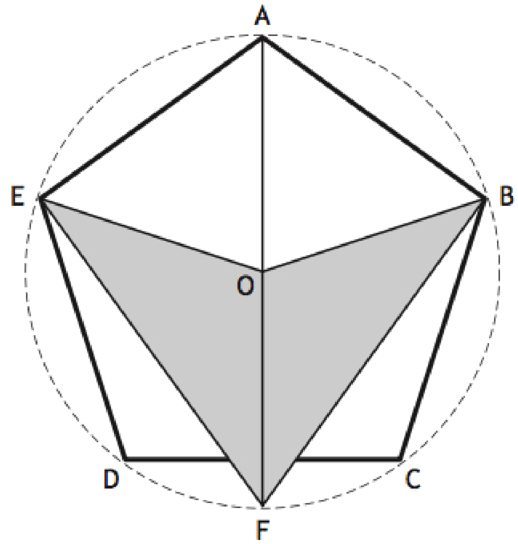
Solution

$$
\begin{aligned}
\overrightarrow{MQ} &= \overrightarrow{MR} + \overrightarrow{RQ} \\
&= \tfrac{1}{2}\overrightarrow{PQ} + \overrightarrow{RQ} \\
&= \tfrac{1}{2}\begin{pmatrix} 6 \\ -4 \end{pmatrix} + \begin{pmatrix} -1 \\ 8 \end{pmatrix} \\
&= \begin{pmatrix} 3 \\ -2 \end{pmatrix} + \begin{pmatrix} -1 \\ 8 \end{pmatrix} \\
&= \underline{\underline{\begin{pmatrix} 2 \\ 6 \end{pmatrix}}}.
\end{aligned}
$$

11. Pam is designing a company logo. (3)
She starts by drawing a regular pentagon $ABCDE$.
The vertices of the pentagon lie on the circumference of a circle with centre O.



She then adds to the design as shown in the diagram below.



AF is a diameter of the circle.
Calculate the size of angle OFB.

Solution

$$\begin{aligned}\angle AOB &= \frac{360}{5} \\ &= 72^\circ\end{aligned}$$

and

$$\angle FOB = 180 - 72 = 108^\circ \text{ (supplementary angles).}$$

Finally,

$$\begin{aligned}\angle OFB (= \angle OBF) &= \tfrac{1}{2}(180 - 108) \\ &= \tfrac{1}{2}(72) \\ &= \underline{\underline{36^\circ}}.\end{aligned}$$

12. Express (3)

$$\frac{\sqrt{2}}{\sqrt{40}}$$

as a fraction with a rational denominator.
Give your answer in its simplest form.

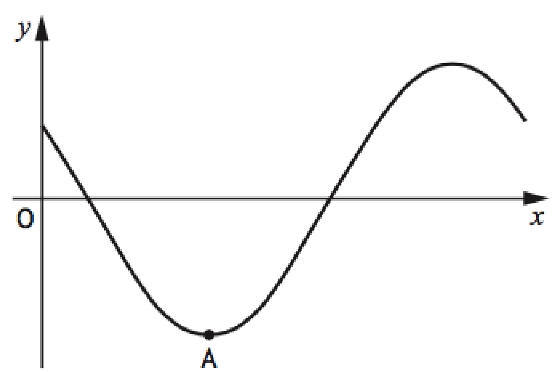
Solution

$$\begin{aligned}\frac{\sqrt{2}}{\sqrt{40}} &= \frac{\sqrt{2}}{\sqrt{40}} \times \frac{\sqrt{40}}{\sqrt{40}}\\ &= \frac{\sqrt{2}\times\sqrt{40}}{40}\\ &= \frac{\sqrt{2}\times\sqrt{4\times 2\times 5}}{40}\\ &= \frac{\sqrt{2}\times\sqrt{4}\times\sqrt{2}\times\sqrt{5}}{40}\\ &= \frac{2\times 2\times\sqrt{5}}{40}\\ &= \frac{4\times\sqrt{5}}{40}\\ &= \underline{\underline{\frac{\sqrt{5}}{10}}}.\end{aligned}$$

13. Part of the graph of (2)

$$y = 3\cos(x+45)^\circ$$

is shown in the diagram.



The graph has a minimum turning point at A.
State the coordinates of A.

Solution

$\underline{\underline{A(135, -3)}}$.

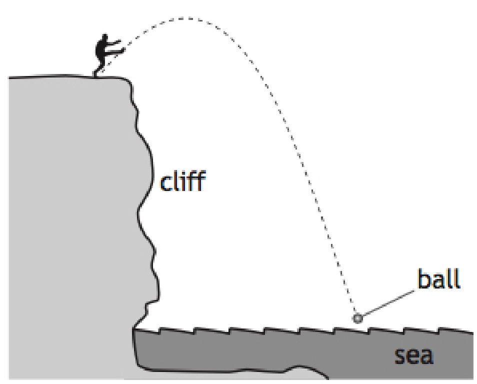
14. Solve the equation (3)

$$\tfrac{1}{2}x - 1 = \frac{3 - x}{5}.$$

Solution

$$\begin{aligned}\tfrac{1}{2}x - 1 = \frac{3 - x}{5} &\Rightarrow \tfrac{1}{2}x - 1 = \tfrac{3}{5} - \tfrac{1}{5}x \\ &\Rightarrow \tfrac{7}{10}x = \tfrac{8}{5} \\ &\Rightarrow 7x = 16 \\ &\Rightarrow \underline{\underline{x = \tfrac{16}{7} \text{ or } 2\tfrac{2}{7}}}.\end{aligned}$$

15. A ball is kicked from a clifftop.



The height, h metres, of the ball relative to the clifftop after t seconds is given by

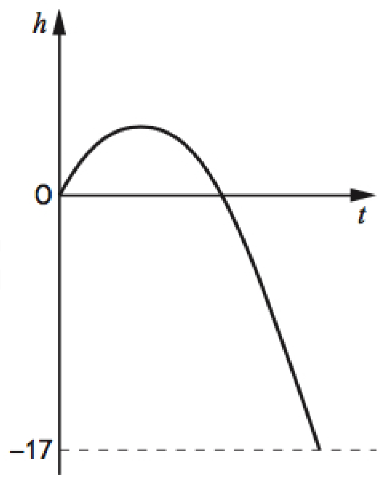
$$h = 12t - 5t^2.$$

(a) Calculate the height of the ball above the clifftop after 2 seconds. (1)

Solution

$$
\begin{aligned}
h &= 12(2) - 5(2^2) \\
&= 24 - 20 \\
&= \underline{\underline{4 \text{ metres}}}.
\end{aligned}
$$

The graph below represents the height, h metres, of the ball relative to the clifftop after t seconds.



The sea is 17 metres below the clifftop.

(b) After how many seconds will the ball hit the sea? (4)

Solution

$$12t - 5t^2 = -17 \Rightarrow 5t^2 - 12t - 17 = 0$$

$$\left.\begin{array}{ll} \text{add to:} & -12 \\ \text{multiply to:} & (+5) \times (-17) = -85 \end{array}\right\} -17,\ +5$$

$$
\begin{aligned}
&\Rightarrow 5t^2 - 17t + 5t - 17 = 0 \\
&\Rightarrow t(5t - 17) + (5t - 17) = 0 \\
&\Rightarrow (5t - 17)(t + 1) = 0 \\
&\Rightarrow 5t - 17 = 0 \text{ or } t + 1 = 0 \\
&\Rightarrow t = 3\tfrac{2}{5} \text{ or } t = -1;
\end{aligned}
$$

as $t > 0$, $\underline{\underline{t = 3\frac{2}{5} \text{ seconds}}}$.