Dr Oliver Mathematics
OCR FMSQ Additional Mathematics
2025 Paper
2 hours

The total number of marks available is 100.
You must write down all the stages in your working.
You are permitted to use a scientific or graphical calculator in this paper.
Give your final answers to a degree of accuracy that is appropriate to the context.

1. Find the coefficient of x^3 in the expansion of (3)

$$(2 - 3x)^5.$$

Solution

Well, the coefficient of x^3 is

$$\begin{aligned}\binom{5}{3}(2)^2(-3)^3 &= 10 \times 4 \times (-27) \\ &= \underline{\underline{-1\,080}}.\end{aligned}$$

2. **In this question you must show detailed reasoning.** (3)

Solve the inequality

$$7 < 3(x - 2) + 1 < 16.$$

Solution

Subtract 1:

$$7 < 3(x - 2) + 1 < 16 \Rightarrow 6 < 3(x - 2) < 15$$

divide by 3:

$$\Rightarrow 2 < x - 2 < 5$$

add 2:

$$\Rightarrow \underline{\underline{4 < x < 7}}.$$

3. Determine the exact value of (3)

$$x^2 - x + 5$$

when

$$x = 3 + \sqrt{2}.$$

Solution

Well,

$\times$	3	$+\sqrt{2}$
3	9	$+3\sqrt{2}$
$+\sqrt{2}$	$+3\sqrt{2}$	$+2$

and so

$$x^2 = 11 + 6\sqrt{2}.$$

Finally,

$$\begin{aligned} x = 3 + \sqrt{2} &\Rightarrow \text{exact value} = (3 + \sqrt{2})^2 - (3 + \sqrt{2}) + 5 \\ &\Rightarrow \text{exact value} = 11 + 6\sqrt{2} - 3 - \sqrt{2} + 5 \\ &\Rightarrow \underline{\underline{\text{exact value} = 13 + 5\sqrt{2}}}. \end{aligned}$$

4. Determine the equation of the tangent to the curve (4)

$$y = 4 + x + x^2$$

at the point $(1, 6)$.

Solution

Well,

$$y = 4 + x + x^2 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 1 + 2x$$

and

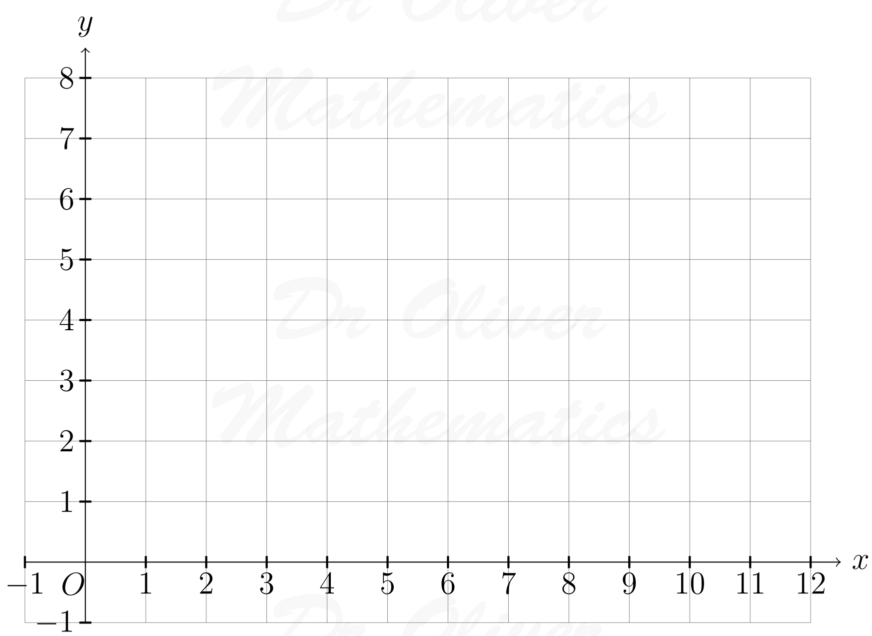
$$x = 1 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 3.$$

Hence, the equation of the tangent is

$$\begin{aligned} y - 6 = 3(x - 1) &\Rightarrow y - 6 = 3x - 3 \\ &\Rightarrow \underline{\underline{y = 3x + 3}}. \end{aligned}$$

5. (a) On the grid below, indicate the region for which the following inequalities hold. (4)
You should shade the region that is **not** satisfied by the inequalities.

$$\begin{aligned} 2x + 3y &\leqslant 12 \\ x + 3y &\geqslant 9 \\ x &\geqslant 0. \end{aligned}$$



Solution

(b) Write down the minimum value of (1)

$$2x + y$$

within this region.

Solution
Well,

$$x = 0, y = 3 \Rightarrow 2x + y = 3$$
$$x = 0, y = 4 \Rightarrow 2x + y = 4$$
$$x = 3, y = 2 \Rightarrow 2x + y = 8;$$

hence, minimum value is $\underline{\underline{3}}$.

6. A car accelerates from rest in a straight line.

At time t seconds the displacement of the car, s metres, is modelled by the equation

$$s = \tfrac{1}{2}t^3 - \tfrac{1}{24}t^4.$$

(a) Determine the maximum velocity of the car. (6)

Solution
Well,

$$s = \tfrac{1}{2}t^3 - \tfrac{1}{24}t^4 \Rightarrow v = \tfrac{3}{2}t^2 - \tfrac{1}{6}t^3$$
$$\Rightarrow a = 3t - \tfrac{1}{2}t^2.$$

Now,

$$\begin{aligned} a = 0 &\Rightarrow 3t - \tfrac{1}{2}t^2 = 0 \\ &\Rightarrow \tfrac{1}{2}t(6-t) = 0 \\ &\Rightarrow t = 0 \text{ or } 6 - t = 0 \\ &\Rightarrow t = 0 \text{ or } t = 6. \end{aligned}$$

Next,

$$t = 0 \Rightarrow v = 0$$

and

$$t = 6 \Rightarrow v = 18.$$

Hence, the maximum velocity of the car is $\underline{\underline{18 \text{ m/s}}}$.

(b) Describe what is happening when $t \geqslant 9$. (2)

Solution
Well,

$$\tfrac{3}{2}t^2 - \tfrac{1}{6}t^3 = \tfrac{1}{6}t^2(9-t),$$

so $t > 9$, the car will be moving in the $\underline{\underline{\text{opposite direction}}}$.

7. A small club consists of 10 members.

(a) In how many ways can 4 members be chosen from the 10 members? (2)

Solution

$$\binom{10}{4} = \underline{\underline{210}}.$$

(b) You are given that half of the members are male and half are female.

In how many ways can 4 members be chosen so that

(i) they are either all male or all female, (2)

Solution
Now,

$$\begin{aligned}\text{different ways} &= \text{all men} + \text{all female}\\ &= \binom{5}{4} + \binom{5}{4}\\ &= 5+5\\ &= \underline{\underline{10}}.\end{aligned}$$

(ii) 2 are male and 2 are female? (2)

Solution
Next,

$$\begin{aligned}\text{2 male, 2 female} &= \binom{5}{2} \times \binom{5}{2}\\ &= 10 \times 10\\ &= \underline{\underline{100}}.\end{aligned}$$

8. **In this question you must show detailed reasoning.**

(a) Solve the simultaneous equations (4)

$$y = 2x^2 - 5x + 4 \text{ and } x + y = 2.$$

Solution
Well,

$$x + y = 2 \Rightarrow y = 2 - x$$

and

$$
\begin{aligned}
y = 2x^2 - 5x + 4 &\Rightarrow 2 - x = 2x^2 - 5x + 4 \\
&\Rightarrow 2x^2 - 4x + 2 = 0 \\
&\Rightarrow 2(x^2 - 2x + 1) = 0 \\
&\Rightarrow 2(x-1)^2 = 0 \\
&\Rightarrow x - 1 = 0 \\
&\Rightarrow \underline{\underline{x = 1}} \\
&\Rightarrow \underline{\underline{y = 1}}.
\end{aligned}
$$

(b) State the geometrical relationship between the graphs of (1)

$$y = 2x^2 - 5x + 4 \text{ and } x + y = 2.$$

Solution

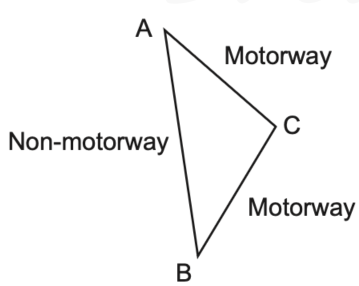
The graph $y = 2x^2 - 5x + 4$ is a $\underline{\underline{\text{tangent}}}$ to the straight line $x + y = 2$.

9. Tom plans to travel by car from a point A to a point B.

There are two routes that he can take.

- He can take either a motorway route or a non-motorway route.
- Tom models the roads used on these two routes as straight lines.
- The line AB represents the non-motorway route.
- The lines AC and CB represent the two motorways used for the motorway route.

The diagram illustrates the two routes where C is the junction of the motorways.



Not to scale

You are given that

- $AC = 50$ km,
- $CB = 40$ km, and
- angle $ACB = 120°$.

(a) Calculate the distance AB. (3)

Solution

Cosine rule:

$$\begin{aligned} & AB^2 = AC^2 + BC^2 - 2 \times AC \times BC \times \cos ACB \\ \Rightarrow\ & AB^2 = 50^2 + 40^2 - 2 \times 50 \times 40 \times \cos 120° \\ \Rightarrow\ & AB^2 = 6100 \\ \Rightarrow\ & \underline{\underline{AB = 10\sqrt{61} \text{ or } 78.1 \text{ km (3 sf)}}}. \end{aligned}$$

In his model Tom assumes that

- he drives at a constant speed,
- he can move through the junction at C without losing speed, and
- his speed on the non-motorway route is kv where v is his speed on the motorway route, k is a constant, and $k < 1$.

(b) Determine the value of k such that the time taken to travel each route is the same. (3)

Solution

Well, the time to travel on the motorway is

$$\frac{90}{v}$$

and the time to travel on the non-motorway is

$$\frac{10\sqrt{61}}{kv}.$$

Now,

$$\begin{aligned} \frac{90}{v} = \frac{10\sqrt{61}}{kv} & \Rightarrow k = \frac{10\sqrt{61}}{90} \\ & \Rightarrow k = 0.867\,805\,519\,5 \text{ (FCD)} \\ & \Rightarrow \underline{\underline{k = 0.868 \text{ (3 sf)}}}. \end{aligned}$$

10. **In this question you must show detailed reasoning.** (5)

Solve the equation

$$\frac{1}{x-5} - \frac{1}{x} + \frac{5}{6} = 0.$$

Solution

Multiply all three terms by $6x(x-1)$:

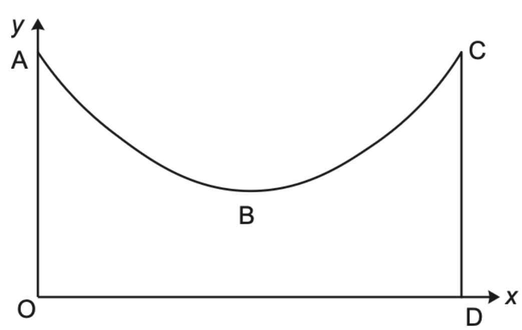
$$\begin{aligned}
\frac{1}{x-5} - \frac{1}{x} + \frac{5}{6} = 0 &\Rightarrow 6x - 6(x-5) + 5x(x-5) = 0 \\
&\Rightarrow 6x - 6x + 30 + 5x^2 - 25x = 0 \\
&\Rightarrow 5x^2 - 25x + 30 = 0 \\
&\Rightarrow 5(x^2 - 5x + 6) = 0
\end{aligned}$$

$$\left.\begin{array}{ll} \text{add to:} & -5 \\ \text{multiply to:} & +6 \end{array}\right\} -2, -3$$

$$\begin{aligned}
&\Rightarrow 5(x-2)(x-3) = 0 \\
&\Rightarrow x - 2 = 0 \text{ or } x - 3 = 0 \\
&\Rightarrow \underline{\underline{x = 2 \text{ or } x = 3}}.
\end{aligned}$$

11. A rectangular metal plate $OACD$ is aligned on a coordinate system such that O is the origin and A, C, and D have coordinates $(0, 16)$, $(6, 16)$, and $(6, 0)$, respectively.

A curved section ABC is removed from $OACD$ to leave the shape $OABCD$ shown in the diagram.



The curved edge ABC has equation

$$y = x^2 - 6x + 16,$$

and B is the stationary point on the curve.

(a) By completing the square, determine the coordinates of B. (3)

Solution

Well,

$$\begin{array}{ll} \text{add to:} & -6 \\ \text{half it:} & -3 \\ \text{square it:} & (-3)^2 = +9 \end{array}$$

and

$$\begin{aligned} y &= x^2 - 6x + 16 \\ &= (x^2 - 6x + 9) + 7 \\ &= (x-3)^2 + 7; \end{aligned}$$

so, the minimum is at $x = 3$ and $y = 7$.

Hence, $\underline{\underline{B(3,7)}}$.

(b) The line $y = k$ is such that it cuts the **area** of the shape $OABCD$ in half. (5)

Find the value of k.

Solution

Well,

$$\begin{aligned} \text{area } OABCD &= \int_0^6 (x^2 - 6x + 16)\,\mathrm{d}x \\ &= \left[\tfrac{1}{3}x^3 - 3x^2 + 16x\right]_{x=0}^{6} \\ &= (72 - 108 + 96) - (0 - 0 + 0) \\ &= 60. \end{aligned}$$

Now, half the area is

$$\frac{60}{2} = 30.$$

Finally,

$$k \times 6 = 30 \Rightarrow \underline{\underline{k = 5}}.$$

12. Heidi and Charlie have been asked to solve the equation

$$\mathrm{f}(x) = 1.3^x - x = 0.$$

(a) Show by sign change that $\mathrm{f}(x) = 0$ has a root, α, in the interval $[1, 2]$. (2)

Solution

Well,

$$\begin{aligned} \mathrm{f}(1) &= 0.3 \\ \mathrm{f}(2) &= -0.31; \end{aligned}$$

since the function $\mathrm{f}(x)$ is continuous and changes sign, we have a $\underline{\underline{\text{root}}}$ in $[1, 2]$.

(b) Heidi attempts to solve the equation $\mathrm{f}(x) = 0$ using the iterative formula (2)

$$x_{r+1} = \frac{\log_{10} x_r}{\log_{10} 1.3}.$$

She starts with $x_0 = 1.4$.

Show that the sequence derived from this iterative formula fails to find α.

Solution

Now,

$$\begin{aligned} x_0 &= 1.4 \\ x_1 &= \frac{\log_{10} 1.4}{\log_{10} 1.3} = 1.282\,462\,142 \text{ (FCD)} \\ x_2 &= \frac{\log_{10} 1.282\ldots}{\log_{10} 1.3} = 0.948\,230\,428\,1 \text{ (FCD)} \\ x_3 &= \frac{\log_{10} 0.948\ldots}{\log_{10} 1.3} = -0.202\,610\,438\,6 \text{ (FCD)}, \end{aligned}$$

and we get an error next iteration.

Hence, the sequence derived from this iterative formula $\underline{\underline{\text{fails}}}$ to find α.

(c) Charlie attempts to solve the equation $\mathrm{f}(x) = 0$ using the iterative formula (3)

$$x_{r+1} = 1.3^{x_r}.$$

Starting with $x_0 = 1.4$, determine the value of α correct to 3 significant figures.

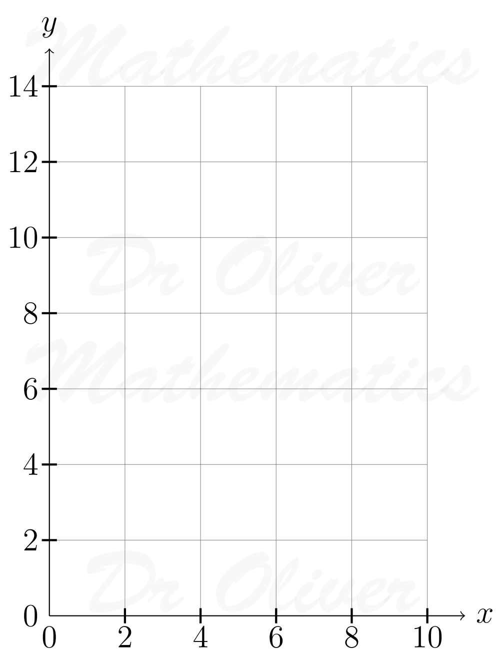
Solution

Next, Now,

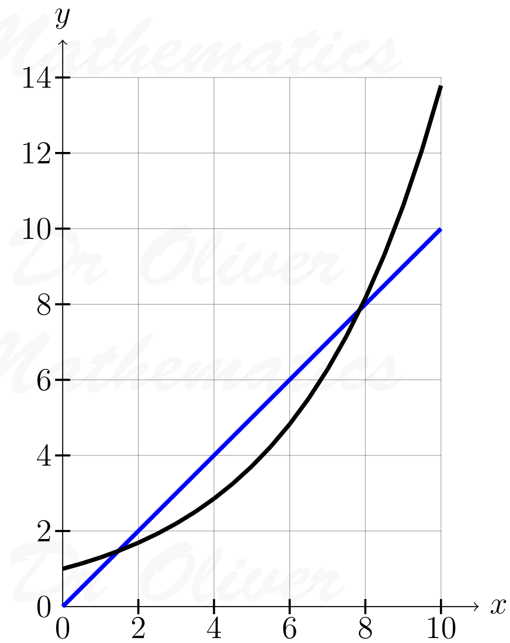
$$\begin{aligned}
x_0 &= 1.4 \\
x_1 &= 1.3^{1.4} = 1.443\,845\,399 \text{ (FCD)} \\
x_2 &= 1.3^{1.443\ldots} = 1.460\,550\,524 \text{ (FCD)} \\
x_3 &= 1.3^{1.460\ldots} = 1.466\,965\,914 \text{ (FCD)} \\
x_4 &= 1.3^{1.466\ldots} = 1.469\,437\,145 \text{ (FCD)} \\
x_5 &= 1.3^{1.469\ldots} = 1.470\,390\,182 \text{ (FCD)};
\end{aligned}$$

hence, $\underline{\underline{x = 1.47 \ \ (3 \text{ sf})}}$.

(d) (i) Plot both the curve $y = 1.3^x$ and the line $y = x$ on the grid below to show that there is another root, β, which you should identify on your graph. (4)



Solution



(ii) Give the integer interval within which β lies. (1)

Solution
$\underline{\underline{7 < \beta < 8}}$.

(e) Without doing any further calculation, use your graph to explain why Charlie will not find β using his iterative formula. (2)

Solution
E.g., because the gradient of the curve is greater than 1, a staircase diagram will either diverge or converge to the lower root, however close to the second root you start.

13. (a) You are given that (3)

$$\sin 2\theta = 0.7.$$

Find the values of θ for $0^\circ \leqslant \theta \leqslant 360^\circ$.
Give your answers correct to 1 decimal place.

Solution
Well,

$$0^\circ \leqslant \theta \leqslant 360^\circ \Rightarrow 0^\circ \leqslant 2\theta \leqslant 720^\circ$$

and

$$\begin{aligned}\sin 2\theta = 0.7 &\Rightarrow 2\theta = 44.427\,004,\ 135.572\,996,\ 404.427\,004,\ 495.572\,996 \text{ (FCD)}\\ &\Rightarrow \theta = 22.213\,502,\ 67.786\,498,\ 202.213\,502,\ 247.786\,498 \text{ (FCD)}\\ &\Rightarrow \underline{\underline{\theta = 22.2,\ 67.8,\ 202.2,\ 247.8 \text{ (1 dp)}}}.\end{aligned}$$

(b) Given that $0^\circ \leqslant \theta \leqslant 90^\circ$, show that (3)

$$\frac{1}{\sin^2\theta} - \frac{1}{\tan^2\theta} \equiv 1.$$

Solution

Now,

$$\begin{aligned}\frac{1}{\sin^2\theta} - \frac{1}{\tan^2\theta} &\equiv \frac{1}{\sin^2\theta} - \frac{1}{\frac{\sin^2\theta}{\cos^2\theta}}\\ &\equiv \frac{1}{\sin^2\theta} - \frac{\cos^2\theta}{\sin^2\theta}\\ &\equiv \frac{1-\cos^2\theta}{\sin^2\theta}\\ &\equiv \frac{\sin^2\theta}{\sin^2\theta}\\ &\equiv \underline{\underline{1}},\end{aligned}$$

as required.

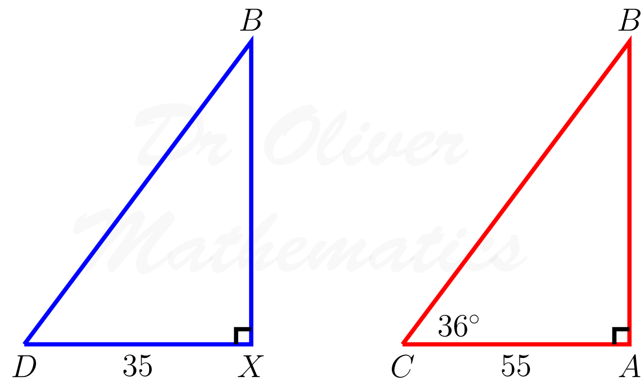
14. A tower AB stands vertically at a point A. (5)

- C is a point due south of A and on the same horizontal level.
- The distance AC is 55 m.
- From C the angle of elevation of B is 36°.
- D is a point due west of the tower and 15 m higher than A and C.
- The horizontal distance of D from the tower is 35 m.

Find the angle of elevation of B from D.

Solution

Let X be the point on AB which is 15 m above A.



Now,

$$\tan = \frac{\text{opp}}{\text{adj}} \Rightarrow \tan 36^\circ = \frac{AB}{55}$$
$$\Rightarrow AB = 55\tan 36^\circ$$

and

$$BX = AB - AX$$
$$= 55\tan 36^\circ - 15.$$

Let θ° be the angle of elevation of B from D. Then,

$$\tan = \frac{\text{opp}}{\text{adj}} \Rightarrow \tan\theta^\circ = \frac{55\tan 36^\circ - 15}{35}$$
$$\Rightarrow \theta = 35.494\,120\,72 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{\theta = 35.5 \text{ (3 sf)}}}.$$

15. The table shows population data for a country.
t is the number of years after 1920.

Years after 1920 (t)	0	20	40	60	80	100
Population in millions (p)	20	31.7	51.3	79.6	126.2	200

Jane believes that this population growth may be modelled by the equation

$$p = c \times 10^{kt},$$

where c and k are constants.

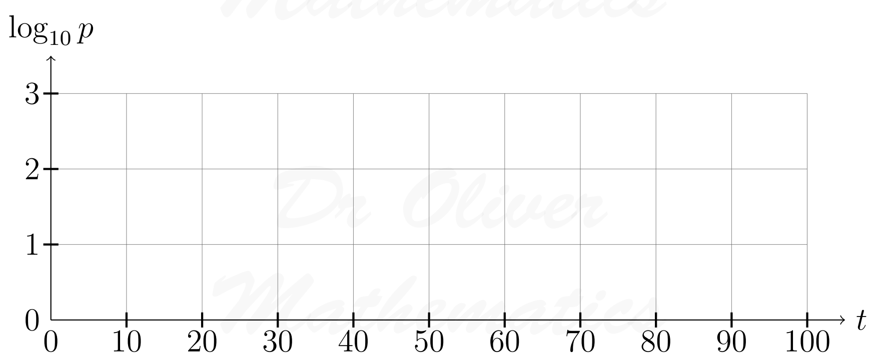
(a) Derive an equation for $\log_{10} p$ in terms of c, k, and t. (3)

Solution

Well,

$$\begin{aligned} p = c \times 10^{kt} &\Rightarrow \log_{10} p = \log_{10}(c \times 10^{kt}) \\ &\Rightarrow \log_{10} p = \log_{10} c + \log_{10} 10^{kt} \\ &\Rightarrow \underline{\underline{\log_{10} p = \log_{10} c + kt}}. \end{aligned}$$

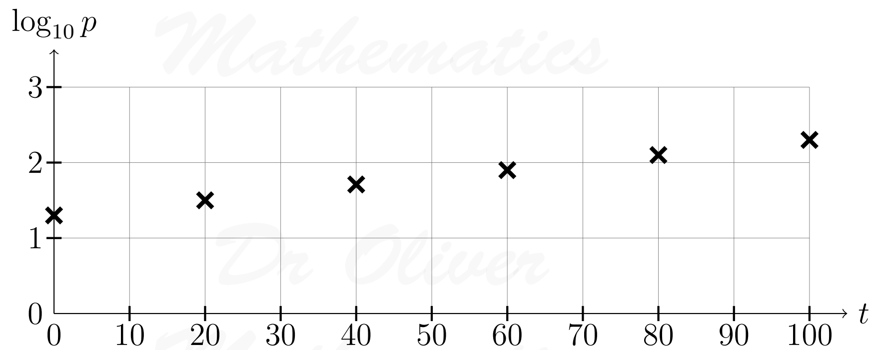
(b) Plot the points $(t, \log_{10} p)$ for the 6 values of t on the grid. (3)
(You may use the blank row in the table above for your values of $\log_{10} p$.)



Solution

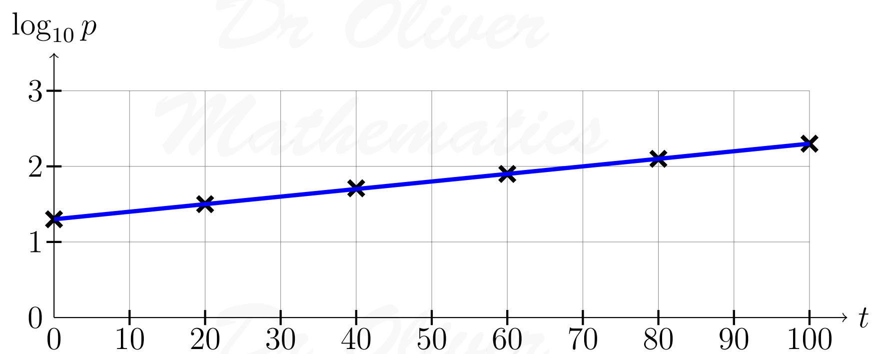
We will use 3 significant figures:

Years after 1920 (t)	0	20	40	60	80	100
Population in millions (p)	20	31.7	51.3	79.6	126.2	200
$\log_{10} p$	1.30	1.50	1.71	1.90	2.10	2.30



(c) On the grid draw a line of best fit. (1)

Solution



(d) (i) State the value of c. (1)

Solution
$c = 20$.

(ii) Using your line, or otherwise, estimate a value for k. (1)

Solution
The line of best fit goes through $(0, 1.30)$ and $(100, 2.30)$ and

$$k = \frac{2.30 - 1.30}{100 - 1} = \tfrac{1}{100}.$$

(e) According to the model what will be the population in 2040? (2)

Solution

Now,

$$\begin{aligned} t = 120 \Rightarrow p &= 20 \times 10^{\frac{1}{100} \times 120} \\ \Rightarrow p &= 20 \times 10^{1.2} \\ \Rightarrow p &= 316.978\,638\,5 \text{ (FCD)} \\ \Rightarrow \underline{\underline{p} &= \underline{\underline{317 \text{ (3 sf)}}}. \end{aligned}$$

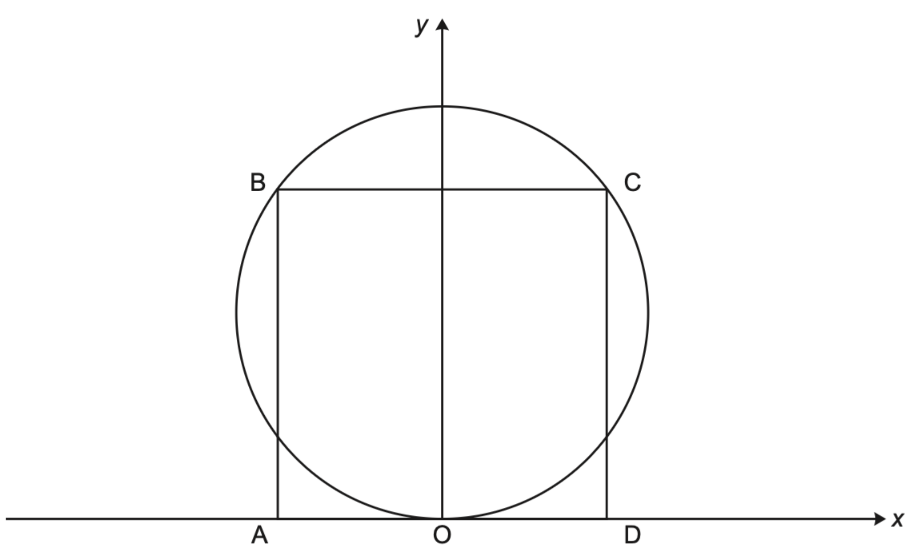
(f) Give a reason why this model may **not** be realistic in the long term. (1)

Solution

E.g., because it is governed by an exponential function and, given long enough, the city's population would exceeds the world's population.

16. A square with vertices $ABCD$ is aligned in a coordinate system where the coordinates of A, B, C, and D are $(-8, 0)$, $(-8, 16)$, $(8, 16)$, and $(8, 0)$ respectively.

- The origin, O, is the midpoint of AD.
- A circle is such that it passes through the points B and C and touches the line AD at the origin as shown in the diagram.



(a) the radius of the circle, (4)

Solution

Let P be the centre of the circle, radius r. Then

$$r = OP = BP = CP$$

and

$$\begin{aligned} OP = BP &\Rightarrow OP^2 = BP^2 \\ &\Rightarrow r^2 = 8^2 + (16 - r)^2 \end{aligned}$$

$\times$	16	$-r$
16	256	$-16r$
$-r$	$-16r$	$+r^2$

$$\begin{aligned} &\Rightarrow r^2 = 64 + (256 - 32r + r^2) \\ &\Rightarrow 32r = 320 \\ &\Rightarrow \underline{\underline{r = 10}}. \end{aligned}$$

(b) the equation of the circle. (3)

Solution

The centre is $(0, 10)$ and the equation is

$$\underline{\underline{x^2 + (y - 10)^2 = 100}}.$$