Dr Oliver Mathematics
GCSE Mathematics
2019 Paper 1H: Non-Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. There are only blue cubes, red cubes, and yellow cubes in a box.
   The table shows the probability of taking at random a blue cube from the box.

Colour	Blue	Red	Yellow
Probability	0.2		

The number of red cubes in the box is the same as the number of yellow cubes in the box.

(a) Complete the table. (2)

Solution
Now,

$$\tfrac{1}{2}(1 - 0.2) = \tfrac{1}{2} \times 0.8 = 0.4$$

and so we complete the table:

Colour	Blue	Red	Yellow
Probability	0.2	0.4	0.4

There are 12 blue cubes in the box.

(b) Work out the total number of cubes in the box. (2)

Solution
The total number of cubes in the box is

$$\frac{1}{\frac{1}{5}} \times 12 = 5 \times 12 = \underline{\underline{60}}.$$

2. Deon needs 50 g of sugar to make 15 biscuits.
She also needs three times as much flour as sugar and two times as much butter as sugar.
Deon is going to make 60 biscuits.

(a) Work out the amount of flour she needs. (3)

Solution

$$\frac{60}{15} = 4$$

so the amount of flour she needs is

$$\begin{aligned} 4 \times 3 \times 50 &= 4 \times 150 \\ &= \underline{\underline{600 \text{ g}}}. \end{aligned}$$

Deon has to buy all the butter she needs to make 60 biscuits.
She buys the butter in 250 g packs.

(b) How many packs of butter does Deon need to buy? (2)

Solution

The amount of butter she needs is

$$\begin{aligned} 4 \times 2 \times 50 &= 4 \times 100 \\ &= 400 \text{ g} \end{aligned}$$

so she needs to buy $\underline{\underline{\text{2 packs}}}$.

3. Find the highest common factor (HCF) of 72 and 90. (2)

Solution

$$\begin{array}{r|l} & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 2 & 9 \\ \hline 3 & 3 \\ \hline 3 & 1 \\ \hline \end{array}$$

So

$$72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2.$$

$$\begin{array}{r|l} & 90 \\ \hline 2 & 45 \\ \hline 3 & 15 \\ \hline 3 & 5 \\ \hline 5 & 1 \\ \hline \end{array}$$

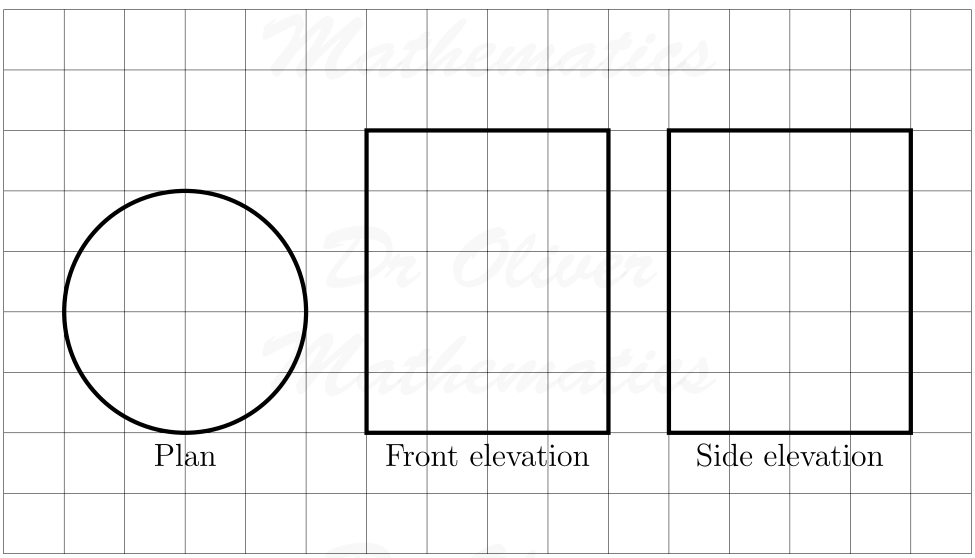
So

$$90 = 2 \times 3 \times 3 \times 5 = 2 \times 3^2 \times 5.$$

Finally,

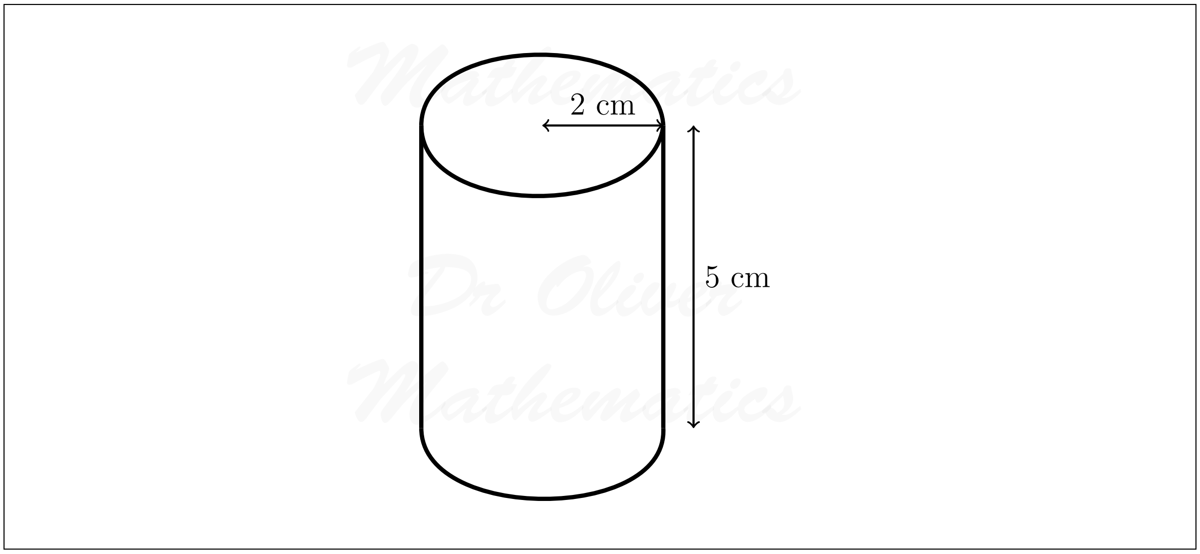
$$\text{HCF}(72, 90) = 2 \times 3^2 = \underline{\underline{18}}.$$

4. The diagram shows the plan, front elevation and side elevation of a solid shape, drawn on a centimetre grid. (2)

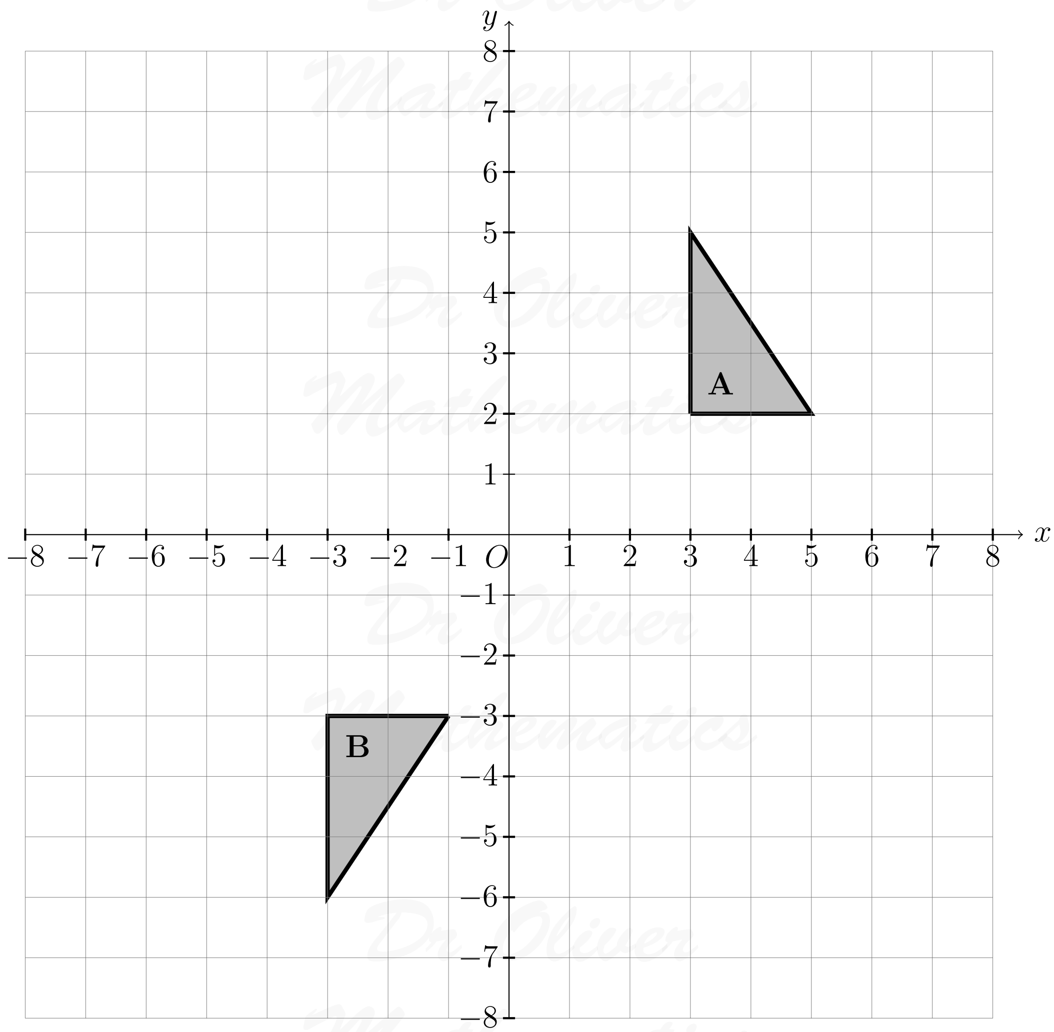


In the space below, draw a sketch of the solid shape.
Give the dimensions of the solid on your sketch.

Solution

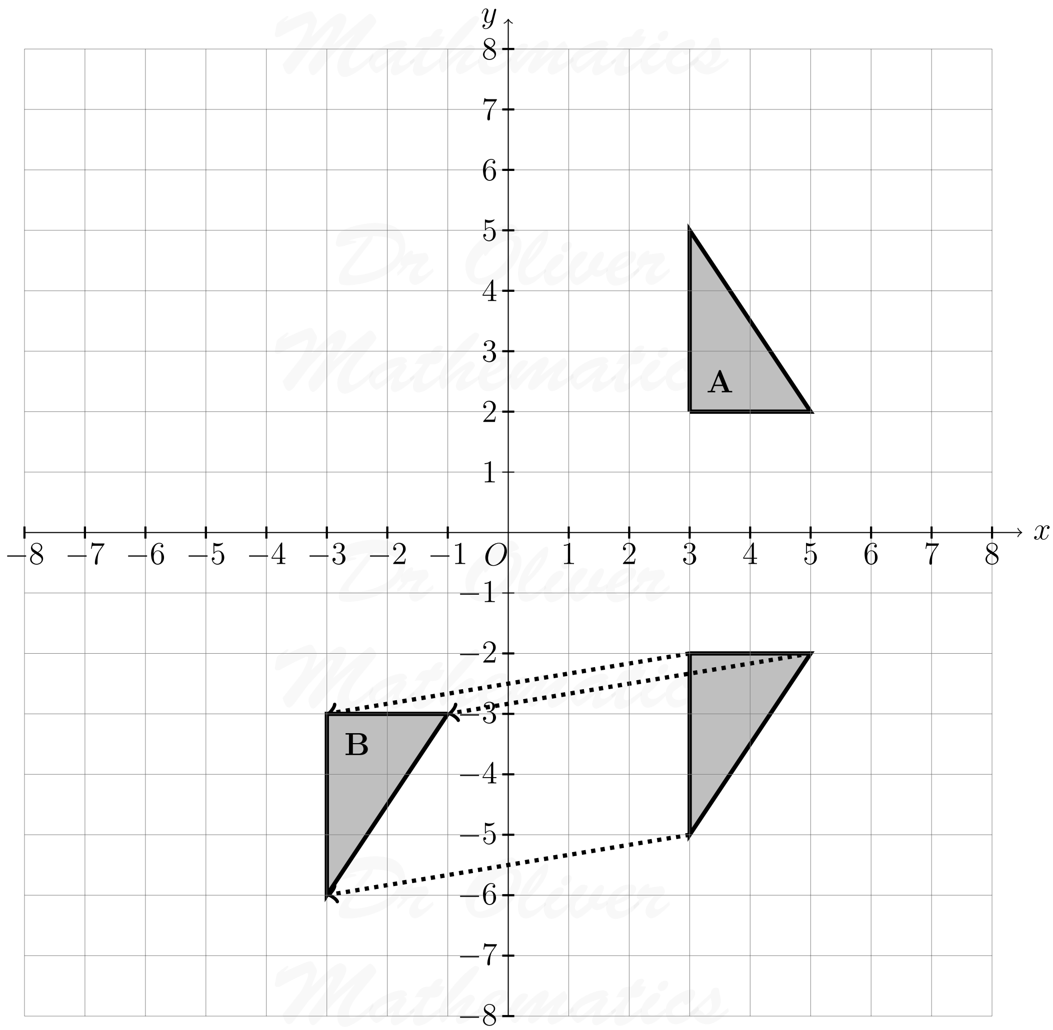


5. Shape **A** can be transformed to shape **B** by a reflection in the x-axis followed by a translation $\begin{pmatrix} c \\ d \end{pmatrix}$. (3)



Find the value of c and the value of d.

Solution



So $\underline{\underline{c = -6}}$ and $\underline{\underline{d = -1}}$

6. A shop sells packs of black pens, packs of red pens, and packs of green pens. (4)
There are:
2 pens in each pack of black pens,
5 pens in each pack of red pens, and
6 pens in each pack of green pens.

On Monday,

$$\begin{array}{c}\text{number of packs}\\ \text{of black pens sold}\end{array} : \begin{array}{c}\text{number of packs}\\ \text{of red pens sold}\end{array} : \begin{array}{c}\text{number of packs}\\ \text{of green pens sold}\end{array} = 7 : 3 : 4.$$

A total of 212 pens were sold.

Work out the number of green pens sold.

Solution

There are

$$2 \times 7 : 5 \times 3 : 6 \times 4 = 14 : 15 : 24$$

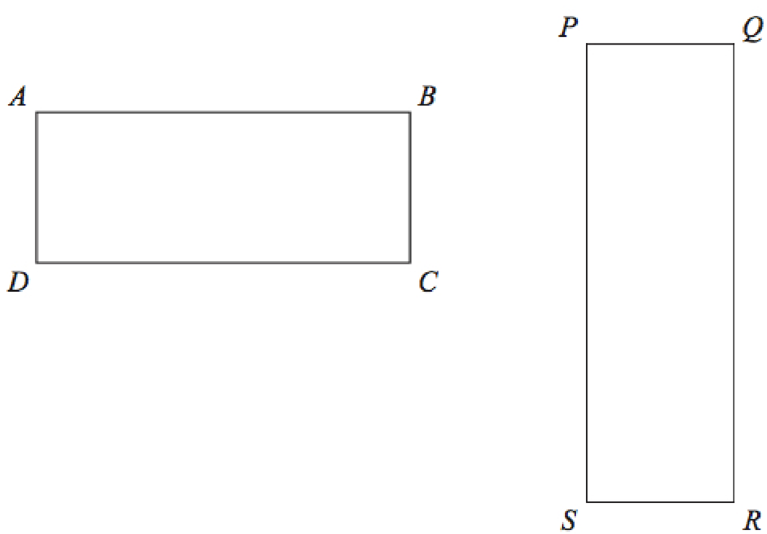
and there are

$$14 + 15 + 24 = 53$$

'shares.' Finally, the number of green pens sold is

$$\begin{aligned} \frac{24}{53} \times 212 &= 24 \times 4 \\ &= \underline{\underline{96}}. \end{aligned}$$

7. Here are two rectangles. (4)



$QR = 10$ cm.
$BC = PQ$.

The perimeter of $ABCD$ is 26 cm.
The area of $PQRS$ is 45 cm^2.

Find the length of AB.

Solution

The other side of $PQRS$ (PQ or RS) is

$$\frac{45}{10} = 4.5 \text{ cm}$$

and this is also BC.
Now, let the length of $AB = x$ cm. Next, the length of AB is

$$\begin{aligned} 2x + (2 \times 4.5) = 26 &\Rightarrow 2x + 9 = 26 \\ &\Rightarrow 2x = 17 \\ &= \underline{\underline{x = 8\tfrac{1}{2} \text{ cm}}}. \end{aligned}$$

8. (a) Work out an estimate for the value of (2)

$$\sqrt{63.5 \times 101.7}.$$

Solution

We round 63.5 to 64 and 101.7 to 100:

$$\begin{aligned} \sqrt{63.5 \times 101.7} &\approx \sqrt{64 \times 100} \\ &= \sqrt{64} \times \sqrt{100} \\ &= 8 \times 10 \\ &= \underline{\underline{80}}. \end{aligned}$$

$(2.3)^6 = 148$, correct to 3 significant figures.

(b) Find the value of $(0.23)^6$ correct to 3 significant figures. (1)

Solution

$$\begin{aligned}(0.23)^6 &= [(0.1) \times (2.3)]^6 \\ &= (0.1)^6 \times (2.3)^6 \\ &= (1 \times 10^{-6}) \times 148 \\ &= (1 \times 10^{-6}) \times (1.48 \times 10^2) \\ &= \underline{\underline{1.48 \times 10^{-4} \text{ or } 0.000\,148 \text{ (3 sf)}}}.\end{aligned}$$

(c) Find the value of 5^{-2}. (1)

Solution

$$5^{-2} = \frac{1}{5^2} = \underline{\underline{\tfrac{1}{25}}}.$$

9. Work out (3)

$$3\tfrac{1}{2} \times 1\tfrac{3}{5}.$$

Give your answer as a mixed number in its simplest form.

Solution

$$\begin{aligned}3\tfrac{1}{2} \times 1\tfrac{3}{5} &= \tfrac{7}{2} \times \tfrac{8}{5} \\ &= \tfrac{7}{1} \times \tfrac{4}{5} \\ &= \tfrac{28}{5} \\ &= \underline{\underline{5\tfrac{3}{5}}}.\end{aligned}$$

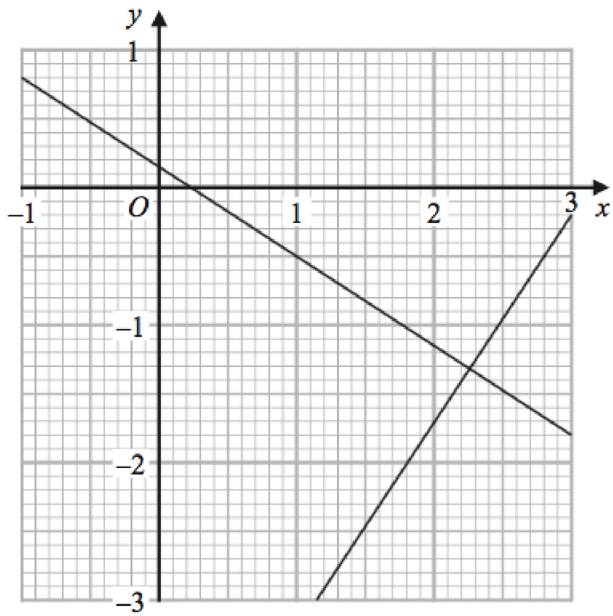
10. The graphs with equations (2)

$$3y + 2x = \tfrac{1}{2}$$

and

$$2y - 3x = -\tfrac{113}{12}$$

have been drawn on the grid below.



Using the graphs, find estimates of the solutions of the simultaneous equations

$$3y + 2x = \tfrac{1}{2}$$
$$2y - 3x = -\tfrac{113}{12}.$$

Solution

Correct read-off: approximately $x = 2.25$ and $y = -1.32$.

11. A bus company recorded the ages, in years, of the people on coach A and the people on coach B.

Here are the ages of the 23 people on coach A.

41	42	44	48	52	53	53	53	56	57	57	59
60	61	63	64	64	66	67	69	74	77	79	

(a) Complete the table below to show information about the ages of the people on coach A (2)

Median	
Lower quartile	
Upper quartile	
Least age	41
Least age	79

Solution

There are

$$\frac{23+1}{2} = 12\text{th}$$

pieces of information and the $\underline{\underline{\text{median} = 59}}$.

There are

$$\frac{23+1}{4} = 6\text{th}$$

pieces of information and the $\underline{\underline{\text{LQ} = 53}}$.

There are

$$\frac{3(23+1)}{4} = 18\text{th}$$

pieces of information and the $\underline{\underline{\text{UQ} = 66}}$.

Here is some information about the ages of the people on coach B.

Median	70
Lower quartile	54
Upper quartile	73
Least age	42
Least age	85

Richard says that the people on coach A are younger than the people on coach B.

(b) Is Richard correct? (1)

You must give a reason for your answer.

Solution

$\underline{\underline{\text{Yes}}}$: the median age for those on coach A is 59 and the median age for those on coach B is 70 so the median for coach A is lower.

Richard says that the people on coach A vary more in age than the people on coach B.

(c) Is Richard correct? (1)

You must give a reason for your answer.

Solution

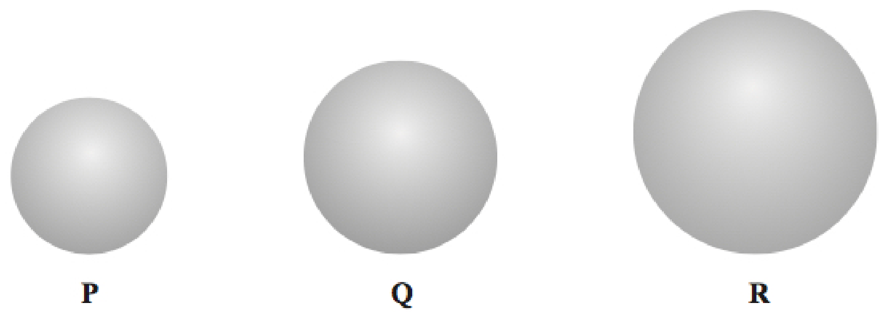
No.

Range: the range for those on coach A is $79 - 41 = 38$ and the range for those on coach B is $85 - 42 = 43$ so the range is more consistent for coach A.

OR

IQR: the IQR for those on coach A is $66 - 53 = 13$ and the IQR for those on coach B is $73 - 54 = 19$ so the IQR is more consistent for coach A.

12. Here are three spheres. (3)



The volume of sphere **Q** is 50% more than the volume of sphere **P**.
The volume of sphere **R** is 50% more than the volume of sphere **Q**.

Find the volume of sphere **P** as a fraction of the volume of sphere **R**.

Solution

The volume of sphere **P** as a fraction of the volume of sphere **R** is

$$\begin{aligned}\frac{1}{1.5^2} &= \frac{1}{2.25}\\ &= \frac{1}{\frac{9}{4}}\\ &= \tfrac{4}{9}.\end{aligned}$$

13. Given that n can be any integer such that $n > 1$, prove that (2)

$$n^2 - n$$

is never an odd number.

Solution

Suppose that n is even, i.e., $n = 2m$ for some integer m. Then

$$\begin{aligned}(2m)^2 - (2m) &= 4m^2 - 2m \\ &= 2(2m^2 - m),\end{aligned}$$

which is clearly an even integer.

Suppose that n is even, i.e., $n = 2p + 1$ for some integer p. Then

$$\begin{aligned}(2p+1)^2 - (2p+1) &= (4p^2 + 4p + 1) - (2p + 1) \\ &= 4p^2 + 2p \\ &= 2(2p^2 + p),\end{aligned}$$

which is clearly an even integer.

Hence, $n^2 - n$ is never an odd number.

14. Find the exact value of (2)

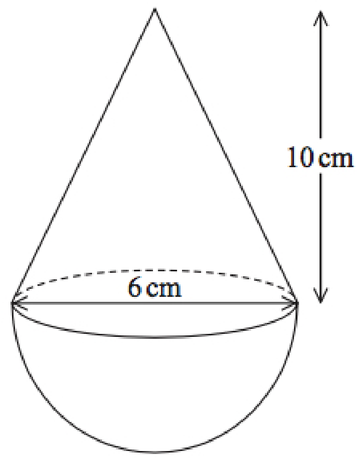
$$\tan 30^\circ \times \sin 60^\circ.$$

Give your answer in its simplest form.

Solution

$$\begin{aligned}\tan 30^\circ \times \sin 60^\circ &= \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{2} \\ &= \tfrac{1}{2}.\end{aligned}$$

15. The diagram shows a solid shape. (4)
The shape is a cone on top of a hemisphere.



The height of the cone is 10 cm.
The base of the cone has a diameter of 6 cm.
The hemisphere has a diameter of 6 cm.

The total volume of the shape is $k\pi$ cm^3, where k is an integer.

Work out the value of k.

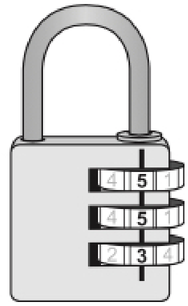
Solution

The radius of the base is 3 cm and so total volume of the shape is

$$\begin{aligned}\text{cone} + \text{hemisphere} &= \left(\tfrac{1}{3} \times \pi \times 3^2 \times 10\right) + \left(\tfrac{2}{3} \times \pi \times 3^3\right) \\ &= 30\pi + 18\pi \\ &= 48\pi;\end{aligned}$$

hence, $\underline{\underline{k = 48}}$.

16. There are three dials on a combination lock.
Each dial can be set to one of the numbers 1, 2, 3, 4, and 5.
The three digit number 553 is one way the dials can be set, as shown in the diagram.



(a) Work out the number of different three digit numbers that can be set for the combination lock. (2)

Solution

$$5^3 = \underline{\underline{125}}.$$

(b) How many of the possible three digit numbers have three different digits? (2)

Solution

$$\begin{aligned} 5 \times 4 \times 3 &= 5 \times 12 \\ &= \underline{\underline{60}}. \end{aligned}$$

17. Given that (4)

$$x^2 : (3x + 5) = 1 : 2,$$

find the possible values of x.

Solution

$$\begin{aligned} x^2 : (3x + 5) = 1 : 2 &\Rightarrow 2x^2 = 3x + 5 \\ &\Rightarrow 2x^2 - 3x - 5 = 0 \end{aligned}$$

$$\left.\begin{array}{ll}\text{add to:} & -3 \\ \text{multiply to:} & (+2)\times(-5) = -10\end{array}\right\} -5,\ +2$$

$$\begin{aligned}
&\Rightarrow 2x^2 - 5x + 2x - 5 = 0 \\
&\Rightarrow x(2x-5) + (2x-5) = 0 \\
&\Rightarrow (2x-5)(x+1) = 0 \\
&\Rightarrow 2x-5 = 0 \text{ or } x+1 = 0 \\
&\Rightarrow \underline{\underline{x = 2\tfrac{1}{2} \text{ or } x = -1}}.
\end{aligned}$$

18. (a) Express (2)

$$\sqrt{3} + \sqrt{12}$$

in the form $a\sqrt{3}$ where a is an integer.

Solution

$$\begin{aligned}
\sqrt{3} + \sqrt{12} &= \sqrt{3} + \sqrt{4 \times 3} \\
&= \sqrt{3} + \sqrt{4} \times \sqrt{3} \\
&= \sqrt{3} + 2\sqrt{3} \\
&= \underline{\underline{3\sqrt{3}}}.
\end{aligned}$$

(b) Express (3)

$$\left(\frac{1}{\sqrt{3}}\right)^7$$

in the form $\dfrac{\sqrt{b}}{c}$ where b and c are integers.

Solution

$$\begin{aligned}\left(\frac{1}{\sqrt{3}}\right)^7 &= \left(\frac{1}{\sqrt{3}}\right)^2 \times \left(\frac{1}{\sqrt{3}}\right)^2 \times \left(\frac{1}{\sqrt{3}}\right)^2 \times \left(\frac{1}{\sqrt{3}}\right) \\ &= \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{\sqrt{3}} \\ &= \frac{1}{27\sqrt{3}} \\ &= \frac{1}{27\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\ &= \frac{\sqrt{3}}{81};\end{aligned}$$

hence, $\underline{\underline{b = 3}}$ and $\underline{\underline{c = 81}}$.

19. Given that

$$x^2 - 6x + 1 \equiv (x - a)^2 - b$$

for all values of x,

(a) find the value of a and the value of b. (2)

Solution

$$\begin{aligned}x^2 - 6x + 1 &\equiv (x^2 - 6x + 9) - 8 \\ &\equiv (x - 3)^2 - 8;\end{aligned}$$

hence, $\underline{\underline{a = 3}}$ and $\underline{\underline{b = 8}}$.

(b) Hence write down the coordinates of the turning point on the graph of (1)

$$y = x^2 - 6x + 1.$$

Solution

The coordinates of the turning point on the graph are $\underline{\underline{(3, -8)}}$.

20. h is inversely proportional to p. (4)
p is directly proportional to $\sqrt{t}$.

Given that $h = 10$ and $t = 144$ when $p = 6$, find a formula for h in terms of t.

Solution

$$h \propto \frac{1}{p} \Rightarrow h = \frac{k}{p}$$

for some value of k. Now,

$$10 = \frac{k}{6} \Rightarrow k = 60$$

and so

$$h = \frac{60}{p}.$$

Next,

$$p \propto \sqrt{t} \Rightarrow p = l\sqrt{t}$$

for some value of l. Now,

$$6 = l\sqrt{144} \Rightarrow l = \tfrac{1}{2}$$

and so

$$p = \tfrac{1}{2}\sqrt{t}.$$

Finally,

$$h = \frac{60}{p} \Rightarrow h = \frac{60}{\frac{1}{2}\sqrt{t}}$$
$$\Rightarrow h = \frac{120}{\sqrt{t}}.$$

21. The functions f and g are such that

$$\mathrm{f}(x) = 3x - 1 \text{ and } \mathrm{g}(x) = x^2 + 4.$$

(a) Find $\mathrm{f}^{-1}(x)$. (2)

Solution

$$y = 3x - 1 \Rightarrow y + 1 = 3x$$
$$\Rightarrow x = \frac{y+1}{3}$$

and hence

$$\mathrm{f}^{-1}(x) = \frac{x+1}{3}.$$

Given that

$$\mathrm{f\,g}(x) = 2\,\mathrm{g\,f}(x),$$

(b) show that (5)

$$15x^2 - 12x - 1 = 0.$$

Solution

$$\begin{aligned}\mathrm{f\,g}(x) &= \mathrm{f}(\mathrm{g}(x)) \\ &= \mathrm{f}(x^2+4) \\ &= 3(x^2+4) - 1 \\ &= 3x^2 + 11\end{aligned}$$

and

$$\begin{aligned}\mathrm{g\,f}(x) &= \mathrm{g}(\mathrm{f}(x)) \\ &= \mathrm{g}(3x-1) \\ &= (3x-1)^2 + 4\end{aligned}$$

$\times$	$3x$	-1
$3x$	$9x^2$	$-3x$
-1	$-3x$	$+1$

$$\begin{aligned}&= (9x^2 - 6x + 1) + 4 \\ &= 9x^2 - 6x + 5.\end{aligned}$$

Finally,

$$\begin{aligned}\mathrm{f\,g}(x) = 2\,\mathrm{g\,f}(x) &\Rightarrow 3x^2 + 11 = 2(9x^2 - 6x + 5) \\ &\Rightarrow 3x^2 + 11 = 18x^2 - 12x + 10 \\ &\Rightarrow \underline{\underline{15x^2 - 12x - 1 = 0}},\end{aligned}$$

as required.

22. There are only r red counters and g green counters in a bag. (5)

A counter is taken at random from the bag.

The probability that the counter is green is $\frac{3}{7}$.
The counter is put back in the bag.

2 more red counters and 3 more green counters are put in the bag.
A counter is taken at random from the bag.
The probability that the counter is green is $\frac{6}{13}$.

Find the number of red counters and the number of green counters that were in the bag originally.

Solution

For the first counter,

$$\begin{aligned}\frac{g}{r+g} = \tfrac{3}{7} &\Rightarrow 7g = 3(r+g)\\ &\Rightarrow 7g = 3r + 3g\\ &\Rightarrow 4g - 3r = 0 \quad (1).\end{aligned}$$

For the second counter,

$$\begin{aligned}\frac{g+3}{(r+2)+(g+3)} = \tfrac{6}{13} &\Rightarrow 13(g+3) = 6(r+g+5)\\ &\Rightarrow 13g + 39 = 6r + 6g + 30\\ &\Rightarrow 7g - 6r = -9 \quad (2).\end{aligned}$$

Now,

$$4g - 3r = 0 \Rightarrow 8g - 6r = 0 \quad (3)$$

and subtract (3) – (2):

$$\begin{aligned}g = 9 &\Rightarrow 36 - 3r = 0\\ &\Rightarrow 3r = 36\\ &\Rightarrow r = 12;\end{aligned}$$

hence, there are 12 red counters and 9 green counters.