Dr Oliver Mathematics
GCSE Mathematics
2017 November Paper 2H: Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. Solve (3)

$$5x - 6 = 3(x - 1).$$

Solution

$$\begin{aligned} 5x - 6 = 3(x-1) &\Rightarrow 5x - 6 = 3x - 3 \\ &\Rightarrow 2x = 3 \\ &\Rightarrow \underline{\underline{x = 1\tfrac{1}{2}}}. \end{aligned}$$

2. Emily buys a pack of 12 bottles of water. (3)
The pack costs £5.64.

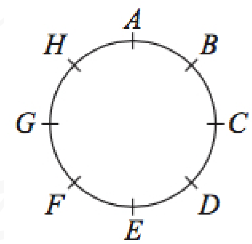
Emily sells all 12 bottles for 50p each.

Work out Emily's percentage profit.
Give your answer correct to 1 decimal place.

Solution

$$\begin{aligned} \text{Emily's percentage profit} &= \left[\frac{(12 \times 0.5) - 5.64}{5.64}\right] \times 100\% \\ &= \left[\frac{6 - 5.64}{5.64}\right] \times 100\% \\ &= \frac{0.36}{5.64} \times 100\% \\ &= 6.382\,978\,723 \text{ (FCD)} \\ &= \underline{\underline{6.4\% \text{ (1 dp)}}}. \end{aligned}$$

3. Hasmeet walks once round a circle with diameter 80 metres.



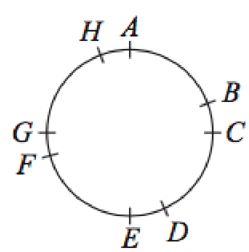
There are 8 points equally spaced on the circumference of the circle.

(a) Find the distance Hasmeet walks between one point and the next point. (2)

Solution

$$\begin{aligned}\text{Distance} &= \frac{\pi \times 80}{8} \\ &= \underline{\underline{10\pi \text{ m}}}.\end{aligned}$$

Four of the points are moved, as shown in the diagram below.



Hasmeet walks once round the circle again.

(b) Has the mean distance that Hasmeet walks between one point and the next point changed? (1)
You must give a reason for your answer.

Solution

No: e.g., total distance remains unchanged.

4. There are only blue cubes, yellow cubes, and green cubes in a bag. (3)

There are

$$\text{twice as many blue cubes as yellow cubes}$$

and

$$\text{four times as many blue cubes as green cubes.}$$

Hannah takes at random a cubes from the bag.

Work out the probability that Hannah takes a yellow cube.

Solution

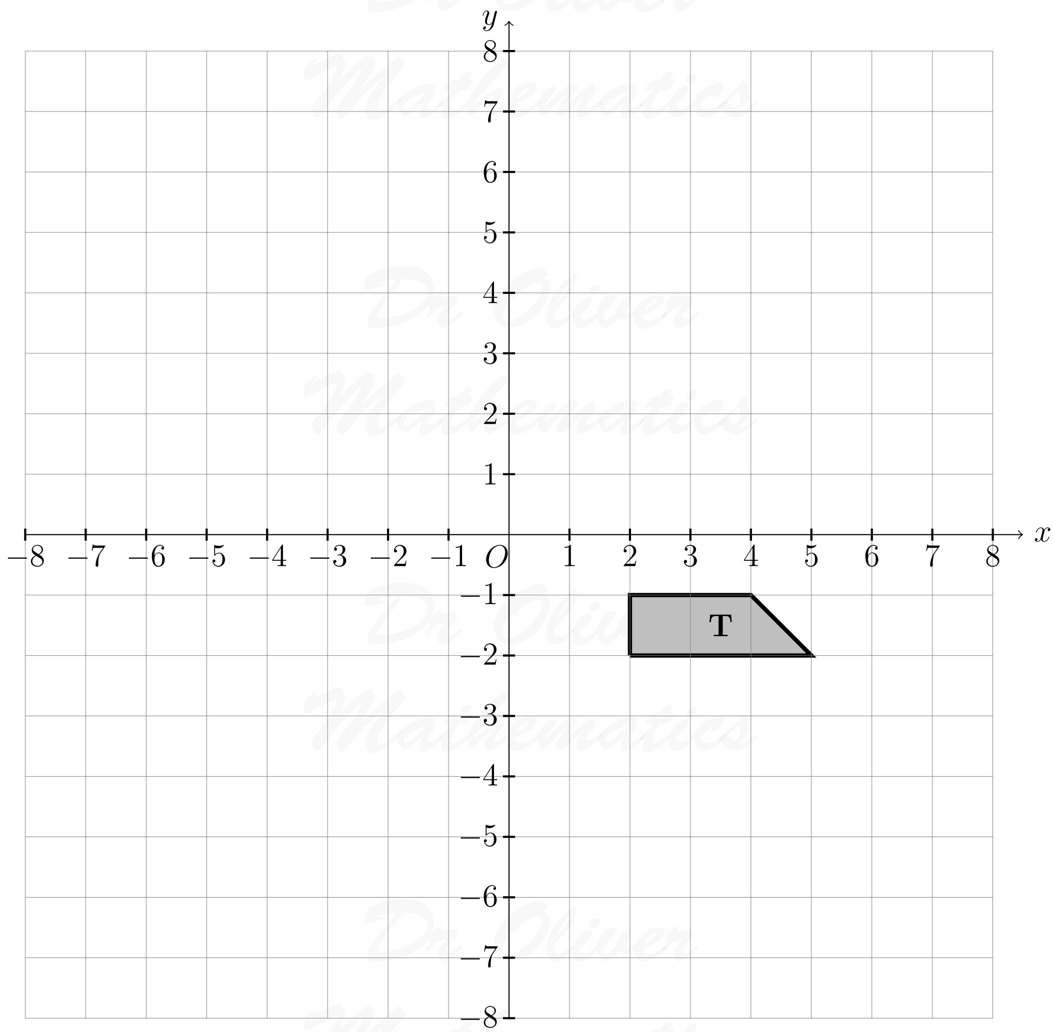
In all,

$$\text{blue} : \text{yellow} : \text{green} = 2 : 1 : 8$$

so

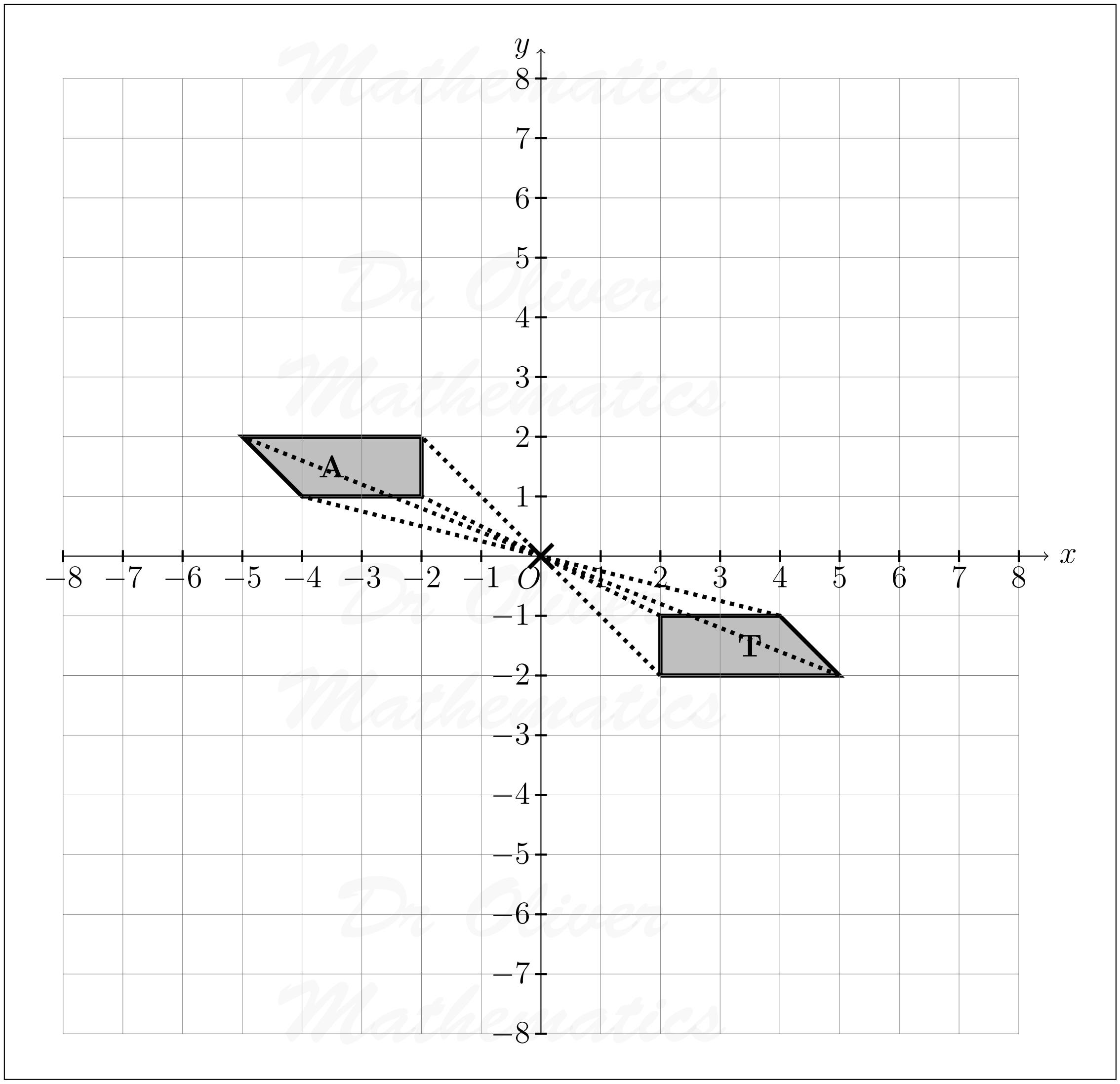
$$\begin{aligned} \text{P(yellow cube)} &= \frac{1}{2+1+8} \\ &= \underline{\underline{\tfrac{1}{11}}}. \end{aligned}$$

5. Here is a picture.



(a) Rotate **T** 180° about the origin. (1)
Label the new trapezium **A**.

Solution

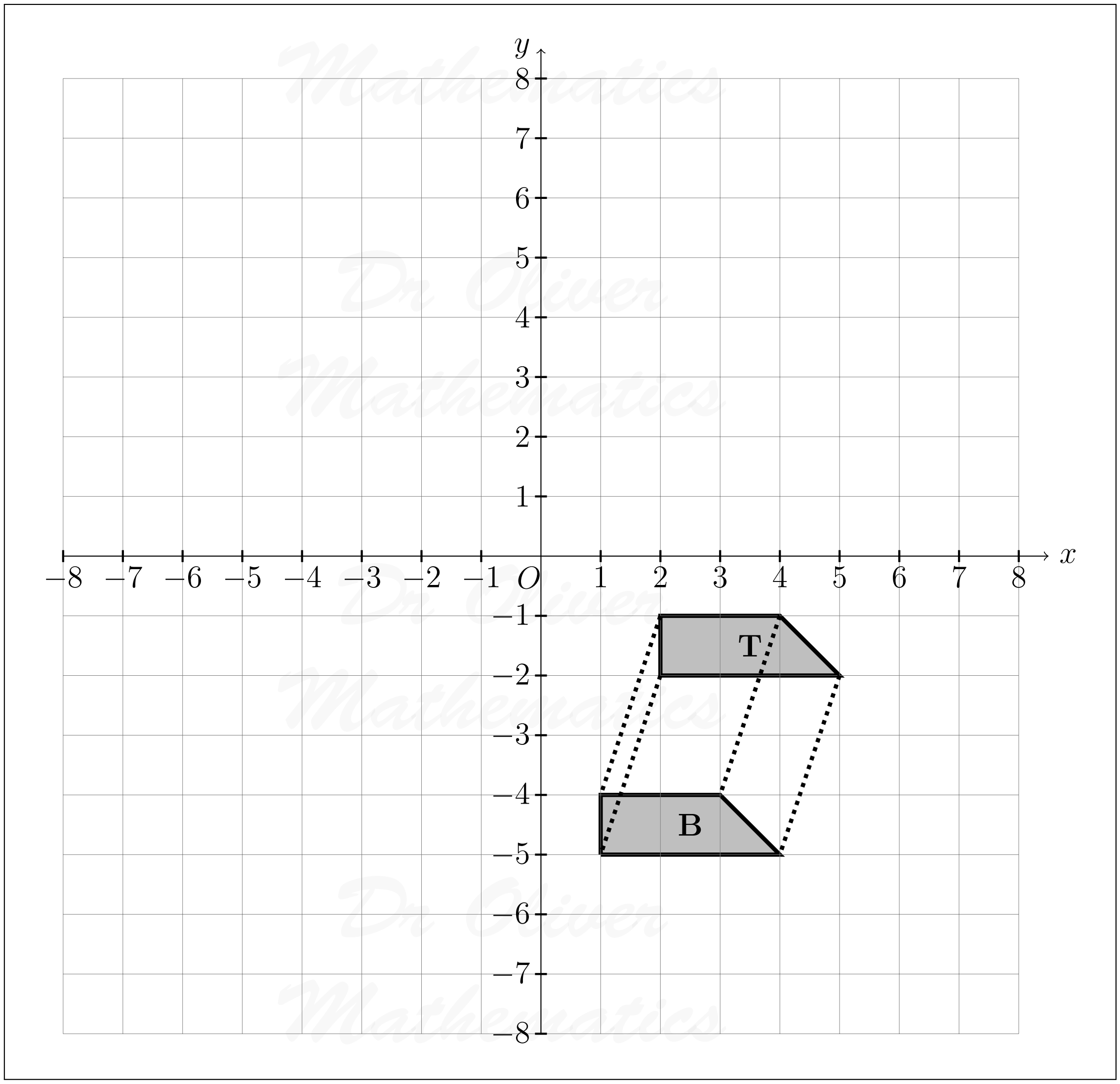


(b) Rotate **T** by the vector (1)

$$\begin{pmatrix} -1 \\ -3 \end{pmatrix}.$$

Label the new trapezium **B**.

Solution



6.

$$p^3 \times p^x = p^9.$$

(a) Find the value of x. (1)

Solution

$$3 + x = 9 \Rightarrow \underline{\underline{x = 6}}.$$

$$(7^2)^y = 7^{10}.$$

(b) Find the value of y. (1)

Solution

$$\begin{aligned}(7^2)^y = 7^{10} &\Rightarrow 7^{2y} = 7^{10}\\ &\Rightarrow 2y = 10\\ &\Rightarrow \underline{\underline{y = 5}}.\end{aligned}$$

$$100^a \times 1\,000^b$$

can be written in the form

$$10^w.$$

(c) Show that (2)

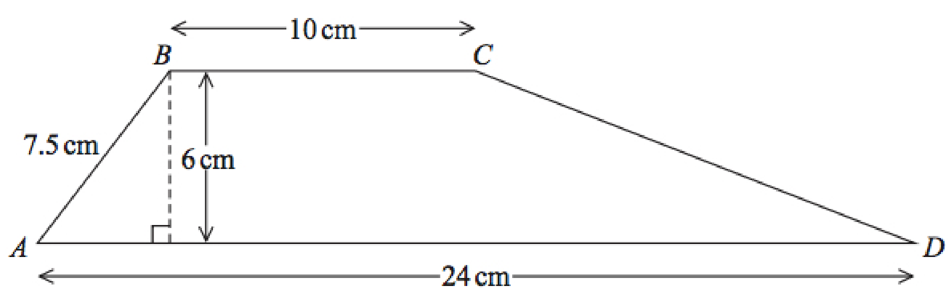
$$w = 2a + 3b.$$

Solution

$$\begin{aligned}100^a \times 1\,000^b = 10^w &\Rightarrow (10^2)^a \times (10^3)^b = 10^w\\ &\Rightarrow 10^{2a} \times 10^{3b} = 10^w\\ &\Rightarrow 10^{2a+3b} = 10^w\\ &\Rightarrow \underline{\underline{w = 2a + 3b}},\end{aligned}$$

as required.

7. $ABCD$ is trapezium. (5)



Work out the size of angle CDA.
Give your answer correct to 1 decimal place.

Solution

Let E be the point on AD directly below B. Then

$$\begin{aligned} AE^2 + BE^2 = AB^2 &\Rightarrow AE^2 + 6^2 = 7.5^2 \\ &\Rightarrow AE^2 = 20.25 \\ &\Rightarrow AE = 4.5 \text{ cm.} \end{aligned}$$

Let F be the point on AD directly below C. Then

$$AF = 4.5 + 10 = 14.5 \text{ cm}$$

and

$$FD = 24 - 14.5 = 9.5 \text{ cm.}$$

Finally,

$$\begin{aligned} \tan = \frac{\text{opp}}{\text{adj}} &\Rightarrow \tan CDA = \frac{6}{9.5} \\ &\Rightarrow \angle CDA = 32.275\,644\,31 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{\angle CDA = 32.3^\circ \text{ (1 dp)}}}. \end{aligned}$$

8. Use your calculator to work out

$$\sqrt{\frac{\sin 25^\circ + \sin 40^\circ}{\cos 25^\circ - \cos 40^\circ}}.$$

(a) Write down all the figures on your calculator display. (2)

Solution

$$\sqrt{\frac{\sin 25^\circ + \sin 40^\circ}{\cos 25^\circ - \cos 40^\circ}} = \underline{\underline{2.756\,039\,57 \text{ (FCD}}}.$$

(b) Write your answer to part (a) correct to 2 decimal places. (1)

Solution

$\underline{\underline{2.76 \text{ (2 dp)}}}$.

9. Yesterday, it took 5 cleaners $4\frac{1}{2}$ hours to clean all the rooms in a hotel. (3)

There were only 3 cleaners to clean all the rooms in a hotel today.

Each cleaner is paid £8.20 for each hour or part of an hour they work.

How much will each cleaner be paid today?

Solution

Let x be the amount of time. Now,

$$5 \times 4\tfrac{1}{2} = 22.5$$

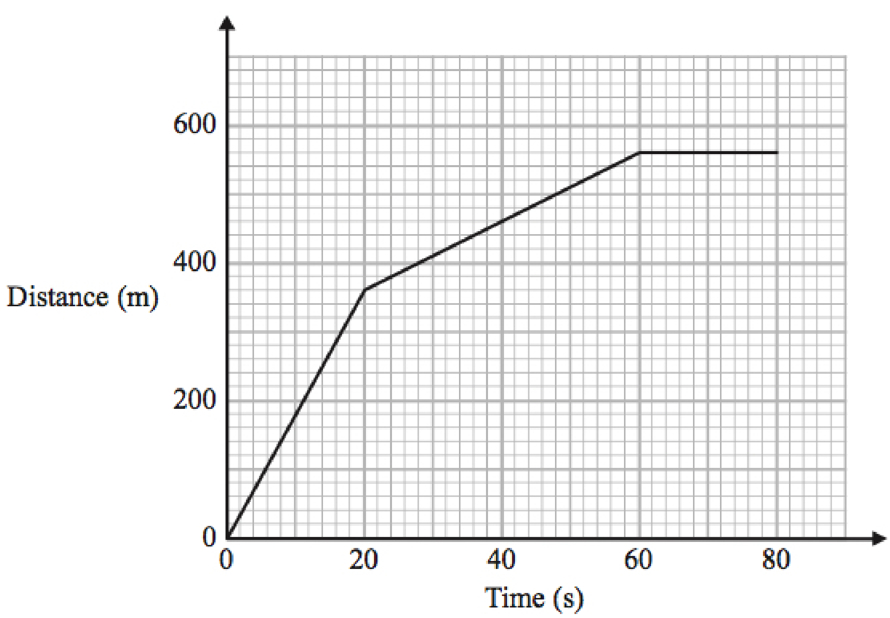
and

$$x = \frac{22.5}{3} = 7.5 \text{ hours.}$$

Let us round it up to 8 ("for each hour or part of an hour they work"). Finally,

$$8.20 \times 8 = \underline{\underline{£65.60}}.$$

10. Here is part of a distance-time graph for a car's journey.



(a) Between which two times does the car travel at its greatest speed? Give a reason for your answer. (2)

Solution

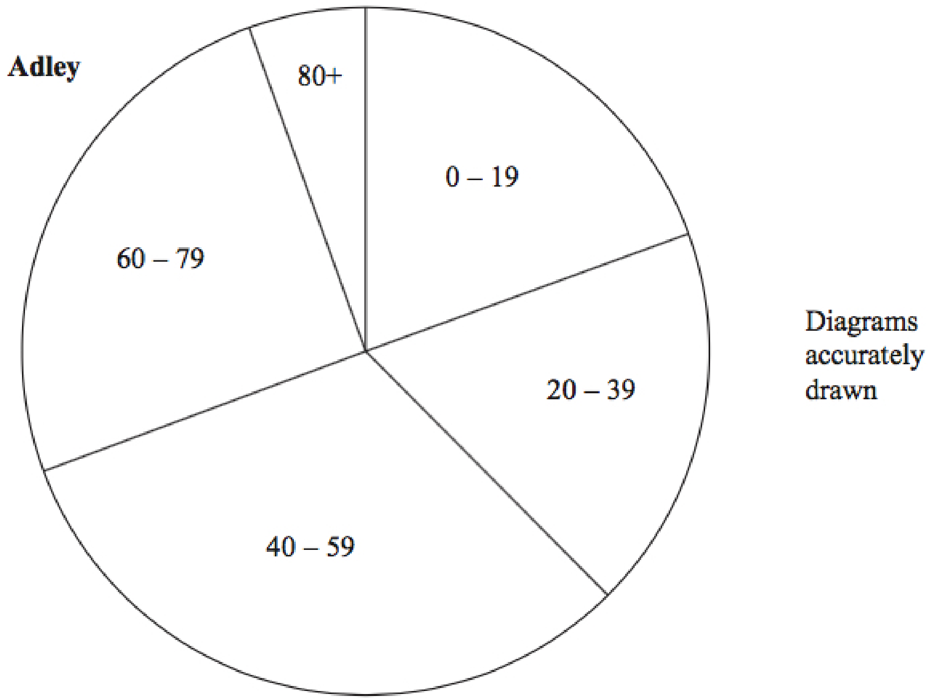
$\underline{\underline{(0-20)\text{ s}}}$ as the gradient is $\underline{\underline{\text{steepest}}}$.

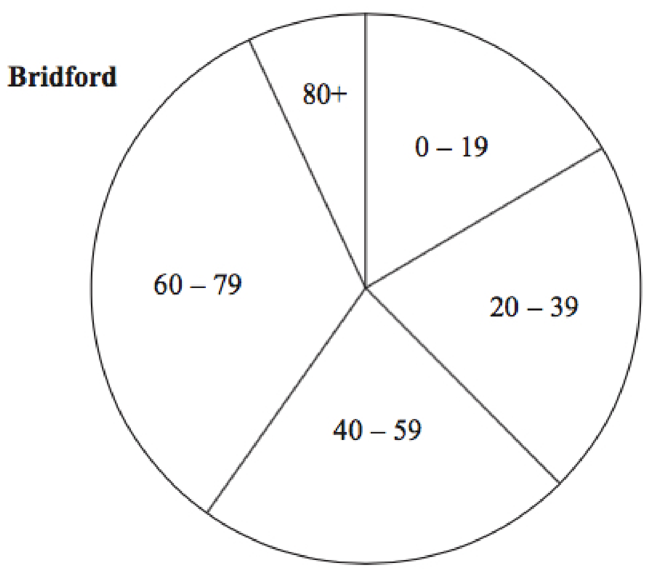
(b) Work out this greatest speed. (1)

Solution

$$\begin{aligned}\text{Greatest speed} &= \frac{360-0}{20-0}\\ &= \underline{\underline{18\text{ m/s}}}.\end{aligned}$$

11. The pie charts give information about the ages, in years, of people living in two towns, Adley and Bridford. (3)





[Note: the radius of Adley's pie chart is 5 cm and the radius of Bridford's pie chart is 4 cm.]

The ratio of the number of people living in Adley to the number of people living in

Bridford is given by the ratio of the areas of the pie charts.

What proportion of the total number of people living in these two towns live in Adley **and** are aged 0-19?
Give your answers correct to 3 significant figures.

Solution
Adley's pie chart, the total number is

$$\pi \times 5^2 = 25\pi$$

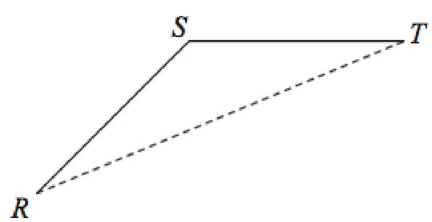
and, on Bridford's pie chart, the total number is

$$\pi \times 4^2 = 16\pi.$$

On Adley's pie chart, $0-19$ is 70°. Hence,

$$\begin{aligned}\text{proportion} &= \frac{70}{360} \times \left(\frac{25\pi}{25\pi + 16\pi}\right) \\ &= \frac{70}{360} \times \frac{25\pi}{41\pi} \\ &= 0.118\,563\,685\,6 \text{ (FCD)} \\ &= \underline{\underline{0.119 \text{ (3 sf)}}}.\end{aligned}$$

12. RS and ST are 2 sides of a regular 12-sided polygon. (3)
RT is a diagonal of the polygon.



Work out the size of angle STR.
You must show your working.

Solution

Now, the angle RST is

$$180 - \frac{360}{12} = 150^\circ$$

and

$$\begin{aligned} \angle STR &= \tfrac{1}{2}(180 - 150) \\ &= \underline{\underline{15^\circ}} \end{aligned}$$

because $\angle STR = \angle SRT$.

13. At the beginning of 2009, Mr Veale bought a company.
The value of the company was £50 000.

Each year the value of the company increased by 2%.

(a) Calculate the value of the company at the beginning of 2017. (2)
Give your answer correct to the nearest £100.

Solution

$$\begin{aligned} \text{Value} &= 50\,000 \times 1.02^8 \\ &= 58\,582.969\,05 \text{ (FCD)} \\ &= \underline{\underline{\pounds 58\,600 \text{ (nearest 100)}}}. \end{aligned}$$

At the beginning of 2009 the value of a different company was £250 000.
In 6 years the value of the company increased to £320 000.

This is equivalent to an increase of x% each year.

(b) Find the value of x. (3)
Give your answer correct to 2 significant figures.

Solution

$$\sqrt[6]{\frac{325\,000}{250\,000}} = 1.044\,697\,508 \text{ (FCD)}$$

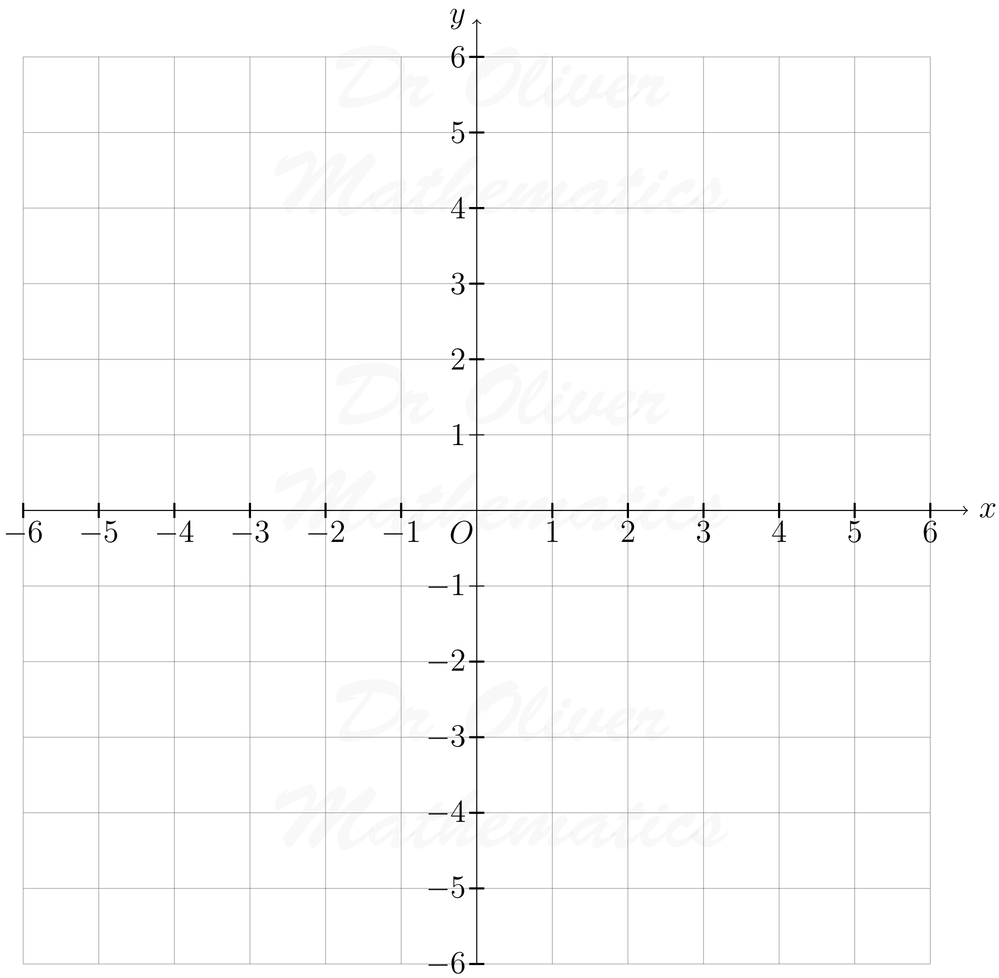
and so

$$\underline{\underline{x = 4.5 \text{ (2 sf)}}}.$$

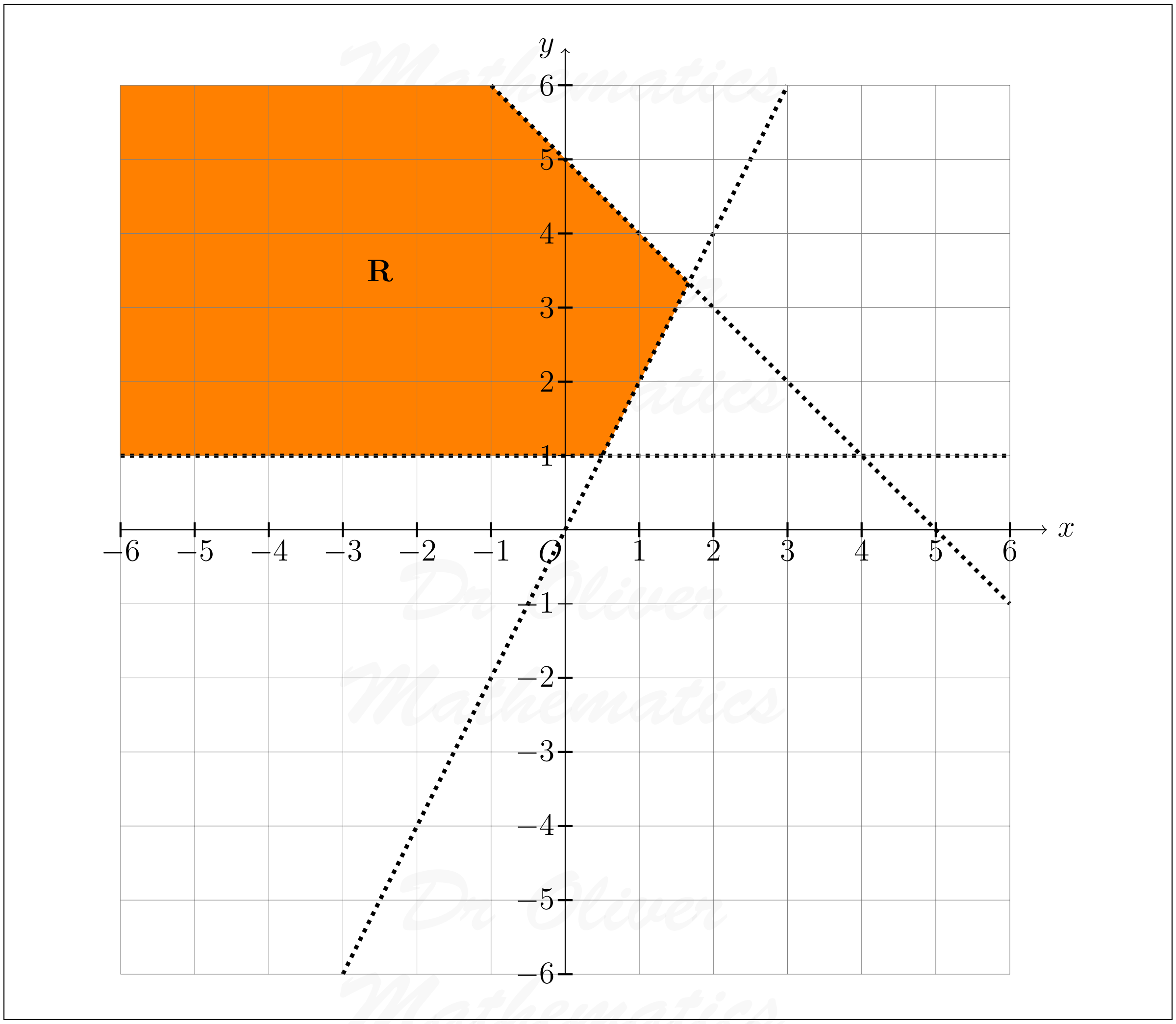
14. On the grid, shade the region that satisfies all these inequalities. (3)

$$y > 1 \qquad x + y < 5 \qquad y > 2x.$$

Label the region **R**.



Solution



15. Tracey is going to choose a main course and a dessert in a cafe.
She can choose from 8 main courses and 7 desserts.

Tracey says that to work out the number of different way of choosing a main course and a dessert you add 8 and 7.

(a) Is Tracey correct? (1)
You must give a reason for your answer.

> **Solution**
>
> No: you multiply rather than adding them.

12 teams play in a competition.
Each team plays each other team exactly once.

(b) Work out the total number of games played. (2)

Solution

$$\tfrac{1}{2} \times 12 \times 11 = \underline{\underline{66}}.$$

16. Solve (2)

$$(x-2)^2 = 3.$$

Give your solutions correct to 3 significant figures.

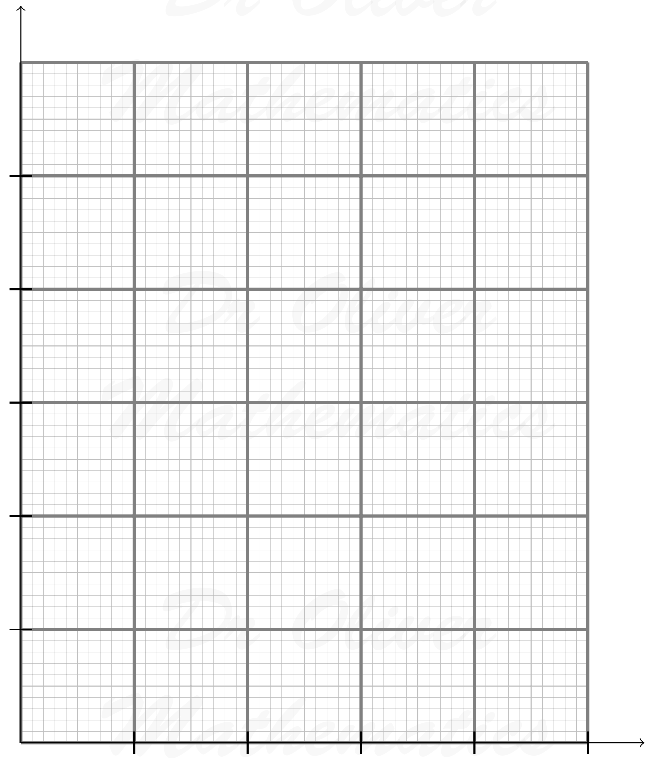
Solution

$$\begin{aligned}(x-2)^2 = 3 &\Rightarrow x - 2 = \pm\sqrt{3} \\ &\Rightarrow x = 2 \pm \sqrt{3} \\ &\Rightarrow x = 0.267\,949\,192\,4,\ 3.732\,050\,808 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{x = 0.268,\ 3.73 \text{ (3 sf)}}}.\end{aligned}$$

17. The table gives information about the heights of 150 students.

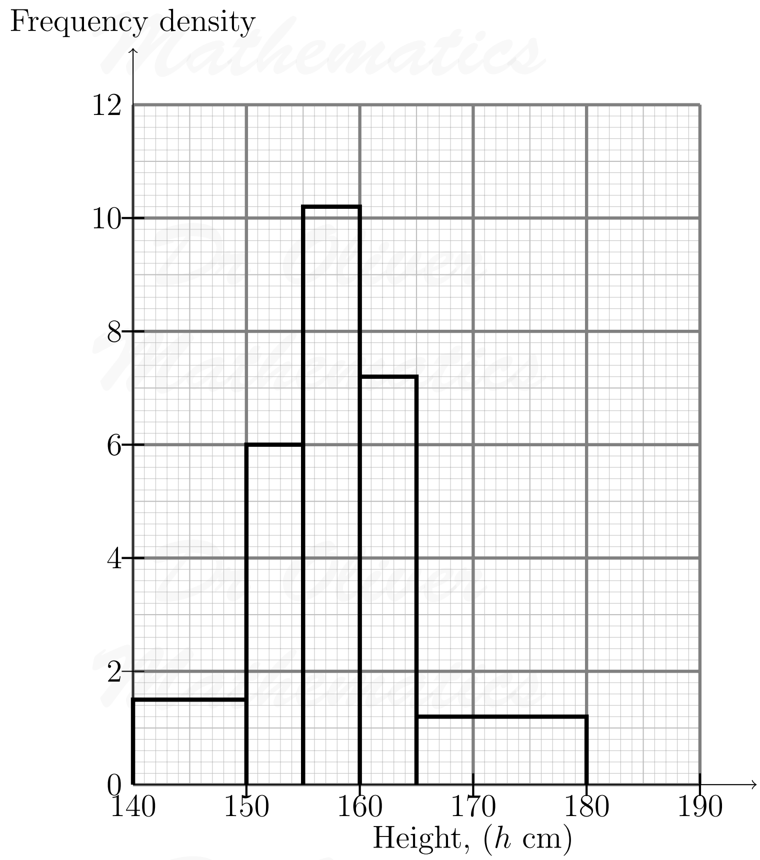
Height, (h cm)	Frequency
$140 < h \leqslant 150$	15
$150 < h \leqslant 155$	30
$155 < h \leqslant 160$	51
$160 < h \leqslant 165$	36
$165 < h \leqslant 180$	18

(a) On the grid, draw a histogram for this information. (3)



Solution

Height, (h cm)	Frequency	Width	Freq Den
$140 < h \leqslant 150$	15	10	1.5
$150 < h \leqslant 155$	30	5	6
$155 < h \leqslant 160$	51	5	10.2
$160 < h \leqslant 165$	36	5	7.2
$165 < h \leqslant 180$	18	15	1.2



(b) Work out an estimate for the fraction of the students who have a height between 150 cm and 170 cm. (2)

Solution

The number of students is

$$\begin{aligned} 30 + 51 + 36 + (\tfrac{1}{3} \times 18) &= 30 + 51 + 36 + 6 \\ &= 123 \end{aligned}$$

and

$$\text{fraction} = \frac{123}{150} = \underline{\underline{0.82}}.$$

18. At time $t = 0$ hours, a tank is full of water. (1)

Water leaks from the tank.
At the end of every hour, there is 2% less water in the tank than at the start of the hour.

The volume of water, in litres, in the tank at time t is V_t.

Given that

$$\begin{aligned} V_0 &= 2\,000, \text{ and} \\ V_{t+1} &= kV_t, \end{aligned}$$

write down the value of k.

Solution

$k = 1 - 0.02 = \underline{\underline{0.98}}$.

19. A triangle has vertices P, Q, and R. (4)

The coordinates of P are $(-3, -6)$.
The coordinates of Q are $(1, 4)$.
The coordinates of R are $(5, -2)$.

M is the midpoint of PQ.
N is the midpoint of QR.

Prove that MN is parallel to PR.
You must show each stage of your working.

Solution

M has coordinates

$$\left(\frac{-3+1}{2}, \frac{-6+4}{2}\right) = (-1, -1)$$

and N has coordinates

$$\left(\frac{1+5}{2}, \frac{4+(-2)}{2}\right) = (3, 1).$$

Now, the gradient of MN is

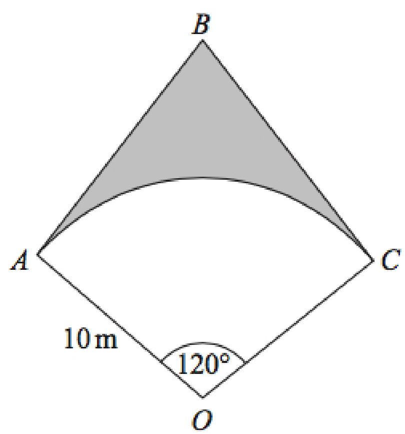
$$\begin{aligned} \frac{1-(-1)}{3-(-1)} &= \frac{2}{4} \\ &= \tfrac{1}{2} \end{aligned}$$

and the the gradient of PR is

$$\frac{-2-(-6)}{5-(-3)} = \frac{4}{8} = \tfrac{1}{2};$$

hence, MN is parallel to PR.

20. OAC is a sector of a circle, centre O, radius 10 m. (5)



BA is the tangent to the circle at point A.
BC is the tangent to the circle at point C.

Angle $AOC = 120°$.

Calculate the area of the shaded region.
Give your answer correct to 3 significant figures.

Solution

Split the triangle down the middle: clearly, $\angle AOB = 60°$, $\angle OAB = 90°$, and $\angle ABO = 30°$.

$$\tan = \frac{\text{opp}}{\text{adj}} \Rightarrow \tan 60° = \frac{BA}{10}$$
$$\Rightarrow BA = 10 \tan 60°.$$

Finally,

$$\begin{aligned}\text{shaded region} &= \text{whole region} - \text{area of the sector} \\ &= (2 \times \tfrac{1}{2} \times 10 \times 10 \tan 60^\circ) - (\tfrac{120}{360} \times \pi \times 10^2) \\ &= 100\sqrt{3} - \tfrac{100}{3}\pi \\ &= 68.485\,325\,64 \text{ (FCD)} \\ &= \underline{\underline{68.5 \text{ m}^2 \text{ (3 sf)}}}.\end{aligned}$$

21. There are 12 counters in a bag.
There is an equal number of red counters, blue counters, and yellow counters in the bag.
There are no other counters in the bag.

3 counters are taken at random from the bag.

(a) Work out the probability of taking 3 red counters. (2)

Solution

$$\begin{aligned}\mathrm{P}(RRR) &= \tfrac{4}{12} \times \tfrac{3}{11} \times \tfrac{2}{10} \\ &= \underline{\underline{\tfrac{1}{55}}}.\end{aligned}$$

The 3 counters are put back in the bag.

Some counters are now put into the bag.
There is still an equal number of red counters, blue counters, and yellow counters in the bag.
There are no other counters in the bag.

3 counters are taken at random from the bag.

(b) Is it now less likely or equally likely or more likely that 3 counters will be red? (2)
You must show how you get your answer.

Solution

Suppose there were double of counters (8 of each). Then

$$\begin{aligned} \mathrm{P}(RRR) &= \tfrac{8}{24} \times \tfrac{7}{23} \times \tfrac{6}{22} \\ &= \tfrac{7}{253} \\ &> \tfrac{1}{55}. \end{aligned}$$

Hence, it is $\underline{\underline{\text{more likely}}}$.

22. The functions f and g are such that (5)

$$\mathrm{f}(x) = 5x + 3 \text{ and } \mathrm{g}(x) = ax + b,$$

where a and b are constants.

$$\mathrm{g}(3) = 20 \text{ and } \mathrm{f}^{-1}(33) = \mathrm{g}(1).$$

Find the value of a and the value of b.

Solution

$$\mathrm{g}(3) = 20 \Rightarrow 20 = 3a + b \quad (1).$$

Now,

$$\mathrm{f}^{-1}(x) = \tfrac{1}{5}(x - 3)$$

and

$$\begin{aligned} \mathrm{f}^{-1}(33) = \mathrm{g}(1) &\Rightarrow \tfrac{1}{5}(33 - 3) = a(1) + b \\ &\Rightarrow 6 = a + b \quad (2). \end{aligned}$$

Do (1) − (2):

$$\begin{aligned} 14 = 2a &\Rightarrow \underline{\underline{a = 7}} \\ &\Rightarrow \underline{\underline{b = -1}}. \end{aligned}$$

23. S is a geometric sequence.

(a) Given that (3)

$$(\sqrt{x} - 1),\ 1,\ \text{and } (\sqrt{x} + 1)$$

are the first three terms of S, find the value of x.
You must show all of your working.

Solution

If S is a geometric sequence,

$$\frac{1}{\sqrt{x}-1} = \frac{\sqrt{x}+1}{1} \Rightarrow (\sqrt{x}-1)(\sqrt{x}+1) = 1$$

$\times$	$\sqrt{x}$	$+1$
$\sqrt{x}$	x	$+\sqrt{x}$
-1	$\sqrt{x}$	-1

$$\Rightarrow x - 1 = 1$$
$$\Rightarrow \underline{\underline{x = 2}}.$$

(b) Show that the 5th term of S is (2)

$$7 + 5\sqrt{2}.$$

Solution

Well,

$$(\sqrt{2}-1),\ 1,\ \text{and } (\sqrt{2}+1)$$

are the first three terms of S.

$\times$	$\sqrt{2}$	$+1$
$\sqrt{2}$	2	$+\sqrt{2}$
$+1$	$+\sqrt{2}$	$+1$

and so

$$(\sqrt{2}+1)^2 = 3 + 2\sqrt{2}.$$

Hence,

$$(\sqrt{2}+1)(\sqrt{2}+1)^2 = (\sqrt{2}+1)(3+2\sqrt{2})$$

$\times$	$\sqrt{2}$	$+1$
3	$3\sqrt{2}$	$+3$
$+2\sqrt{2}$	$+4$	$+2\sqrt{2}$

$$= \underline{\underline{7 + 5\sqrt{2}}},$$

as required.