Dr Oliver Mathematics
AQA Further Maths Level 2
June 2016 Paper 1
1 hour 30 minutes

The total number of marks available is 70.
You must write down all the stages in your working.
You are **not** permitted to use a scientific or graphical calculator in this paper.

1. (3)

$$y = x^2(x - 10).$$

Work out $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

Solution

$$\begin{aligned} y = x^2(x-10) &\Rightarrow y = x^3 - 10x^2 \\ &\Rightarrow \underline{\underline{\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 20x}}. \end{aligned}$$

2. (3)

$$4\begin{pmatrix} 1-2a \\ a \end{pmatrix} = \begin{pmatrix} b \\ 12 \end{pmatrix}.$$

Work out the values of a and b.

Solution

$$4\begin{pmatrix} 1-2a \\ a \end{pmatrix} = \begin{pmatrix} b \\ 12 \end{pmatrix} \Rightarrow \begin{pmatrix} 4-8a \\ 4a \end{pmatrix} = \begin{pmatrix} b \\ 12 \end{pmatrix}.$$

Now,

$$4a = 12 \Rightarrow \underline{\underline{a = 3}}$$

and

$$\begin{aligned} 4 - 8a = b &\Rightarrow 4 - 24 = b \\ &\Rightarrow \underline{\underline{b = -20}}. \end{aligned}$$

3. The nth term of a sequence is

$$\frac{3n}{5n+12}.$$

(a) Work out the position of the term that has a value of $\frac{1}{2}$. (2)

Solution

$$\frac{3n}{5n+12} = \tfrac{1}{2} \Rightarrow 2(3n) = 5n + 12$$
$$\Rightarrow 6n = 5n + 12$$
$$\Rightarrow \underline{\underline{n = 12}}.$$

(b) Write down the limiting value of (1)

$$\frac{3n}{5n+12}$$

as $n \to \infty$.

Solution

$$\frac{3n}{5n+12} = \frac{3}{5+\frac{12}{n}}$$
$$\to \underline{\underline{\tfrac{3}{5}}},$$

as $n \to \infty$.

4. The equation of a circle is

$$(x+5)^2 + (y-8)^2 = 10.$$

(a) What are the coordinates of the centre of the circle? (1)
Circle your answer.

$(-5,-8)$ $(-5,8)$ $(5,8)$ $(5,-8)$

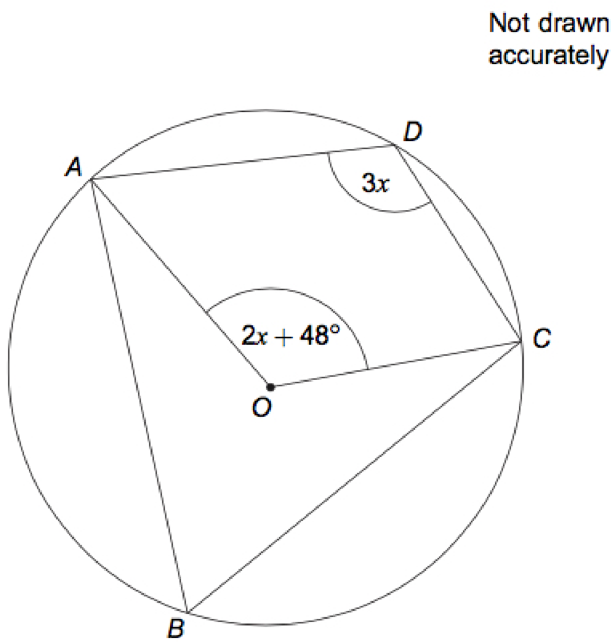
Solution

$(-5,-8)$ $\underline{\underline{(-5,8)}}$ $(5,8)$ $(5,-8)$

(b) Write down the radius of the circle. (1)

> **Solution**
> $\underline{\underline{\sqrt{10}}}$.

5. A, B, C, and D are points on a circle, centre O. (3)



Work out the value of x.

> **Solution**
> $\angle ABC = \frac{1}{2}(2x + 48) = x + 24$ (angle at the centre is twice the angle at the circumference)
> $x + 24 = 3x$ (opposite angle in a cyclic quadrilateral add up to $180°$)
> So
> $$\begin{aligned} x + 24 + 3x = 180 &\Rightarrow 4x = 156 \\ &\Rightarrow \underline{\underline{x = 39°}}. \end{aligned}$$

6. (4)

$$mx + 4 - 2(x + p) \equiv 6(x + 1),$$

where m and p are integers.

Work out the values of m and p.

Solution

$$\begin{aligned} mx + 4 - 2(x+p) \equiv 6(x+1) &\Rightarrow mx + 4 - 2x - 2p \equiv 6x + 6 \\ &\Rightarrow (m-2)x + (4-2p) \equiv 6x + 6. \end{aligned}$$

So,

$$m - 2 = 6 \Rightarrow \underline{\underline{m = 8}}$$

and

$$\begin{aligned} 4 - 2p = 6 &\Rightarrow 2p = -2 \\ &\Rightarrow \underline{\underline{p = -1}}. \end{aligned}$$

7. Work out the integer values of x for which (3)

$$x^2 - 20x + 96 < 0.$$

Solution

$$\left.\begin{array}{ll} \text{add to:} & -20 \\ \text{multiply to:} & +96 \end{array}\right\} -12, -4$$

$$\begin{aligned} x^2 - 20x + 96 < 0 &\Rightarrow (x-12)(x-8) < 0 \\ &\Rightarrow 8 < x < 12; \end{aligned}$$

hence, $\underline{\underline{x = 9,\ 10,\ \text{or}\ 11}}$.

8. Solve (3)

$$(3 - \sqrt{x})^{\frac{1}{3}} = -2.$$

Solution

$$\begin{aligned}(3-\sqrt{x})^{\frac{1}{3}} = -2 &\Rightarrow 3-\sqrt{x} = (-2)^3 \\ &\Rightarrow 3-\sqrt{x} = -8 \\ &\Rightarrow \sqrt{x} = 11 \\ &\Rightarrow x = 11^2 \\ &\Rightarrow \underline{\underline{x = 121}}.\end{aligned}$$

9. Expand and simplify (3)

$$(x-5)^3.$$

Solution

Binomial theorem:

$$\begin{aligned}(x-5)^3 &= x^3 + \binom{3}{1}x^2(-5) + \binom{3}{2}x[(-5)^2] + [(-5)^3] \\ &= \underline{\underline{x^3 - 15x^2 + 75x - 125}}.\end{aligned}$$

10. (4)

$$\sqrt[4]{x} = 2 \text{ and } y^{-2} = 25.$$

$x > 0$ and $y < 0$.

Work out the value of

$$\frac{x}{y}.$$

Solution

$$\sqrt[4]{x} = 2 \Rightarrow x = 2^4 = 16$$

and

$$y^{-2} = 25 \Rightarrow y = -\tfrac{1}{5},$$

because $y < 0$. Finally,

$$\frac{x}{y} = \frac{16}{-\frac{1}{5}} = \underline{\underline{-80}}.$$

11. $A(1\frac{1}{5}, 3\frac{4}{5})$, $B(2, 1\frac{4}{5})$, and $C(5, 3)$ are points on a coordinate grid. (3)

Show that the line segments AB and BC are perpendicular.

Solution

$$\begin{aligned} m_{AB} \times m_{BC} &= \frac{1\frac{4}{5} - 3\frac{4}{5}}{2 - 1\frac{1}{5}} \times \frac{3 - 1\frac{4}{5}}{5 - 2} \\ &= \frac{-2}{\frac{4}{5}} \times \frac{1\frac{1}{5}}{3} \\ &= \tfrac{-5}{2} \times \tfrac{2}{5} \\ &= -1; \end{aligned}$$

hence, AB and BC are $\underline{\underline{\text{perpendicular}}}$.

12. You are given that

$$x^2 + 6x + 2 \equiv (x + h)^2 + k.$$

(a) Work out the values of h and k. (2)

Solution

$\times$	x	$+h$
x	x^2	$+hx$
$+h$	$+hx$	$+h^2$

$$\begin{aligned} x^2 + 6x + 2 &\equiv (x^2 + 2hx + h^2) + k \\ &\equiv x^2 + 2hx + (h^2 + k). \end{aligned}$$

So,

$$2h = 6 \Rightarrow \underline{\underline{h = 3}}$$

and

$$h^2 + k = 2 \Rightarrow 9 + k = 2$$
$$\Rightarrow \underline{\underline{k = -7}}.$$

(b) Write down the coordinates of the minimum point on the curve (1)

$$y = x^2 + 6x + 2.$$

Solution

$\underline{\underline{(-3, -7)}}$.

(c) Solve the equation (1)

$$x^2 + 6x + 2 = 0.$$

Give your answers in the form $a \pm \sqrt{b}$.

Solution

$$\begin{aligned} x^2 + 6x + 2 = 0 &\Rightarrow (x + 3)^2 - 7 = 0 \\ &\Rightarrow (x + 3)^2 = 7 \\ &\Rightarrow x + 3 = \pm\sqrt{7} \\ &\Rightarrow \underline{\underline{x = -3 \pm \sqrt{7}}}; \end{aligned}$$

hence, $\underline{\underline{a = -3}}$ and $\underline{\underline{b = \sqrt{7}}}$.

13. Solve (3)

$$\sqrt{125} + \sqrt{20} = \sqrt{80} + \sqrt{x}.$$

Solution

$$\begin{aligned}\sqrt{125}+\sqrt{20}=\sqrt{80}+\sqrt{x} &\Rightarrow \sqrt{25\times 5}+\sqrt{4\times 5}=\sqrt{16\times 5}+\sqrt{x}\\ &\Rightarrow \sqrt{25}\times\sqrt{5}+\sqrt{4}\times\sqrt{5}=\sqrt{16}\times\sqrt{5}+\sqrt{x}\\ &\Rightarrow 5\sqrt{5}+2\sqrt{5}=4\sqrt{5}+\sqrt{x}\\ &\Rightarrow \sqrt{x}=3\sqrt{5}\\ &\Rightarrow x=(3\sqrt{5})^2\\ &\Rightarrow \underline{\underline{x=45}}.\end{aligned}$$

14. $(x-3)$ is a factor of

$$x^3-8x^2+ax+42,$$

where a is an integer.

(a) Show that the value of a is 1. (2)

Solution
Lef

$$f(x)=x^3-8x^2+ax+42.$$

Now,

$$\begin{aligned}f(3)=0 &\Rightarrow 27-72+3a+42=0\\ &\Rightarrow 3a=3\\ &\Rightarrow \underline{\underline{a=1}},\end{aligned}$$

as required.

(b) Hence, factorise fully (3)

$$x^3-8x^2+x+42.$$

Solution
We use synthetic division:

3	1	−8	1	42
	↓	3	−15	−42
	1	−5	−14	0

Now,

$$x^3 - 8x^2 + x + 42 = (x-3)(x^2 - 5x - 14)$$

$$\left.\begin{array}{ll} \text{add to:} & -5 \\ \text{multiply to:} & -14 \end{array}\right\} -7,\ +2$$

$$= \underline{\underline{(x-3)(x-7)(x+2)}}.$$

15. Rationalise the denominator and simplify fully (3)

$$\frac{6}{\sqrt{7}+2}.$$

Solution

$$\frac{6}{\sqrt{7}+2} = \frac{6}{\sqrt{7}+2} \times \frac{\sqrt{7}-2}{\sqrt{7}-2}$$

$\times$	$\sqrt{7}$	$+2$
$\sqrt{7}$	7	$+2\sqrt{7}$
-2	$-2\sqrt{7}$	-4

$$= \frac{6(\sqrt{7}-2)}{7-4}$$
$$= \frac{6(\sqrt{7}-2)}{3}$$
$$= \underline{\underline{2(\sqrt{7}-2)}}.$$

16. Angle θ is obtuse and (4)

$$\sin\theta = \frac{\sqrt{11}}{6}.$$

Work out the value of $\cos\theta$.

Solution

Angle θ is obtuse which means $\cos\theta < 0$. Now,

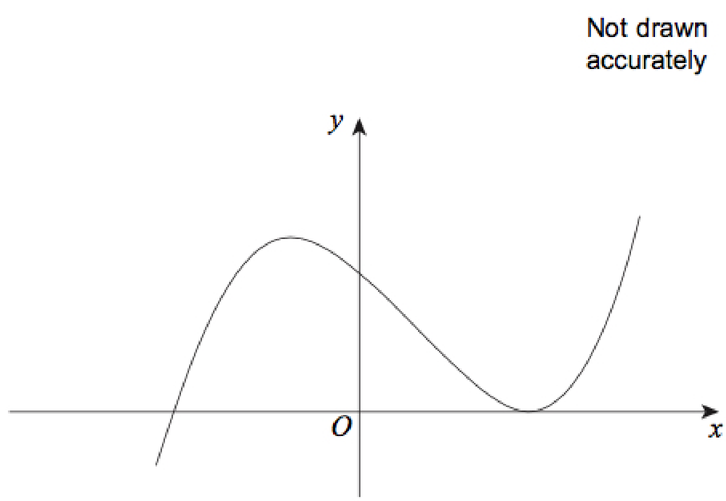
$$\begin{aligned}\sin^2\theta + \cos^2\theta = 1 &\Rightarrow \left(\frac{\sqrt{11}}{6}\right)^2 + \cos^2\theta = 1\\ &\Rightarrow \tfrac{11}{36} + \cos^2\theta = 1\\ &\Rightarrow \cos^2\theta = \tfrac{25}{36}\\ &\Rightarrow \underline{\underline{\cos\theta = -\tfrac{5}{6}}}.\end{aligned}$$

17. The diagram shows a sketch of the cubic curve (5)

$$y = \tfrac{1}{3}x^3 - x^2 - 3x + k,$$

where k is a constant.

The x-axis is a tangent to the curve at its minimum point.



Work out the value of k.

Solution

$$y = \tfrac{1}{3}x^3 - x^2 - 3x + k \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = x^2 - 2x - 3$$

and

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow x^2 - 2x - 3 = 0$$

$$\left.\begin{array}{ll}\text{add to:} & -2 \\ \text{multiply to:} & -3\end{array}\right\} -3,\ +1$$

$$\Rightarrow (x+1)(x-3) = 0$$
$$\Rightarrow x = -1 \text{ or } x = 3.$$

Finally,

$$x = 3, y = 0 \Rightarrow 9 - 9 - 9 + k = 0$$
$$\Rightarrow \underline{\underline{k = 9}}.$$

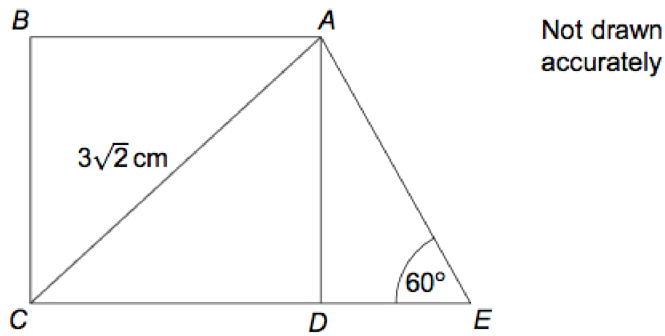
18. Factorise fully (2)

$$x^4 - 81.$$

Solution

$$\begin{aligned} x^4 - 81 &= (x^2)^2 - 9^2 \text{ (difference of two squares)} \\ &= (x^2 - 9)(x^2 + 9) \\ &= \underline{\underline{(x+3)(x-3)(x^2+9)}} \text{ (difference of two squares).} \end{aligned}$$

19. $ABCD$ is a square.
CDE is a straight line.



AC is $3\sqrt{2}$ cm and angle $DEA = 60°$.

(a) Show that the side of the square is 3 cm. (2)

Solution

$$\begin{aligned} AB^2 + BC^2 = AC^2 &\Rightarrow 2\,AB^2 = (3\sqrt{2})^2 \\ &\Rightarrow 2\,AB^2 = 18 \\ &\Rightarrow AB^2 = 9 \\ &\Rightarrow \underline{\underline{AB = 3 \text{ cm}}}, \end{aligned}$$

as required.

(b) Show that the perimeter of trapezium $ABCE$ is (4)

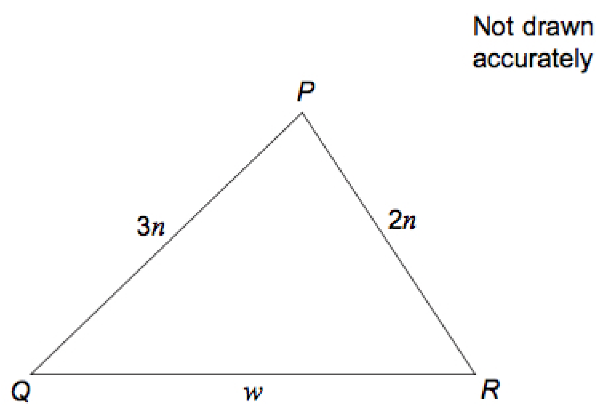
$$3(3 + \sqrt{3}) \text{ cm}.$$

Solution

$$
\begin{aligned}
\text{Perimeter} &= BA + AE + ED + DC + CB \\
&= 3 + \frac{3}{\sin 60^\circ} + \frac{3}{\tan 60^\circ} + 3 + 3 \\
&= 9 + \frac{3}{\frac{\sqrt{3}}{2}} + \frac{3}{\sqrt{3}} \\
&= 9 + \frac{6}{\sqrt{3}} + \sqrt{3} \\
&= 9 + \frac{2\sqrt{3} \times \sqrt{3}}{\sqrt{3}} + \sqrt{3} \\
&= 9 + 2\sqrt{3} + \sqrt{3} \\
&= 9 + 3\sqrt{3} \\
&= \underline{\underline{3(3 + \sqrt{3})\ \text{cm}}},
\end{aligned}
$$

as required.

20. In triangle PQR, $\cos P = \frac{1}{3}$. (4)



Show that triangle PQR is isosceles.

Solution

$$\begin{aligned}
\cos P = \tfrac{1}{3} &\Rightarrow \frac{(3n)^2 + (2n)^2 - w^2}{2 \times 3n \times 2n} = \tfrac{1}{3} \\
&\Rightarrow \frac{9n^2 + 4n^2 - w^2}{12n^2} = \tfrac{1}{3} \\
&\Rightarrow \frac{13n^2 - w^2}{12n^2} = \tfrac{1}{3} \\
&\Rightarrow 13n^2 - w^2 = 4n^2 \\
&\Rightarrow w^2 = 9n^2 \\
&\Rightarrow w = 3n;
\end{aligned}$$

hence, triangle PQR is isosceles because $PQ = QR$.