Dr Oliver Mathematics
AQA Further Maths Level 2
June 2017 Paper 2
2 hours

The total number of marks available is 105.
You must write down all the stages in your working.
You are permitted to use a scientific or graphical calculator in this paper.

1. The nth term of a sequence is

$$\frac{3-5n}{2}.$$

(a) Work out the difference between the 20th term and the 8th term. (2)

Solution

$$\begin{aligned}\text{8th term} &= \frac{3-5(8)}{2} \\ &= \frac{3-40}{2} \\ &= -18\tfrac{1}{2}\end{aligned}$$

and

$$\begin{aligned}\text{20th term} &= \frac{3-5(20)}{2} \\ &= \frac{3-100}{2} \\ &= -48\tfrac{1}{2}.\end{aligned}$$

Finally,

$$\begin{aligned}\text{difference} &= \text{8th term} - \text{20th term} \\ &= -18\tfrac{1}{2} - (-48\tfrac{1}{2}) \\ &= \underline{\underline{30}}.\end{aligned}$$

The nth term of another sequence is

$$\frac{3n}{1-2n}.$$

(b) Write down the limiting value of the sequence as $n \to \infty$. (1)

Solution

$$\begin{aligned}\frac{3n}{1-2n} &= \frac{3}{\frac{1}{n}-2} \\ &\to \frac{3}{0-2} \text{ (as } n \to \infty) \\ &= \underline{\underline{-1\tfrac{1}{2}}}.\end{aligned}$$

2.

$$\mathbf{A} = \begin{pmatrix} 4 & -1 \\ 3 & -2 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}.$$

(a) Workout $\mathbf{A}^2$. (2)

Solution

$$\begin{aligned}\mathbf{A}^2 &= \begin{pmatrix} 4 & -1 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} 4 & -1 \\ 3 & -2 \end{pmatrix} \\ &= \underline{\underline{\begin{pmatrix} 13 & -2 \\ 6 & 1 \end{pmatrix}}}.\end{aligned}$$

$$k\mathbf{B} = \begin{pmatrix} 11-3k \\ 11-6k \end{pmatrix},$$

where k is a constant.

(b) Work out the value of k. (2)

Solution

$$k\mathbf{B} = \begin{pmatrix} 11-3k \\ 11-6k \end{pmatrix} \Rightarrow \begin{pmatrix} 5k \\ 2k \end{pmatrix} = \begin{pmatrix} 11-3k \\ 11-6k \end{pmatrix}.$$

Now,

$$\begin{aligned}5k = 11 - 3k &\Rightarrow 8k = 11 \\ &\Rightarrow \underline{\underline{k = 1\tfrac{3}{8}}}.\end{aligned}$$

(c) Give a reason why it is not possible to work out $\mathbf{BA}$. (1)

Solution

Well, **B** is a 2×1 matrix and **A** is a 2×2. Can you do $(2 \times 1) \times (2 \times 2)$? No, you can't (why?). Hence, it is not possible to work out **BA**

3. p, q, and r are all integers greater than 1.
$pqr = 1\,365$.

(a) Work out one possible set of values for p, q, and r. (2)

Solution

	1 365
3	455
5	91
7	13
13	1

E.g., $p = 3,\ q = 5,\ \text{and } r = 91$.

a and b are both square numbers greater than 1.
$ab - 11b$ is also a square number.

(b) By factorising $ab - 11b$, work out one possible pair of values for a and b. (2)
You **must** show your working.

Solution

Well,

$$ab - 11b = b(a - 11).$$

E.g., $b = 4$ and $a = p - 11$, for some perfect square p. Go through the perfect squares and, for example, $a = 36 - 11 = 25$.

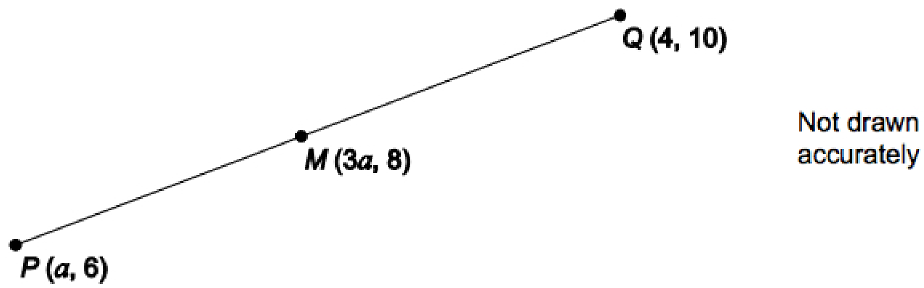
4. Solve (2)

$$\frac{56}{\sqrt[3]{x}} = 4.$$

Solution

$$\frac{56}{\sqrt[3]{x}} = 4 \Rightarrow \sqrt[3]{x} = 14$$
$$\Rightarrow x = 14^3$$
$$\Rightarrow \underline{\underline{x = 2\,744}}.$$

5. M is the midpoint of PQ. (3)



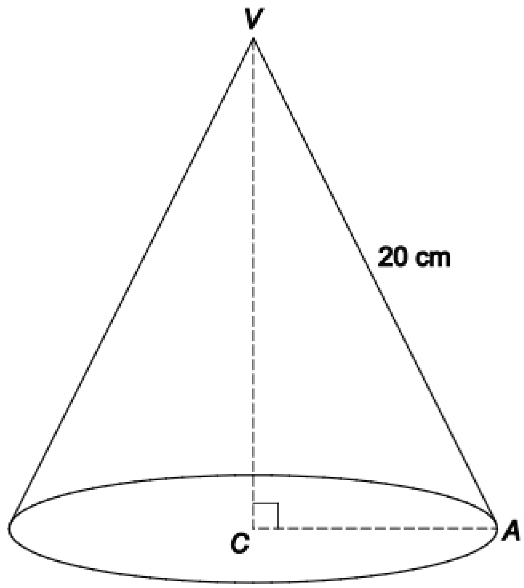
Work out the value of a.

Solution

PM is parallel to PQ:

$$\frac{10-6}{4-a} = \frac{8-6}{3a-a} \Rightarrow \frac{4}{4-a} = \frac{1}{a}$$
$$\Rightarrow 4a = 4 - a$$
$$\Rightarrow 5a = 4$$
$$\Rightarrow \underline{\underline{a = \tfrac{4}{5}}}.$$

6. A cone has vertex V. (3)
C is the centre of the base.
The slant height, VA, is 20 cm.
The angle between VA and VC is 38°.



Work out the radius of the base.

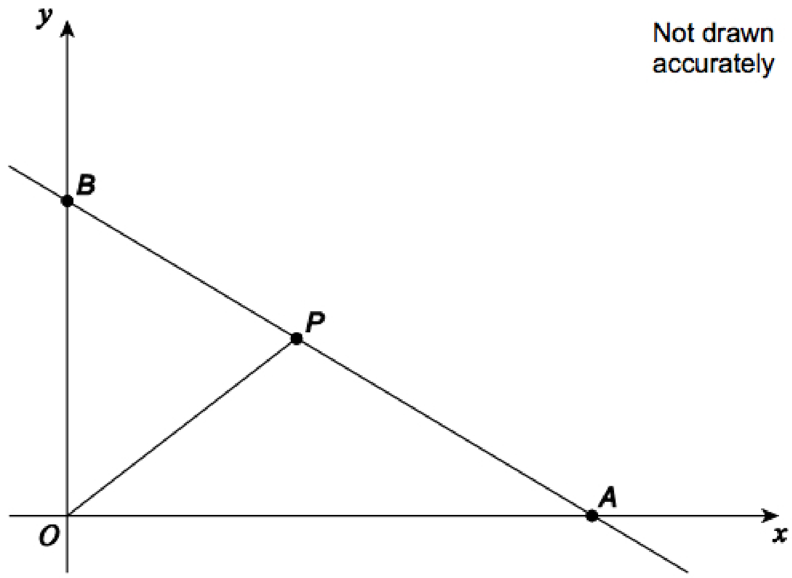
Solution

$$\sin = \frac{\text{opp}}{\text{hyp}} \Rightarrow \sin 38^\circ = \frac{\text{radius}}{20}$$
$$\Rightarrow \text{radius} = 20 \sin 38^\circ$$
$$\Rightarrow \text{radius} = 12.313\,229\,51 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{\text{radius} = 12.3 \text{ cm (3 sf)}}}.$$

7. The equation of the line through B, P, and A is (4)

$$4x + 5y = 40.$$

$BP : PA = 2 : 3.$



Work out the area of triangle OBP.

Solution

Well,

$$x = 0 \Rightarrow 5y = 40 \Rightarrow y = 8$$

and

$$y = 0 \Rightarrow 4x = 40 \Rightarrow x = 10.$$

So $A(10, 0)$ and $B(0, 8)$.

Now, $2 + 3 = 5$ so $BP : BA = 2 : 5$. Next,

$$\begin{aligned} y - \text{coordinate of } P &= 8 - (\tfrac{2}{5} \times 8) \\ &= 4\tfrac{4}{5} \end{aligned}$$

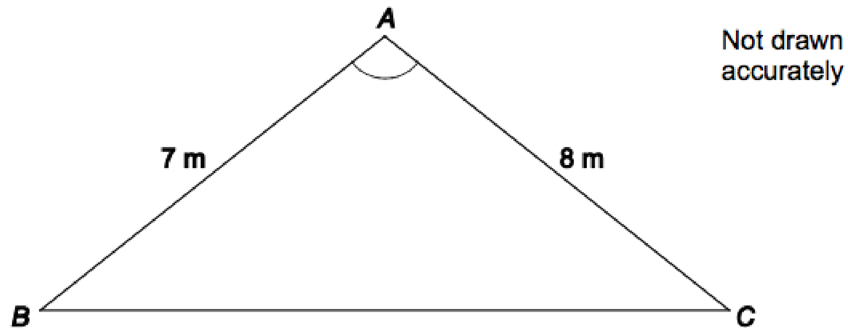
and

$$x - \text{coordinate of } P = \tfrac{1}{4}[40 - 5(4\tfrac{4}{5})] = 4.$$

Finally,

$$\begin{aligned} \text{area} &= \tfrac{1}{2} \times 8 \times 4 \\ &= \underline{\underline{16}}. \end{aligned}$$

8. The perimeter of a triangular flower bed, ABC, is marked out using 27 metres of rope. (4)



Work out the size of angle BAC.

Solution

Well,

$$BC = 27 - 7 - 8 = 12 \text{ m}$$

and we use the cosine rule:

$$\begin{aligned} \cos BAC = \frac{7^2 + 8^8 - 12^2}{2 \times 7 \times 8} &\Rightarrow \cos BAC = -\tfrac{31}{112} \\ &\Rightarrow \angle BAC = 106.068\,459\,4 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{\angle BAC = 106^\circ \text{ (3 sf)}}}. \end{aligned}$$

9. (5)

$$-11 < 5x \leqslant 5 \text{ and } 6x + 7 \leqslant 4x + 4.$$

Show that there is **exactly** one integer that x can be.

Solution

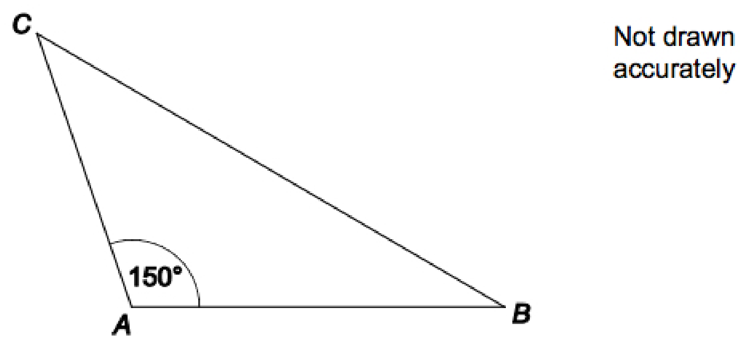
$$\begin{aligned} -11 < 5x \leqslant 5 &\Rightarrow -2\tfrac{1}{5} < x \leqslant 1 \\ &\Rightarrow x = -2, -1, 0, \text{ or } 1 \end{aligned}$$

and

$$\begin{aligned} 6x + 7 \leqslant 4x + 4 &\Rightarrow 2x \leqslant -3 \\ &\Rightarrow x \leqslant -1\tfrac{1}{2}. \end{aligned}$$

Hence, $\underline{\underline{x = -2}}$.

10. ABC is an isosceles triangle with $AB = AC$. (3)
The area of ABC is 57.76 cm^2.



Work out the length of AB.

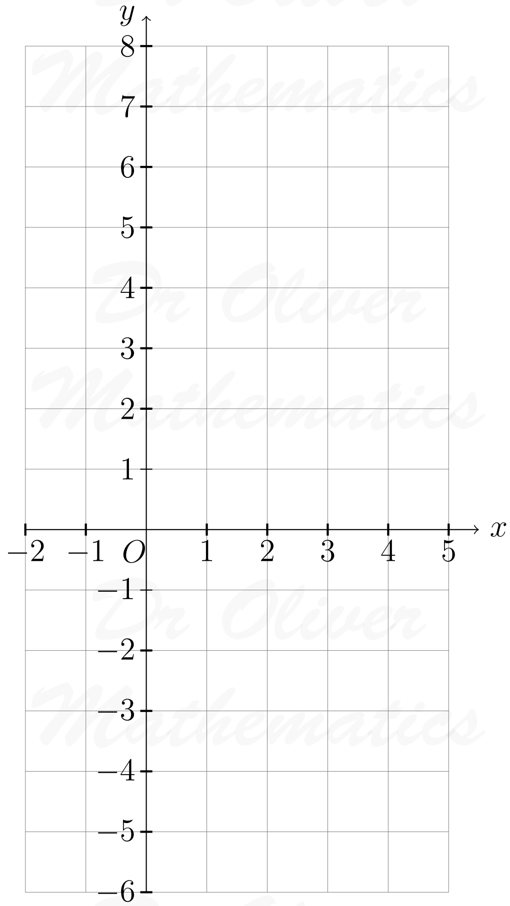
Solution

$$\begin{aligned} \text{Area} = 57.76 &\Rightarrow \tfrac{1}{2} \cdot AB \cdot AC \cdot \sin 150^\circ = 57.76 \\ &\Rightarrow AB^2 = 231.04 \\ &\Rightarrow \underline{\underline{AB = 15.2\text{ cm}}}. \end{aligned}$$

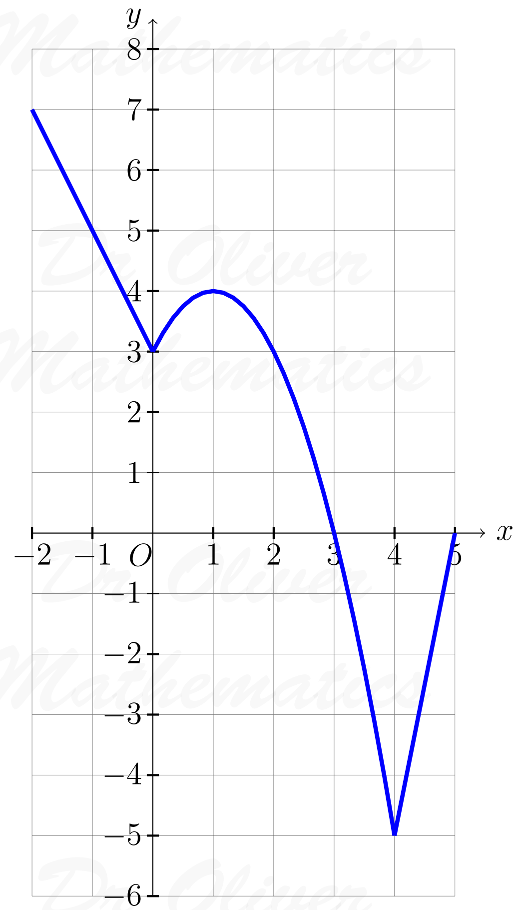
11. A function $f(x)$ is defined as

$$f(x) = \begin{cases} 3 - 2x & \text{for } -2 \leqslant x < 0, \\ (1+x)(3-x) & \text{for } 0 \leqslant x < 4, \\ 5x - 25 & \text{for } 4 \leqslant x \leqslant 5. \end{cases}$$

(a) Draw the graph of $y = f(x)$ on the axes below. (4)



Solution



(b) State the range of $\mathrm{f}(x)$. (2)

Solution

$\underline{\underline{-5 \leqslant \mathrm{f}(x) \leqslant 7}}$.

12. (a) Factorise fully (2)

$$75 - 3x^2.$$

Solution

The difference of two squares:

$$\begin{aligned} 75 - 3x^2 &= 3(25 - x^2) \\ &= \underline{\underline{3(5 + x)(5 - x)}}. \end{aligned}$$

(b) Simplify fully (2)

$$(3n+1)^2 - (3n-1)^2.$$

Solution

The difference of two squares:

$$\begin{aligned}(3n+1)^2 - (3n-1)^2 &= [(3n+1)-(3n-1)][(3n+1)+(3n-1)] \\ &= (2)(6n) \\ &= \underline{\underline{12n}}.\end{aligned}$$

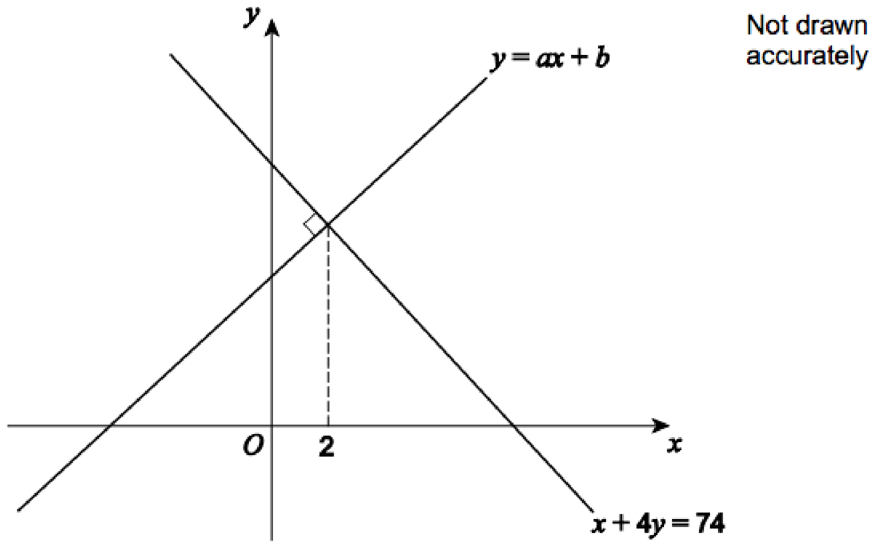
13. Simplify fully (3)

$$\frac{8a}{3a+6} \times \frac{5a+10}{3a^2} \div \frac{4}{15a^3}.$$

Solution

$$\begin{aligned}\frac{8a}{3a+6} \times \frac{5a+10}{3a^2} \div \frac{4}{15a^3} &= \frac{8a}{3(a+2)} \times \frac{5(a+2)}{3a^2} \times \frac{15a^3}{4} \\ &= \frac{2}{1} \times \frac{5}{3} \times \frac{5a^2}{1} \\ &= \underline{\underline{\tfrac{50}{3}a^2}}.\end{aligned}$$

14. The line $y = ax + b$ is perpendicular to the line $x + 4y = 74$. (5)
The lines intersect at the point where $x = 2$.



Work out the values of a and b.

Solution

$$x + 4y = 74 \Rightarrow 4y = -x + 74$$
$$\Rightarrow y = -\tfrac{1}{4}x + \tfrac{37}{2}$$

and so the gradient of the line which is perpendicular to it is

$$m = -\frac{1}{-\frac{1}{4}} = 4;$$

so, $\underline{\underline{a = 4}}$. Now,

$$x = 2 \Rightarrow 2 + 4y = 74$$
$$\Rightarrow 4y = 72$$
$$\Rightarrow y = 18$$

and so the point of intersection is $(2, 18)$. Finally,

$$18 = 4(2) + b \Rightarrow 18 = 8 + b$$
$$\Rightarrow \underline{\underline{b = 10}}.$$

15. Rearrange (3)

$$w = \frac{8x - y}{y}$$

to make y the subject.

Solution

$$\begin{aligned} w = \frac{8x - y}{y} &\Rightarrow wy = 8x - y \\ &\Rightarrow wy + y = 8x \\ &\Rightarrow y(w + 1) = 8x \\ &\Rightarrow \underline{\underline{y = \frac{8x}{w + 1}}}. \end{aligned}$$

16. (a) (1)

$$a = 3^{2b}$$

Circle the correct expression for $\frac{1}{a}$.

$$3^{2b-1} \qquad 3^{-2b} \qquad -3^{2b} \qquad \left(\frac{1}{3}\right)^{-2b}$$

Solution

$$\begin{aligned} \frac{1}{a} &= \frac{1}{3^{2b}} \\ &= \underline{\underline{3^{-2b}}}. \end{aligned}$$

(b) (1)

$$y = 5^x$$

Circle the correct expression for $25y$.

$$5^{x+2} \qquad 25^x \qquad 5^{2x} \qquad 125^x$$

Solution

$$25y = 5^2 \times 5^x$$
$$= \underline{\underline{5^{x+2}}}.$$

(c) (1)

$$w = 2^m$$

Circle the correct expression for w^3.

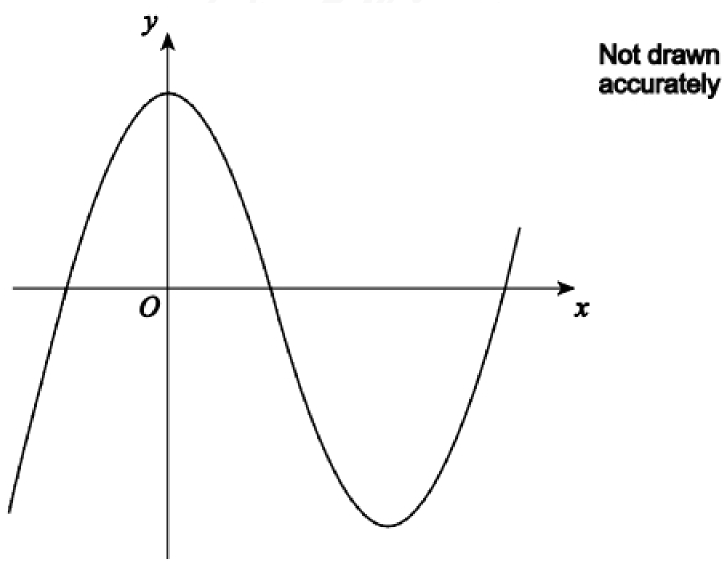
8^{3m} 6^m 2^{m+3} 2^{3m}

Solution

$$w^3 = (2^m)^3$$
$$= \underline{\underline{2^{3m}}}.$$

17. Here is a sketch of

$$y = x^3 - 6x^2 + 7.$$



(a) Use differentiation to work out the coordinates of the stationary point that is a minimum. (4)
You **must** show your working.

Solution

$$y = x^3 - 6x^2 + 7 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 12x$$

and

$$\begin{aligned}\frac{\mathrm{d}y}{\mathrm{d}x} = 0 &\Rightarrow 3x^2 - 12x = 0 \\ &\Rightarrow 3x(x-4) = 0 \\ &\Rightarrow x = 0 \text{ or } x = 4.\end{aligned}$$

Now,

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 12x \Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 6x - 12$$

and

$$x = 4 \Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 12 > 0.$$

Finally,

$$x = 4 \Rightarrow y = -25$$

which means $\underline{\underline{(4, -25)}}$ is a $\underline{\underline{\text{minimum}}}$.

The three roots of

$$x^3 - 6x^2 + 7 = 0$$

are the x-coordinates of the points where the graph intersects the x-axis.

(b) Show that $x = -1$ is one root of (1)

$$x^3 - 6x^2 + 7 = 0.$$

Solution

We use synthetic division:

-1	1	-6	0	7
	$\downarrow$	-1	7	-7
	-1	-7	7	0

Hence, as their is no remainder, $\underline{\underline{x = -1}}$ of a $\underline{\underline{\text{root}}}$ of the cubic.

(c) Hence, work out the other two roots of (5)

$$x^3 - 6x^2 + 7 = 0.$$

Give your answers to 2 decimal places.
You **must** show your working.

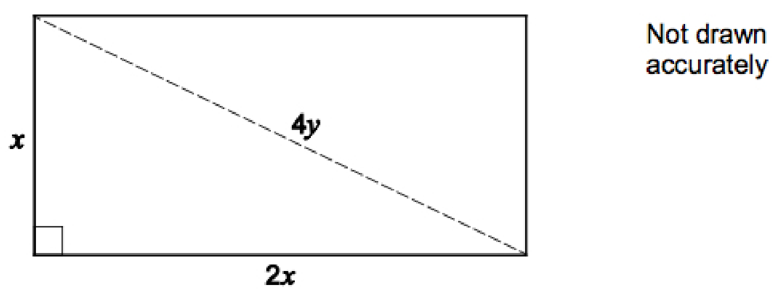
Solution

$$x^3 - 6x^2 + 7 = 0 \Rightarrow (x+1)(x^2 - 7x + 7) = 0:$$

$a = 1$, $b = -7$, and $c = 7$:

$$\begin{aligned} x &= \frac{7 \pm \sqrt{7^2 - 4 \times 1 \times 7}}{2 \times 1} \\ &= \frac{7 \pm \sqrt{21}}{2} \\ &= 1.208\,712\,153,\ 5.791\,287\,847 \text{ (FCD)} \\ &= \underline{\underline{1.21,\ 5.79 \text{ (2 dp)}}}. \end{aligned}$$

18. The diagram shows a rectangle with a diagonal drawn. (4)
The given expressions for the measurements are in centimetres.



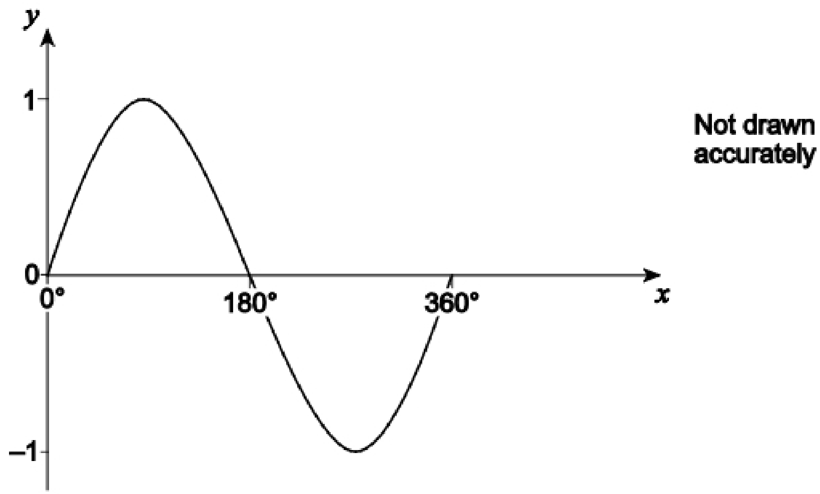
Work out an expression for the area of the rectangle, in cm^2.
Give your answer in its simplest form, in terms of y.

Solution

Pythagoras' theorem:

$$\begin{aligned} (4y)^2 = x^2 + (2x)^2 &\Rightarrow 16y^2 = x^2 + 4x^2 \\ &\Rightarrow 16y^2 = 5x^2 \\ &\Rightarrow x^2 = \tfrac{16}{5}y^2 \\ &\Rightarrow \text{area} = 2x^2 = \underline{\underline{\tfrac{32}{5}y^2}}. \end{aligned}$$

19. Here is a sketch of $y = \sin x$ for $0^\circ \leqslant x \leqslant 360^\circ$.



α is an acute angle measured in degrees.
$\sin \alpha = k$, where k is a constant.
Write the answers to each of the following in terms of k, without involving trigonometric functions.

(a) $\sin(180 - \alpha)^\circ$. (1)

Solution

$$\begin{aligned} \sin(180 - \alpha)^\circ &= \sin 180^\circ \cos \alpha^\circ - \cos 180^\circ \sin \alpha^\circ \\ &= (0)(\cos \alpha^\circ) - (-1) \sin \alpha^\circ \\ &= \sin \alpha^\circ \\ &= \underline{\underline{k}}. \end{aligned}$$

(b) $\sin(360 - \alpha)^\circ$. (1)

Solution

$$\begin{aligned} \sin(360 - \alpha)^\circ &= \sin 360^\circ \cos \alpha^\circ - \cos 360^\circ \sin \alpha^\circ \\ &= (0)(\cos \alpha^\circ) - (1) \sin \alpha^\circ \\ &= -\sin \alpha^\circ \\ &= \underline{\underline{-k}}. \end{aligned}$$

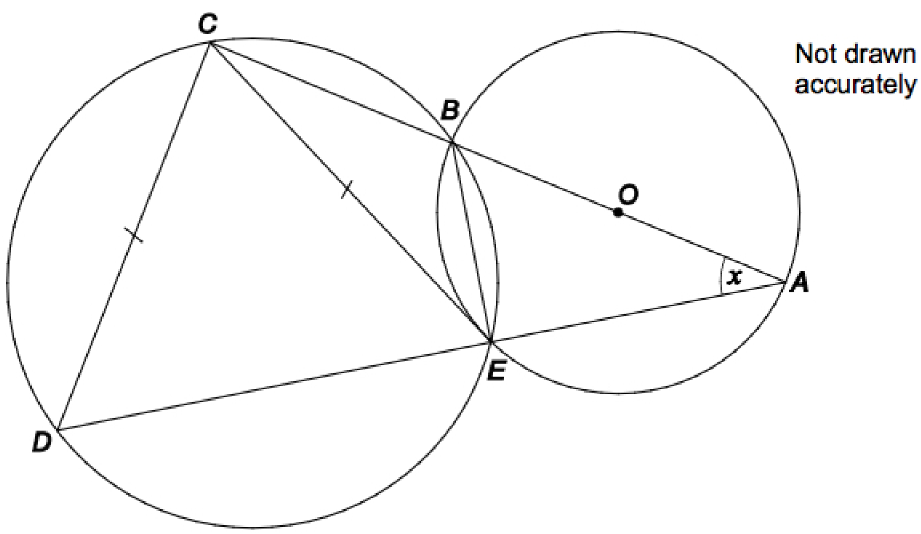
(c) $\cos \alpha^\circ$. (2)

Solution

$$\begin{aligned}\cos \alpha^\circ &= \sqrt{1 - \sin^2 \alpha^\circ} \\ &= \underline{\underline{\sqrt{1 - k^2}}}.\end{aligned}$$

20. Two circles overlap.

- A, B, and E lie on the circle, centre O.
- B, C, D, and E lie on the other circle.
- $AOBC$ and AED are straight lines.
- $CD = CE$.
- Angle $BAE = x$.



(a) Give a reason why angle $BEA = 90^\circ$. (1)

Solution

The angle $\angle BEA$ in a semi-circle is a right-angle.

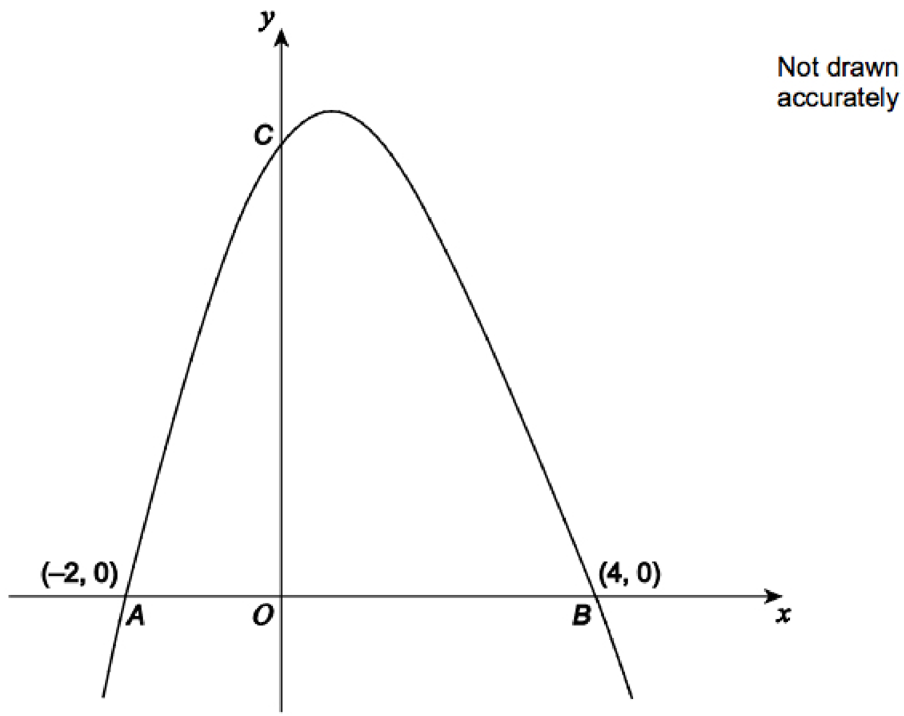
(b) Prove that angle $DCE = 2x$. (4)

Solution
$\angle ABE = 90 - x$ (completing the triangle)
$\angle CDE = 90 - x$ (alternate segment theorem)
$\angle CED = 90 - x$ (base angles)
$\angle DCE = 180 - 2(90 - x) = \underline{\underline{2x}}$ (completing the triangle), as required.

21. Here is a sketch of

$$y = (x + 2)(4 - x).$$

The graph intersects the axes at $A(-2, 0)$, $B(4, 0)$, and C.



(a) Work out the coordinates of C. (1)

Solution

$$x = 0 \Rightarrow y = (0 + 2)(4 - 0) = 8$$

and so $\underline{\underline{C(0, 8)}}$.

(b) Work out the gradient function of the curve. (3)

Solution

$\times$	x	$+2$
4	$4x$	$+8$
$-x$	$-x^2$	$-2x$

$$y = 8 + 2x - x^2 \Rightarrow \underline{\underline{\frac{\mathrm{d}y}{\mathrm{d}x} = 2 - 2x}}.$$

The normal to the curve at C intersects the x-axis at D.

(c) Show that (5)

$$\text{length } BD = 2 \times \text{length } AB.$$

Solution

$$\begin{aligned} x = 0 &\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 2 \\ &\Rightarrow m_{\text{normal}} = -\tfrac{1}{2} \end{aligned}$$

and the equation of CD is

$$y - 8 = -\tfrac{1}{2}(x - 0).$$

Now,

$$\begin{aligned} y = 0 &\Rightarrow -8 = -\tfrac{1}{2}x \\ &\Rightarrow x = 16 \end{aligned}$$

so $D(16, 0)$. Finally,

$$AB = 4 - (-2) = 6$$

and

$$BD = 16 - 4 = 12;$$

hence,

$$\underline{\underline{\text{length } BD = 2 \times \text{length } AB}}.$$

22. The equation of a circle is (5)

$$(x-2)^2+(y-1)^2=16.$$

The equation of a line is $y=2x+1$.
The circle and the line intersect at two points.

Work out the coordinates of the two points.
You **must** show your working.
Do **not** use trial and improvement.

Solution

$$(x-2)^2+(y-1)^2=16 \Rightarrow (x-2)^2+(2x)^2=16$$

$\times$	x	-2
x	x^2	$-2x$
-2	$-2x$	$+4$

$$\Rightarrow (x^2-4x+4)+4x^2=16$$
$$\Rightarrow 5x^2-4x-12=0$$

$$\left.\begin{array}{ll}\text{add to:} & -4 \\ \text{multiply to:} & (+5)\times(-12)=-60\end{array}\right\}-10,\ +6$$

$$\Rightarrow 5x^2-10x+6x-12=0$$
$$\Rightarrow 5x(x-2)+6(x-2)=0$$
$$\Rightarrow (5x+6)(x-2)=0$$
$$\Rightarrow 5x+6=0 \text{ or } x-2=0$$
$$\Rightarrow x=-1\tfrac{1}{5} \text{ or } x=2$$
$$\Rightarrow y=-1\tfrac{2}{5} \text{ or } y=5;$$

hence, the circle and the line intersect at $\underline{\underline{(-1\tfrac{1}{5},-1\tfrac{2}{5}) \text{ and } (2,5)}}$.

23. In this question, $\tan x \neq 0$ and $\sin x \neq 0$. (3)

Show that

$$\frac{1}{\tan^2 x} - \frac{1}{\sin^2 x}$$

is a constant.

Solution

$$\begin{aligned}
\frac{1}{\tan^2 x} - \frac{1}{\sin^2 x} &\equiv \frac{\cos^2 x}{\sin^2 x} - \frac{1}{\sin^2 x} \\
&\equiv \frac{\cos^2 x - 1}{\sin^2 x} \\
&\equiv \frac{-1(1 - \cos^2 x)}{\sin^2 x} \\
&\equiv \frac{-\sin^2 x}{\sin^2 x} \\
&\equiv \underline{\underline{-1}}.
\end{aligned}$$

24. Write (5)

$$12x^2 - 60x + 5$$

in the form

$$a(bx + c)^2 + d,$$

where a, b, c, and d are integers.

Solution

$$\begin{aligned}
1[(12x^2 - \ldots)] &= 1[(\sqrt{12}x - \ldots)^2 + \ldots] \to \text{No} \\
2[(6x^2 - \ldots)] &= 2[(\sqrt{6}x - \ldots)^2 + \ldots] \to \text{No} \\
3[(4x^2 - \ldots)] &= 3[(2x - \ldots)^2 + \ldots] \to \text{Yes}
\end{aligned}$$

So,

$$\begin{aligned}
12x^2 - 60x + 5 &= 3(4x^2 - 20x) + 5 \\
&= 3[(4x^2 - 20x + 25) - 25] + 5 \\
&= 3[(2x - 5)^2 - 25] + 5 \\
&= 3(2x - 5)^2 - 75 + 5 \\
&= \underline{\underline{3(2x - 5)^2 - 70}};
\end{aligned}$$

hence, $\underline{\underline{a = 3,\ b = 2,\ c = -5,\ \text{and}\ d = -70}}$.