Dr Oliver Mathematics
GCSE Mathematics
2013 June Paper 1H: Non-Calculator
1 hour 45 minutes

The total number of marks available is 100.
You must write down all the stages in your working.

1. Given that (2)

$$1\,793 \times 185 = 331\,705,$$

write down the value of

(a) 1.793×185,

Solution

Divide the first number by $1\,000$:

$$1.793 \times 185 = \underline{\underline{331.705}}.$$

(b) $331\,705 \div 1.85$.

Solution

$$\begin{aligned} 1\,793 \times 185 = 331\,705 &\Rightarrow \frac{331\,705}{185} = 1\,793 \\ &\Rightarrow \frac{331\,705}{1.85} = \underline{\underline{179\,300}}. \end{aligned}$$

2. Mr Mason asks 240 Year 11 students what they want to do next year. (4)
15% of the students want to go to college.
$\frac{3}{4}$ of the students want to stay at school.
The rest of the students do not know.
Work out the number of students who do not know.

Solution

The number who want to go to college is

$$24 + 12 = 36 \text{ students}$$

and the number who want to stay at school is

$$\tfrac{3}{4} \times 240 = 3 \times 60 = 180 \text{ students.}$$

Finally, the rest of the students do not know is

$$240 - 36 - 180 = \underline{\underline{24 \text{ students}}}.$$

3. Sixteen babies are born in a hospital. (3)
Here are the weights of the babies in kilograms.

2.4	2.7	3.5	4.4	4.5	4.1	4.4	2.8
4.1	3.8	3.8	4.2	3.3	3.0	3.7	3.3

Show this information in an ordered stem and leaf diagram.

Solution

2	4	7	8				
3	0	3	3	5	7	8	8
4	1	1	2	4	4	5	

Key: 2|4 means 24 kilograms.

4. (a) Expand (1)

$$3(2 + t).$$

Solution

$$3(2 + t) = \underline{\underline{6 + 3t}}.$$

(b) Expand (2)

$$3x(2x + 5).$$

Solution

$$3x(2x+5) = \underline{\underline{6x^2 + 15x}}.$$

(c) Expand and simplify (2)

$$(m+3)(m+10).$$

Solution

$\times$	m	$+3$
m	m^2	$+3m$
$+10$	$+10m$	$+30$

$$(m+3)(m+10) = \underline{\underline{m^2 + 13m + 30}}.$$

5. Write 525 as a product of its prime factors. (3)

Solution

	525
3	175
5	35
5	7
7	1

So

$$525 = 3 \times 5 \times 5 \times 7 = \underline{\underline{3 \times 5^2 \times 7}}.$$

6. Ed has 4 cards. (3)
There is a number on each card.

12	6	15	?

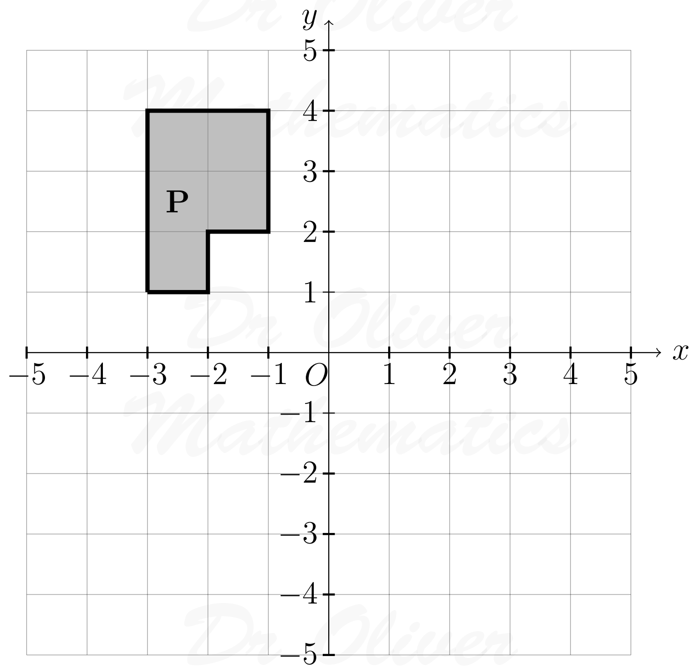
The mean of the 4 numbers on Ed's cards is 10.
Work out the number on the 4th card.

Solution

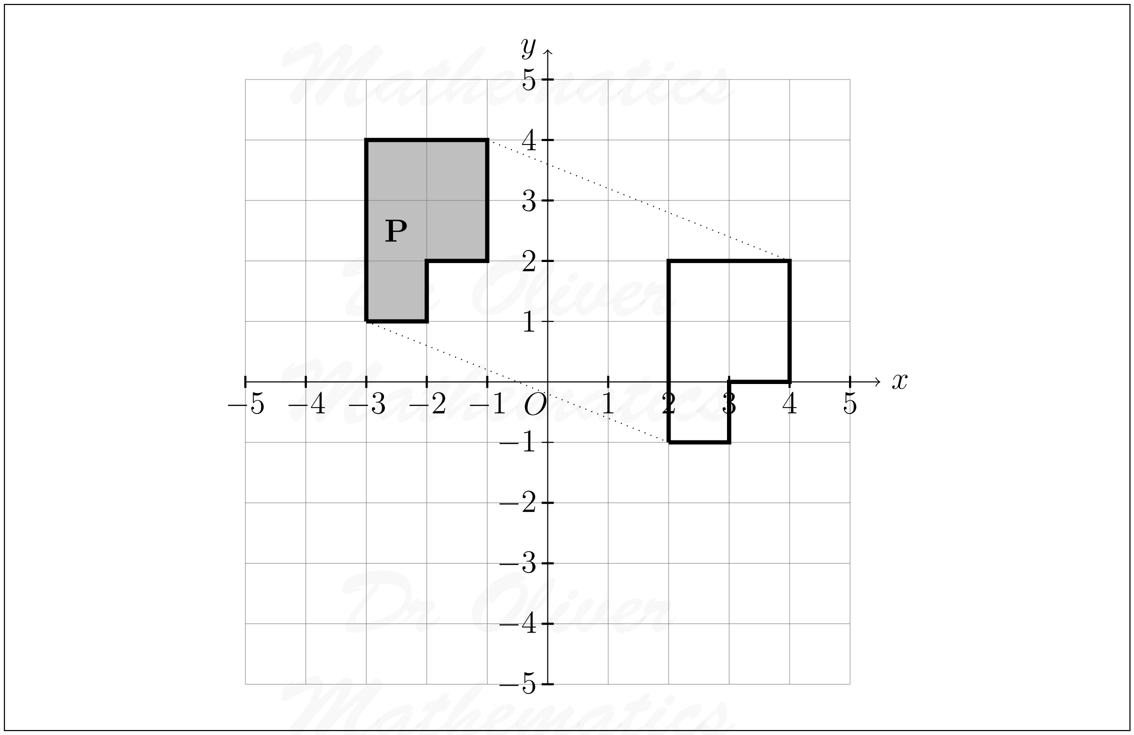
Let x be the number on the fourth card. Now,

$$\frac{12 + 6 + 15 + x}{4} = 10 \Rightarrow 33 + x = 40$$
$$\Rightarrow \underline{\underline{x = 7}}.$$

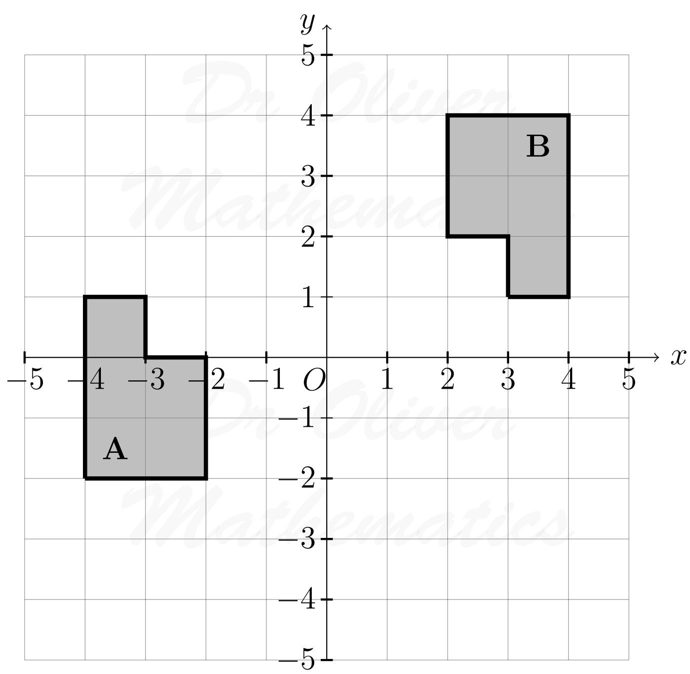
7. (a) Translate shape **P** by the vector $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$. (2)



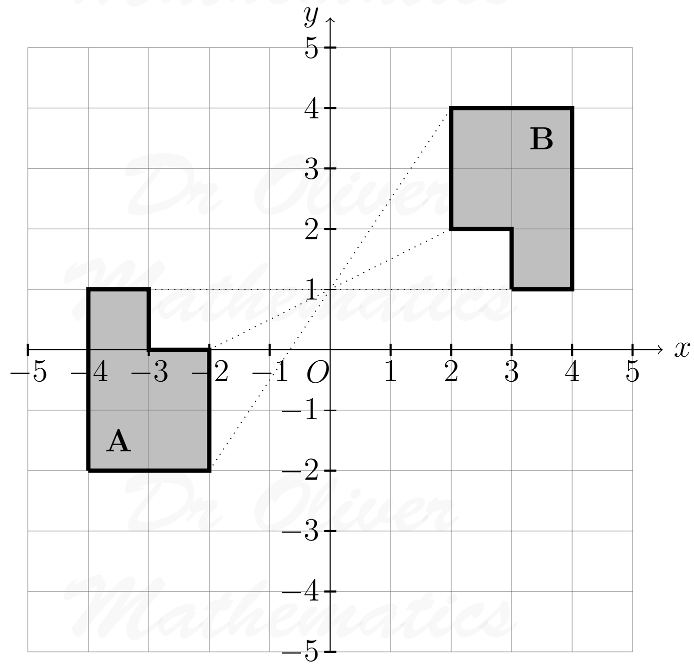
Solution



(b) Describe fully the single transformation that maps shape **A** onto shape **B**. (3)



Solution



It is a rotation, through 180°, about $(0, 1)$.

8. Margaret has some goats. (3)
The goats produce an average total of 21.7 litres of milk per day for 280 days.
Margaret sells the milk in $\frac{1}{2}$ litre bottles.
Work out an estimate for the total number of bottles that Margaret will be able to fill with the milk.
You must show clearly how you got your estimate.

Solution

Round to 1 significant figure:

$$\frac{21.7 \times 280}{\frac{1}{2}} \approx \frac{20 \times 300}{\frac{1}{2}} = \frac{6\,000}{\frac{1}{2}} = \underline{\underline{12\,000 \text{ bottles}}}.$$

9. Matt and Dan cycle around a cycle track. (3)
Each lap Matt cycles takes him 50 seconds.
Each lap Dan cycles takes him 80 seconds.
Dan and Matt start cycling at the same time at the start line.
Work out how many laps they will each have cycled when they are next at the start line together.

Solution

$$\begin{array}{r|l} & 50 \\ \hline 2 & 25 \\ \hline 5 & 5 \\ \hline 5 & 1 \\ \hline \end{array}$$

So
$$50 = 2 \times 5 \times 5 = 2 \times 5^2.$$

$$\begin{array}{r|l} & 80 \\ \hline 2 & 40 \\ \hline 2 & 20 \\ \hline 2 & 10 \\ \hline 2 & 5 \\ \hline 5 & 1 \\ \hline \end{array}$$

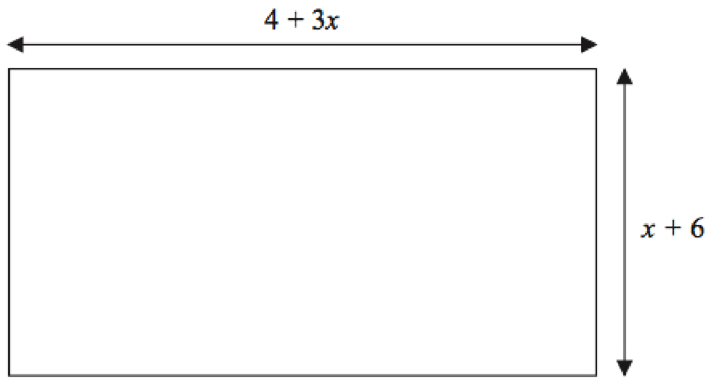
So
$$80 = 2 \times 2 \times 2 \times 2 \times 5 = 2^4 \times 5.$$

Hence,
$$\begin{aligned} \text{LCM}(50, 80) &= 2^4 \times 5^2 \\ &= 16 \times 25 \\ &= 400; \end{aligned}$$

it takes Matt
$$\frac{400}{50} = \underline{\underline{8 \text{ laps}}}$$

and it takes Dan
$$\frac{400}{80} = \underline{\underline{5 \text{ laps}}}.$$

10. The diagram shows a garden in the shape of a rectangle. (4)

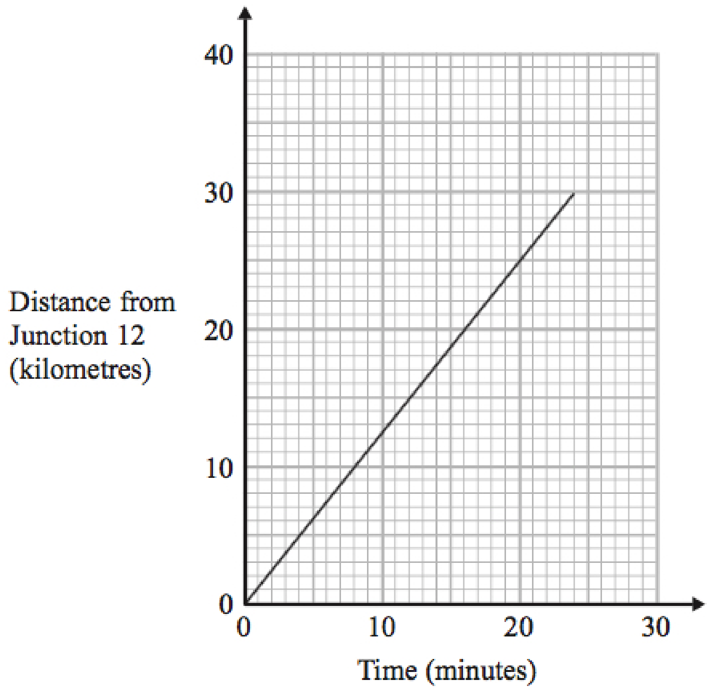


All measurements are in metres.
The perimeter of the garden is 32 metres.
Work out the value of x.

Solution

$$\begin{aligned} 2[(4+3x)+(x+6)] = 32 &\Rightarrow 4x + 10 = 16 \\ &\Rightarrow 4x = 6 \\ &\Rightarrow \underline{\underline{x = 1\tfrac{1}{2} \text{ m}}}. \end{aligned}$$

11. Debbie drove from Junction 12 to Junction 13 on a motorway. (4)
The travel graph shows Debbie's journey.



Ian also drove from Junction 12 to Junction 13 on the same motorway.
He drove at an average speed of 66 km/hour.
Who had the faster average speed, Debbie or Ian?
You must explain your answer.

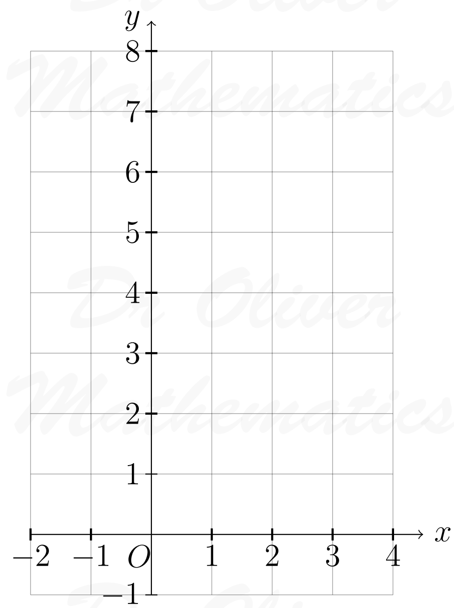
Solution

For Debbie, the time taken is

$$\begin{aligned} \frac{30 \text{ km}}{24 \text{ mins}} &= \frac{30 \text{ km}}{\frac{2}{5} \text{ hr}} \\ &= \frac{30 \times 5}{2} \\ &= \frac{150}{2} \\ &= 75; \end{aligned}$$

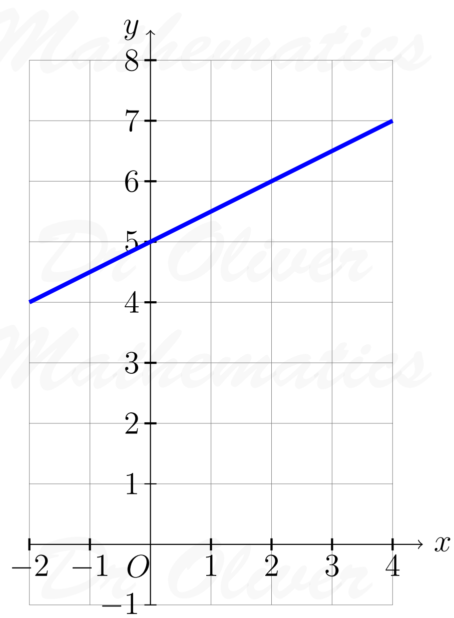
therefore, Debbie did it faster.

12. On the grid, draw the graph of $y = \frac{1}{2}x + 5$ for values of x from -2 to 4. (3)

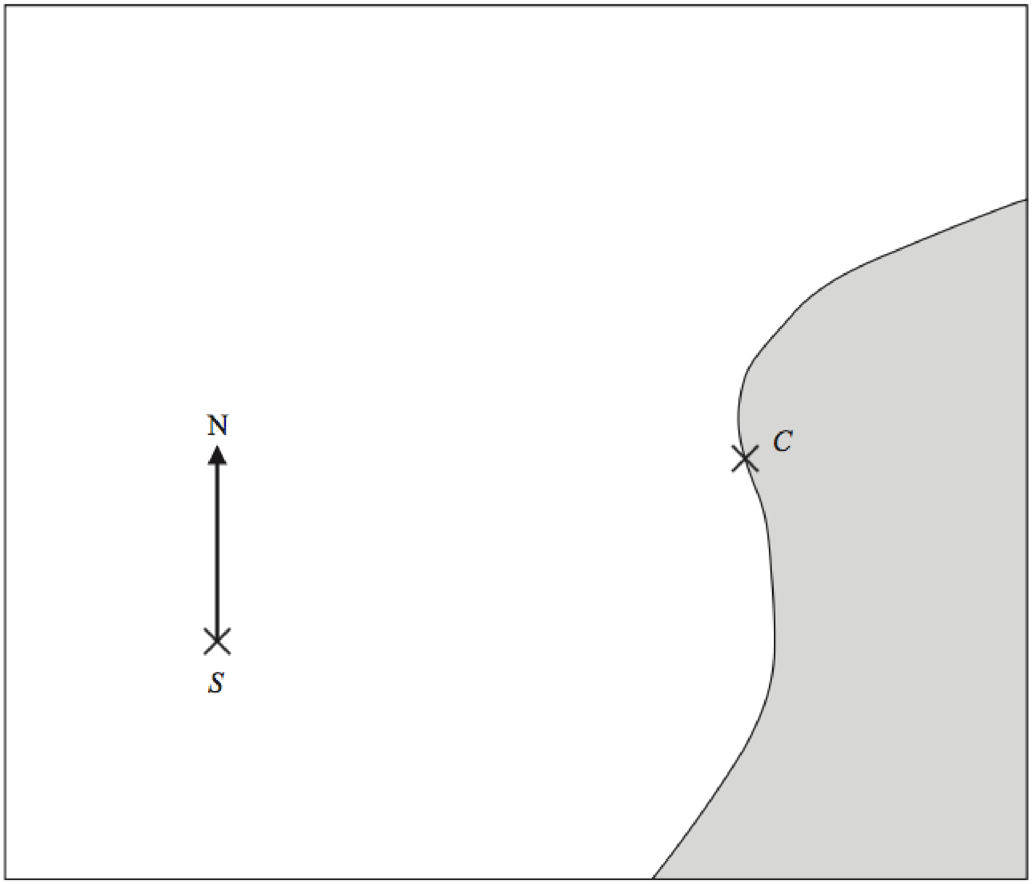


Solution

x	-2	-1	0	1	2	3	4
y	4	4.5	5	5.5	6	6.5	7



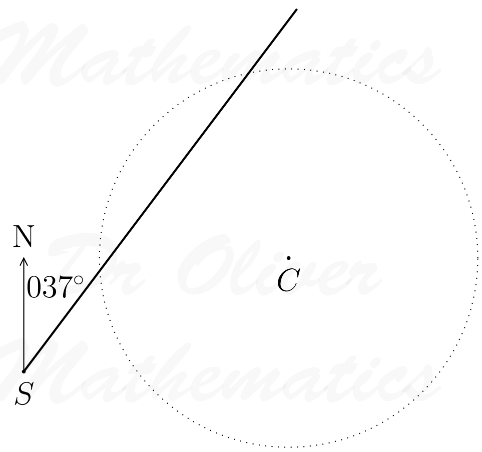
13. Here is a map. (3)
The position of a ship, S, is marked on the map.



Scale 1 cm represents 100 m

Point C is on the coast.
Ships must not sail closer than 500 m to point C.
The ship sails on a bearing of 037°.
Will the ship sail closer than 500 m to point C?
You must explain your answer.

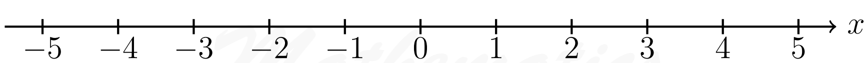
Solution



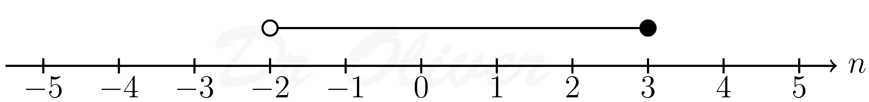
<u>Yes</u>, it does: the ship goes through two different points at distance 500 m as it makes its way.

14. $-2 < n \leqslant 3$.

(a) Represent this inequality on the number line. (2)



Solution



(b) Solve the inequality (2)

$$8x - 3 \geqslant 6x + 4.$$

Solution

$$8x - 3 \geqslant 6x + 4 \Rightarrow 2x \geqslant 7$$
$$\Rightarrow \underline{\underline{x \geqslant 3\tfrac{1}{2}}}.$$

15. One sheet of paper is 9×10^{-3} cm thick. (3)
Mark wants to put 500 sheets of paper into the paper tray of his printer.
The paper tray is 4 cm deep.
Is the paper tray deep enough for 500 sheets of paper?
You must explain your answer.

Solution

$$\frac{500 \times 9 \times 10^{-3}}{4} = \frac{5 \times 9 \times 10^{-1}}{4} = \frac{4.5}{4} = 1.25;$$

hence, $\underline{\underline{\text{no}}}$, it is not deep enough.

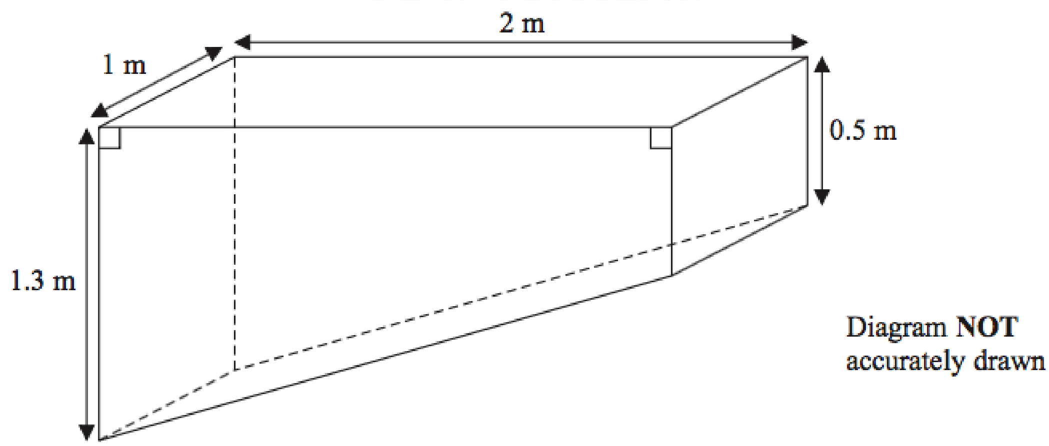
16. The normal price of a television is reduced by 30% in a sale. (3)
The sale price of the television is £350.
Work out the normal price of the television.

Solution

The normal price of the television is

$$\frac{350}{1 - 0.3} = \frac{350}{0.7} = \frac{3\,500}{7} = \underline{\underline{\pounds 500}}.$$

17. Sumeet has a pond in the shape of a prism. (6)



The pond is completely full of water.
Sumeet wants to empty the pond so he can clean it.
Sumeet uses a pump to empty the pond.
The volume of water in the pond decreases at a constant rate.
The level of the water in the pond goes down by 20 cm in the first 30 minutes.
Work out how much more time Sumeet has to wait for the pump to empty the pond completely.

Solution

$$\begin{aligned}\text{Cross-sectional area} &= \tfrac{1}{2} \times (1.3 + 0.5) \times 2\\ &= 1.8 \text{ m}^2\end{aligned}$$

and the

$$\begin{aligned}\text{volume} &= 1.8 \times 1\\ &= 1.8 \text{ m}^3.\end{aligned}$$

Now, it goes down by 20 cm in the first 30 minutes so

$$\begin{aligned}\text{volume} &= 0.2 \times 1 \times 2\\ &= 0.4 \text{ m}^3\end{aligned}$$

and so it goes down by

$$0.8 \text{ m}^3/\text{hr}.$$

When it is full, the tank empties at

$$\frac{1.8}{0.8} = \frac{18}{8} = 2\tfrac{1}{4} \text{ hours}$$

and so Sumeet has to wait for the pump to empty the pond completely is

$$2\tfrac{1}{4} - \tfrac{1}{2} = \underline{\underline{1\tfrac{3}{4}\text{ hours}}}.$$

18. Solve the simultaneous equations (4)

$$\begin{aligned} 4x + 7y &= 1 \\ 3x + 10y &= 15. \end{aligned}$$

Solution

$$\begin{aligned} 4x + 7y &= 1 \quad (1) \\ 3x + 10y &= 15 \quad (2) \end{aligned}$$

$$\begin{aligned} 3 \times (1): &\quad 12x + 21y = 3 \quad (3) \\ 4 \times (2): &\quad 12x + 40y = 60 \quad (4) \end{aligned}$$

Now, (4) − (3):

$$\begin{aligned} 19y = 57 &\Rightarrow \underline{\underline{y = 3}} \\ &\Rightarrow 4x + 21 = 1 \\ &\Rightarrow 4x = -20 \\ &\Rightarrow \underline{\underline{x = -5}}. \end{aligned}$$

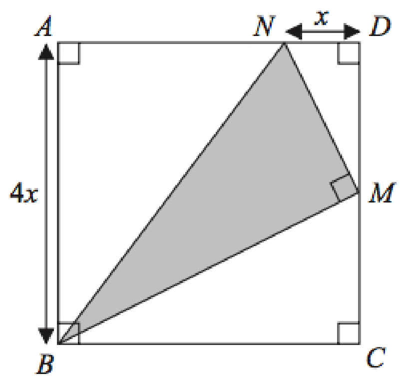
19. Write these numbers in order of size. (2)
Start with the smallest number.

$$5^{-1} \qquad 0.5 \qquad -5 \qquad 5^0$$

Solution

$$\underline{\underline{-5, 5^{-1}, 0.5, 5^0}}.$$

20. $ABCD$ is a square with a side length of $4x$. (4)



M is the midpoint of DC.
N is the point on AD where $ND = x$.
BMN is a right-angled triangle.
Find an expression, in terms of x, for the area of triangle BMN.
Give your expression in its simplest form.

Solution

$$\begin{aligned} MN &= \sqrt{x^2 + (2x)^2} \\ &= \sqrt{5x^2} \\ &= \sqrt{5}x \end{aligned}$$

and

$$\begin{aligned} BM &= \sqrt{(2x)^2 + (4x)^2} \\ &= \sqrt{20x^2} \\ &= 2\sqrt{5}x. \end{aligned}$$

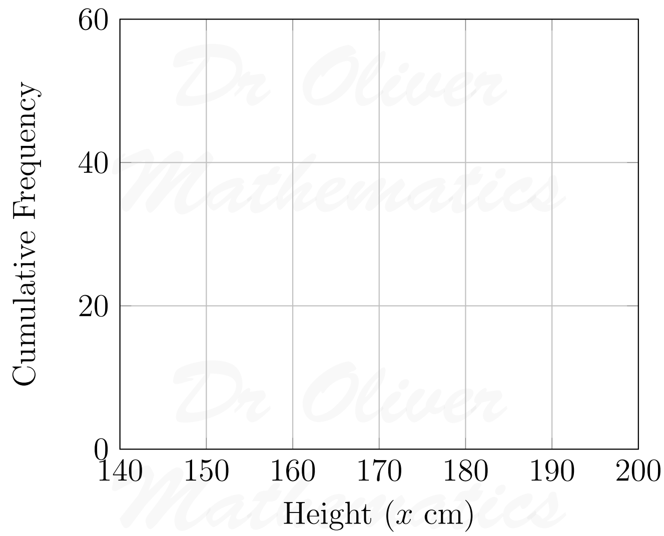
Finally,

$$\begin{aligned} \text{area of } \triangle BMN &= \tfrac{1}{2} \times \sqrt{5}x \times 2\sqrt{5}x \\ &= \underline{\underline{5x^2}}. \end{aligned}$$

21. The table below shows information about the heights of 60 students.

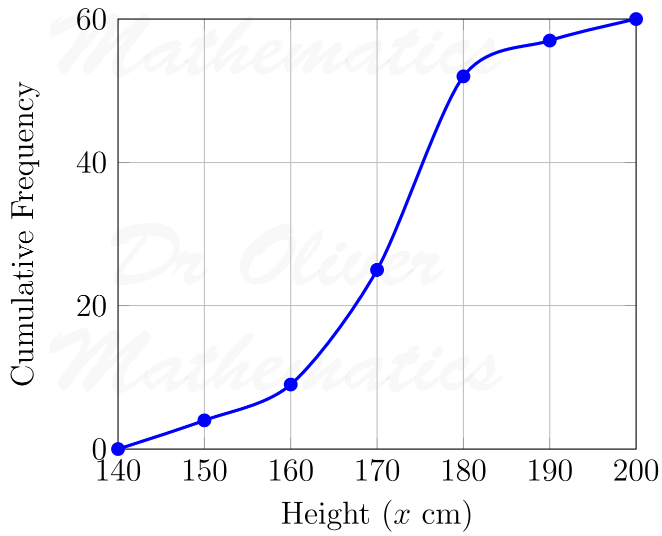
Height (x cm)	Number of students
$140 < x \leqslant 150$	4
$150 < x \leqslant 160$	5
$160 < x \leqslant 170$	16
$170 < x \leqslant 180$	27
$180 < x \leqslant 190$	5
$190 < x \leqslant 200$	3

(a) On the grid opposite, draw a cumulative frequency graph for the information in the table. (3)



Solution

Height (x cm)	Number of students
$140 < x \leqslant 150$	4
$140 < x \leqslant 160$	$5 + 4 = 9$
$140 < x \leqslant 170$	$16 + 9 + 25$
$140 < x \leqslant 180$	$27 + 25 = 52$
$140 < x \leqslant 190$	$5 + 52 = 57$
$140 < x \leqslant 200$	$3 + 57 = 60$

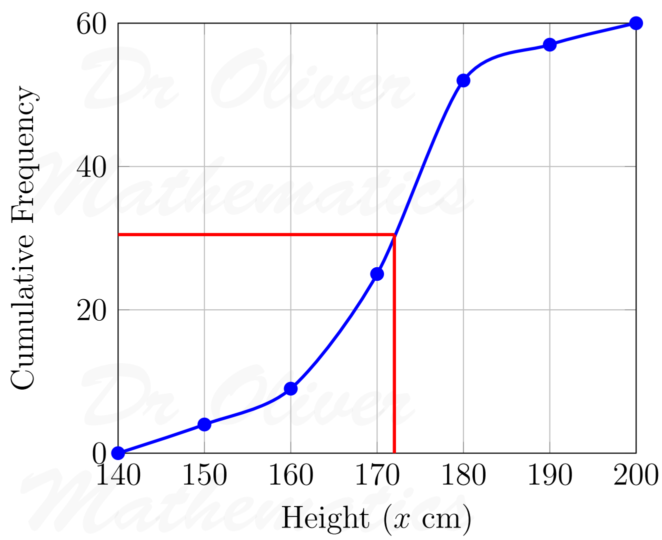


(b) Find an estimate (3)

(i) for the median,

Solution

Draw a horizontal line from 30.5 or 30:

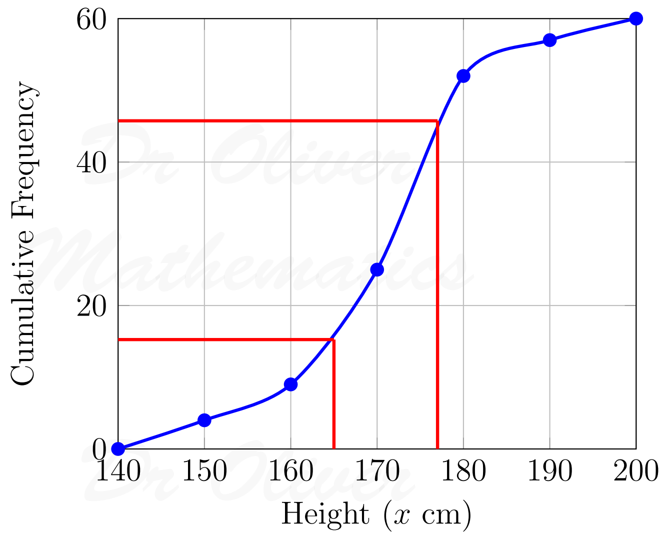


Correct read-off: approximately 172 cm.

(ii) for the interquartile range.

Solution

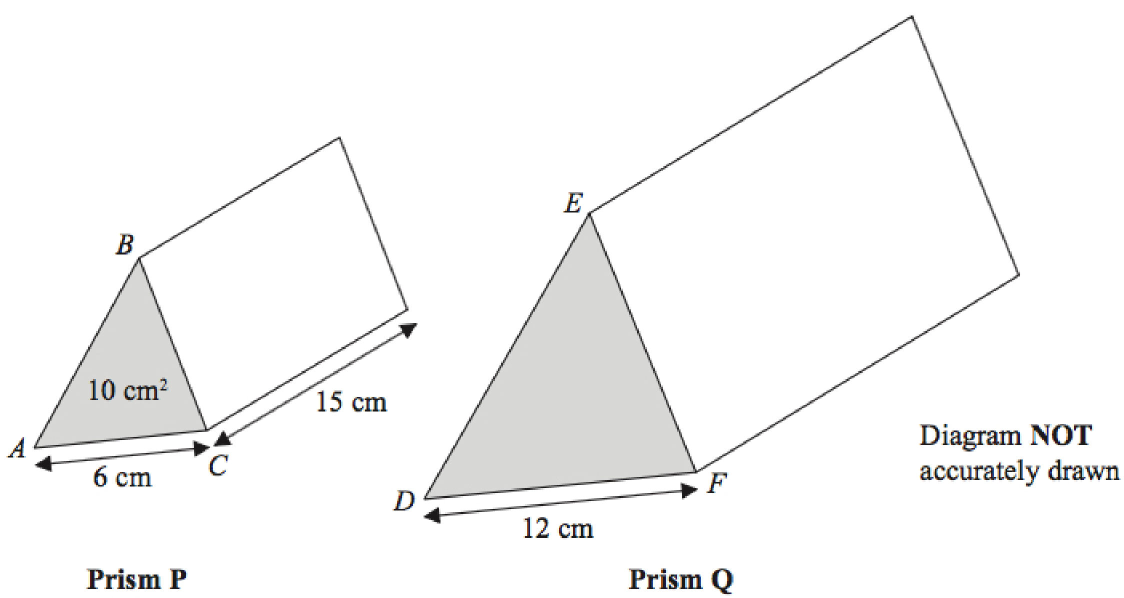
Draw a horizontal lines from 15.25 or 15 and from 45.75 or 45



Correct read-off: approximately

$$177 - 165 = \underline{\underline{12 \text{ cm}}}.$$

22. **P** and **Q** are two triangular prisms that are mathematically similar. (4)



Prism **P** has triangle ABC as its cross section.
Prism **Q** has triangle DEF as its cross section.
$AC = 6$ cm.
$DF = 12$ cm.
The area of the cross section of prism **P** is 10 cm^2.
The length of prism **P** is 15 cm.
Work out the volume of prism **Q**.

Solution

Volume of **P** is

$$10 \times 15 = 150 \text{ cm}^3.$$

Now, the length scale factor (LSF) from **P** to **Q** is

$$\frac{12}{6} = 2$$

and, hence, volume scale factor (VSF) from **P** to **Q** is

$$2^3 = 8.$$

Finally, the volume of **Q** is

$$150 \times 8 = \underline{\underline{1\,200 \text{ cm}^3}}.$$

23. Simplify (2)

$$\frac{4(x+5)}{x^2+2x-15}.$$

Solution

$$\left.\begin{array}{ll}\text{add to:} & +2 \\ \text{multiply to:} & -15\end{array}\right\} -3,\ +5$$

$$x^2 + 2x - 15 = (x-3)(x+5)$$

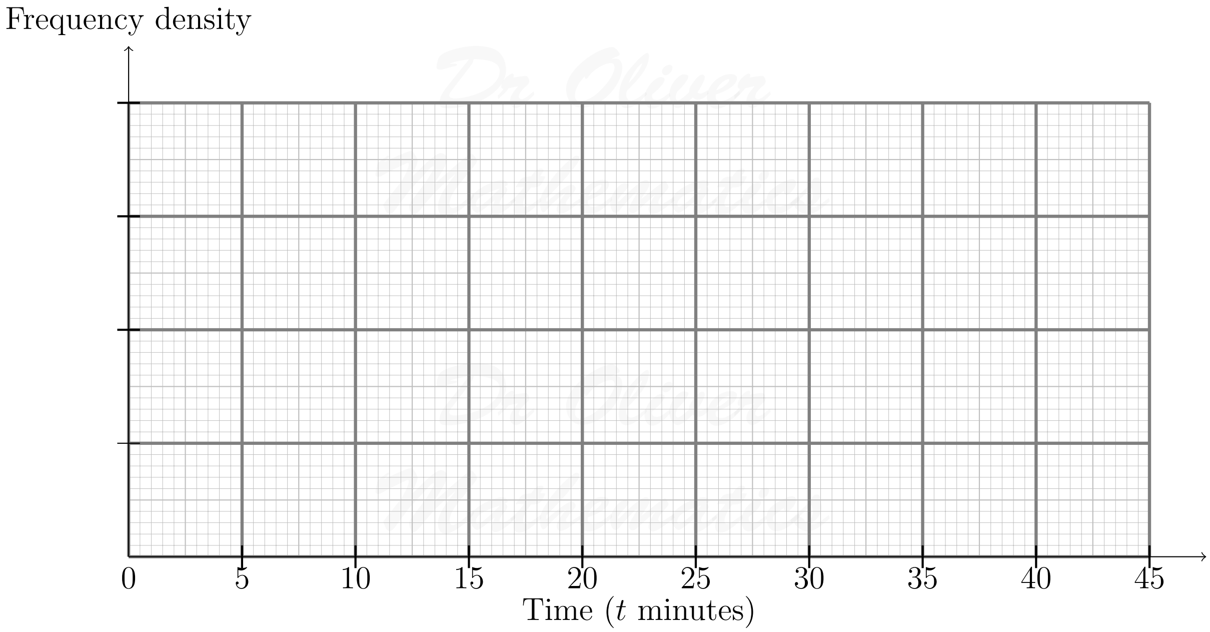
$$\begin{aligned}\frac{4(x+5)}{x^2+2x-15} &= \frac{4(x+5)}{(x-3)(x+5)} \\ &= \underline{\underline{\frac{4}{x-3}}}.\end{aligned}$$

24. Bill works for a computer service centre. (3)

The table shows some information about the length of time, t minutes, of the phone calls Bill had.

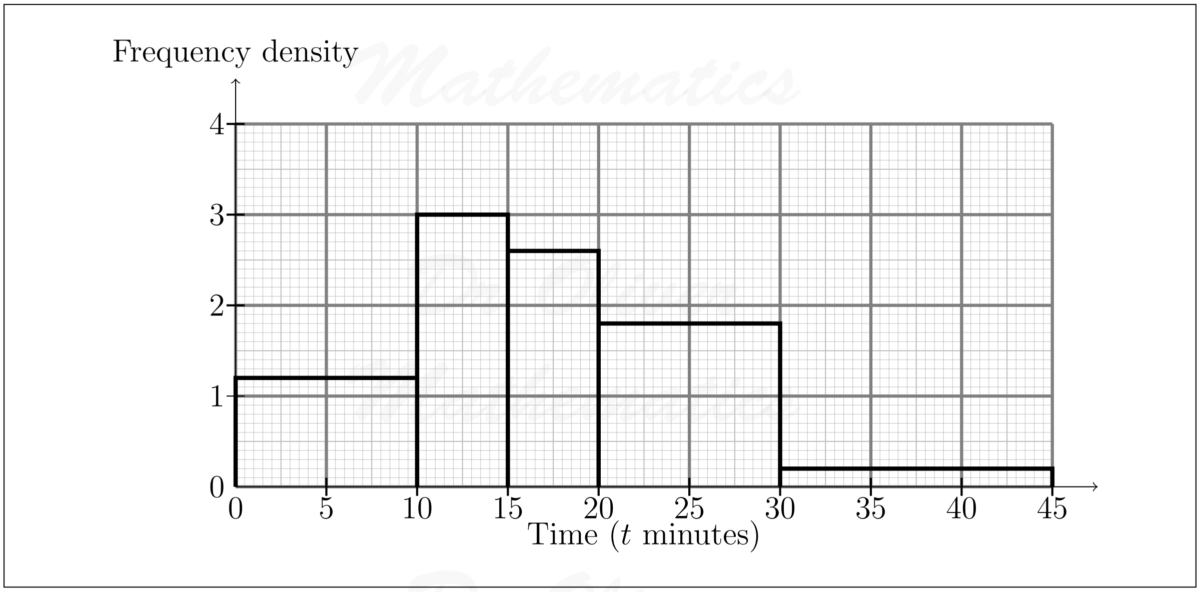
Time (t minutes)	Number of calls
$0 < t \leqslant 10$	12
$10 < t \leqslant 15$	15
$15 < t \leqslant 20$	13
$20 < t \leqslant 30$	18
$30 < t \leqslant 45$	3

On the grid, draw a histogram to show this information.



Solution

Time (t minutes)	Number of calls	Width	Frequency Density
$0 < t \leqslant 10$	12	10	$\frac{12}{10} = 1.2$
$10 < t \leqslant 15$	15	5	$\frac{15}{5} = 3$
$15 < t \leqslant 20$	13	5	$\frac{13}{5} = 2.6$
$20 < t \leqslant 30$	18	10	$\frac{18}{10} = 1.8$
$30 < t \leqslant 45$	3	15	$\frac{3}{15} = 0.2$



25. The expression

$$x^2 - 8x + 21$$

can be written in the form

$$(x - a)^2 + b$$

for all values of x.

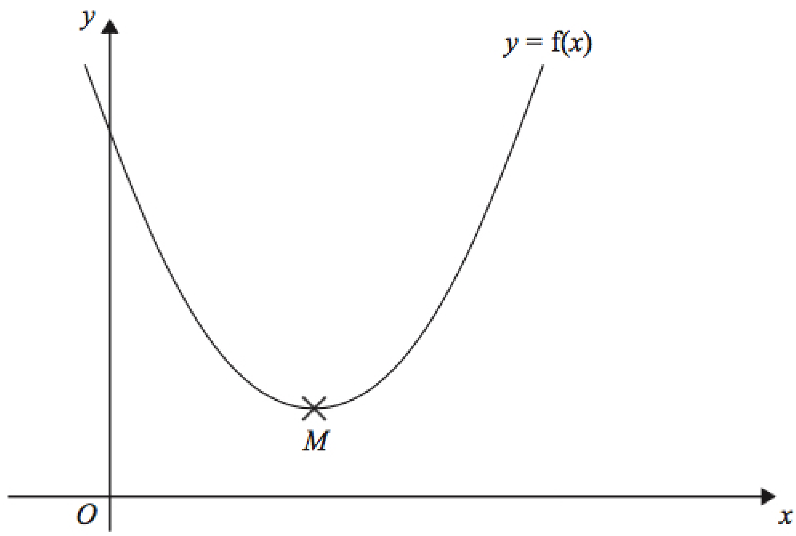
(a) Find the value of a and the value of b. (3)

Solution

$$\begin{aligned} x^2 - 8x + 21 &= (x^2 - 8x + 16) + 5 \\ &= (x - 4)^2 + 5; \end{aligned}$$

hence, $\underline{\underline{a = 4}}$ and $\underline{\underline{b = 5}}$.

The equation of a curve is $y = \mathrm{f}(x)$ where $\mathrm{f}(x) = x^2 - 8x + 21$.
The diagram shows part of a sketch of the graph of $y = \mathrm{f}(x)$.



The minimum point of the curve is M.

(b) Write down the coordinates of M. (1)

Solution

$\underline{\underline{(4, 5)}}$.

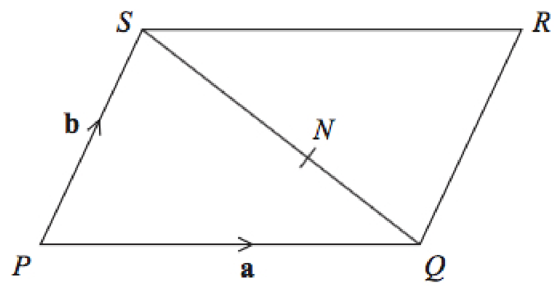
26. Fiza has 10 coins in a bag. (4)
There are three £1 coins and seven 50 pence coins.
Fiza takes at random, 3 coins from the bag.
Work out the probability that she takes exactly £2.50.

Solution

Fiza needs £1, £1, and 50 pence in some order.

$$\begin{aligned}\text{Probability} &= 3 \times \tfrac{3}{10} \times \tfrac{2}{9} \times \tfrac{7}{8} \\ &= 3 \times \tfrac{3}{5} \times \tfrac{1}{9} \times \tfrac{7}{8} \\ &= \tfrac{63}{360} \\ &= \underline{\underline{\tfrac{7}{40}}}.\end{aligned}$$

27. $PQRS$ is a parallelogram.



N is the point on SQ such that $SN : NQ = 3 : 2$.
$\overrightarrow{PQ} = \mathbf{a}$.
$\overrightarrow{PS} = \mathbf{b}$.

(a) Write down, in terms of **a** and **b**, an expression for $\overrightarrow{SQ}$. (1)

Solution

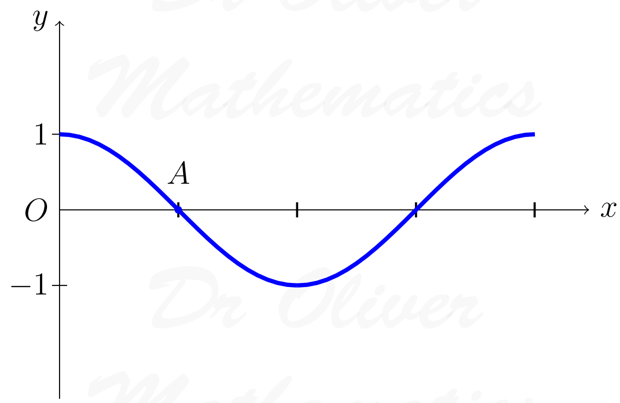
$$\begin{aligned}\overrightarrow{SQ} &= \overrightarrow{SP} + \overrightarrow{PQ} \\ &= \underline{\underline{\mathbf{a} - \mathbf{b}}}.\end{aligned}$$

(b) Express $\overrightarrow{NR}$ in terms of **a** and **b**. (3)

Solution

$$\begin{aligned}\overrightarrow{NR} &= \overrightarrow{NQ} + \overrightarrow{QR} \\ &= \tfrac{2}{5}\overrightarrow{SQ} + \overrightarrow{QR} \\ &= \tfrac{2}{5}(\mathbf{a} - \mathbf{b}) + \mathbf{b} \\ &= \tfrac{2}{5}\mathbf{a} - \tfrac{2}{5}\mathbf{b} + \mathbf{b} \\ &= \tfrac{2}{5}\mathbf{a} + \tfrac{3}{5}\mathbf{b} \\ &= \underline{\underline{\tfrac{1}{5}(2\mathbf{a} + 3\mathbf{b})}}.\end{aligned}$$

28. The diagram shows a sketch of the graph of $y = \cos x^\circ$.



(a) Write down the coordinates of the point A. (1)

Solution

$\underline{\underline{A(90, 0)}}$.

(b) On the same diagram, draw a sketch of the graph of $y = 2\cos x°$. (1)

Solution