

Dr Oliver Mathematics
AQA GCSE Mathematics
2019 November Paper 2: Calculator
1 hour 30 minutes

The total number of marks available is 80.

You must write down all the stages in your working.

1. Expand

$$4x^2(3x + 5).$$

(1)

Circle your answer.

$$32x^3 \quad 12x^3 + 20x^2 \quad 7x^3 + 9x^2 \quad 12x^2 + 5$$

Solution

$$32x^3 \quad \underline{\underline{12x^3 + 20x^2}} \quad 7x^3 + 9x^2 \quad 12x^2 + 5$$

2. How many millimetres are there in a kilometre?

Circle your answer.

$$10^3 \quad 10^5 \quad 10^6 \quad 10^9$$

(1)

Solution

Well,

$$\begin{aligned} 1 \text{ kilometre} &= 1\,000 \text{ metres} \\ &= 1\,000 \times 100 \text{ centimetres} \\ &= 1\,000 \times 100 \times 10 \text{ millimetres} \\ &= 1\,000\,000 \text{ millimetres} \\ &= 10^6 \text{ millimetres} \end{aligned}$$

so

$$10^3 \quad 10^5 \quad \underline{\underline{10^6}} \quad 10^9$$

3. Circle the number half way between

(1)

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$$\frac{7}{12} \text{ and } \frac{3}{4}.$$
$$\frac{7}{32} \quad \frac{5}{8} \quad \frac{2}{3} \quad \frac{1}{2}$$

Solution

Well,

$$\begin{aligned} \frac{1}{2}\left(\frac{7}{12} + \frac{3}{4}\right) &= \frac{1}{2}\left(\frac{7}{12} + \frac{9}{12}\right) \\ &= \frac{1}{2}\left(\frac{16}{12}\right) \\ &= \frac{1}{2}\left(\frac{4}{3}\right) \\ &= \frac{2}{3} \end{aligned}$$

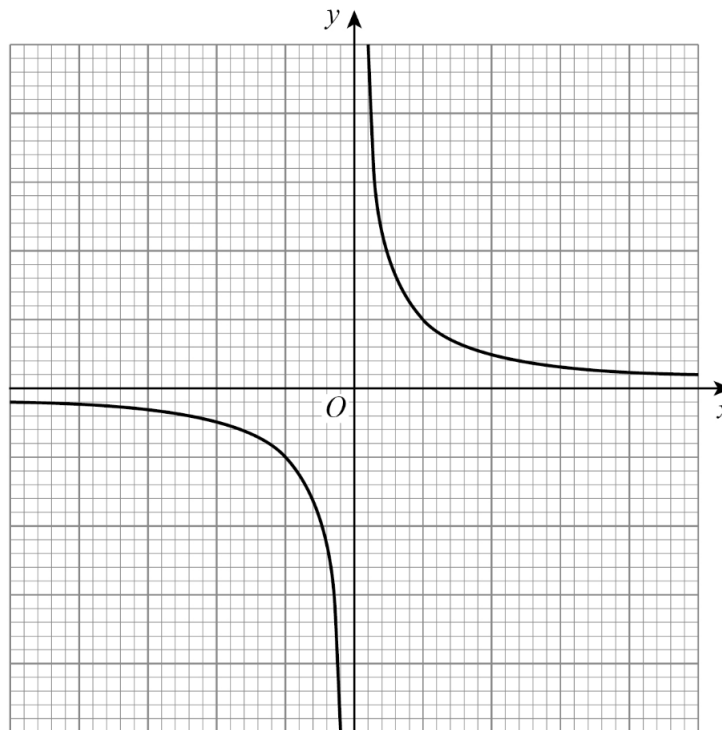
so

$$\frac{7}{32} \quad \frac{5}{8} \quad \underline{\underline{\frac{2}{3}}} \quad \frac{1}{2}$$

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4. Here is the sketch of a graph.

(1)



Mathematics

Circle the equation of the graph.

$$y = x \quad y = -x^2 \quad y = -x^3 \quad y = \frac{1}{x}$$

Solution

$$y = x \quad y = -x^2 \quad y = -x^3 \quad \underline{\underline{y = \frac{1}{x}}}$$

5. Work out the lowest common multiple (LCM) of 120 and 144

(2)

Solution

$$\begin{array}{r|l} & 120 \\ 2 & 60 \\ 2 & 30 \\ 2 & 15 \\ 3 & 5 \\ 5 & 1 \end{array}$$

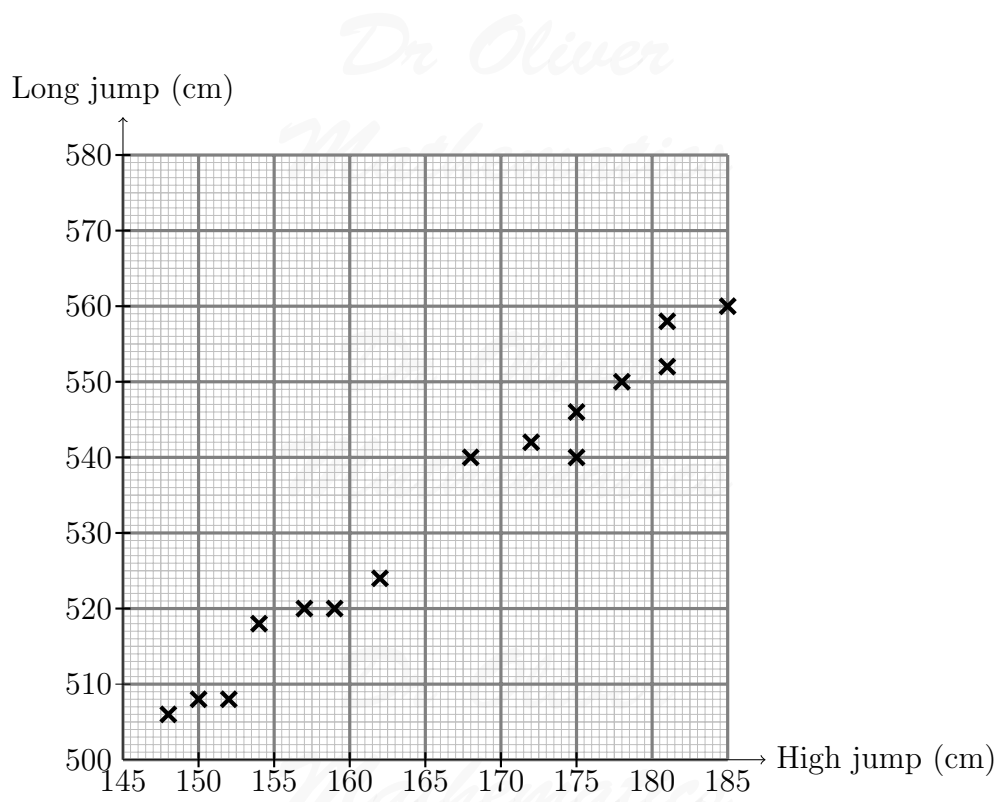
and

$$\begin{array}{r|l} & 144 \\ 2 & 72 \\ 2 & 36 \\ 2 & 18 \\ 2 & 9 \\ 3 & 3 \\ 3 & 1 \end{array}$$

so

$$\text{LCM} = 2^4 \times 3^2 \times 5 = \underline{\underline{720}}.$$

6. The scatter graph shows the best high jump and the best long jump for 15 boys.



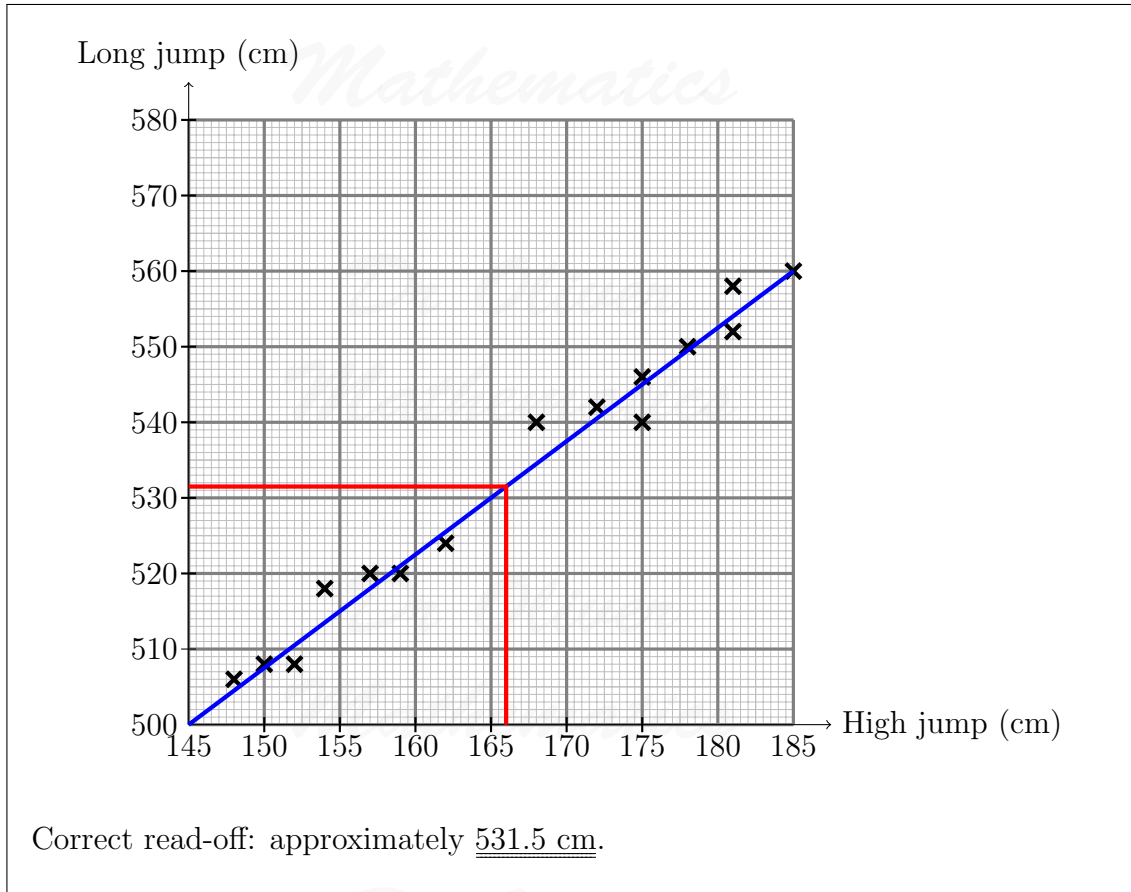
(a) Write down the type of correlation shown. (1)

Solution
Positive correlation.

Liam has a best high jump of 166 cm.

(b) Use a line of best fit to estimate his best long jump. (2)

Solution
 Use a line of best fit:



Another boy has a best high jump of 195 cm.

- (c) Give a reason why you should **not** use a line of best fit to estimate his best long jump. (1)

Solution

E.g., it requires extrapolation, 195 cm is outside the range of values

7. A car journey is in two stages. (3)

- Stage 1: the car travels 110 miles in 2 hours.
- Stage 2: the car travels 44 miles at the same average speed as Stage 1.

Work out the time for Stage 2.

Give your answer in minutes.

Solution

Stage 1:

$$\text{speed} = \frac{110}{2} = 55 \text{ mph.}$$

Stage 2:

$$\begin{aligned} \text{time} &= \frac{44}{55} \text{ hours} \\ &= 0.8 \text{ hours} \\ &= \underline{\underline{48 \text{ minutes}}}. \end{aligned}$$

8. Here is an identity:

$$a(3x - 10) \equiv 21x + 2b.$$

(3)

Work out the values of a and b .

Solution

Well,

$$a(3x - 10) \equiv 21x + 2b \Rightarrow 3ax - 10a \equiv 21x + 2b$$

which gives us

$$3a = 21 \quad (1)$$

$$-10a = 2b \quad (2).$$

From (1),

$$3a = 21 \Rightarrow \underline{\underline{a = 7}}$$

and, from (2),

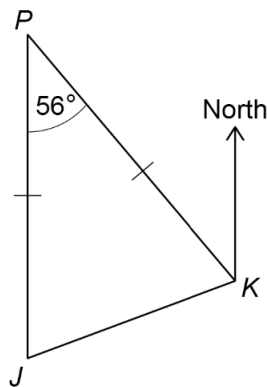
$$2b = -10(7) \Rightarrow \underline{\underline{b = -35}}.$$

9. J and K are ships.

P is a port.

(3)

- J is due South of P .
- Angle $JPK = 56^\circ$.
- $JP = KP$.



Not drawn accurately

Work out the bearing of J from K .

Solution

$$\angle PKJ = \frac{1}{2}(180 - 56) = 62^\circ \text{ (completing the triangles)}$$

$$\angle PKN = 56^\circ \text{ (alternate angles)}$$

$$\text{bearing} = 360 - (62 + 56) = \underline{\underline{242^\circ}}.$$

10. The 5th term of a linear sequence is 17.
The 6th term of the sequence is 21.

(3)

Work out the 100th term of the sequence.

Solution

Let the n th term be $an + b$, where a and b are constants. Then

$$5a + b = 17 \quad (1)$$

$$6a + b = 21 \quad (2).$$

Do (2) - (1):

$$a = 4$$

and substitute into (1):

$$5(4) + b = 17 \Rightarrow 20 + b = 17$$

$$\Rightarrow b = -3;$$

so n th term = $4n - 3$.

Finally,

$$100 \text{ th term} = 4(100) - 3 = \underline{\underline{397}}.$$

11. The value of a house is £120 000.

(4)

The value is expected to increase by 5% each year.

Work out the expected value after 4 years.

Give your answer to 2 significant figures.

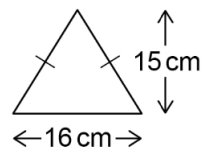
You **must** show your working.

Solution

$$\begin{aligned} \text{Expected value} &= 120\,000 \times 1.05^4 \\ &= 145\,860.75 \text{ (exact)} \\ &= \underline{\underline{\pounds 150\,000}} \text{ (2 sf)}. \end{aligned}$$

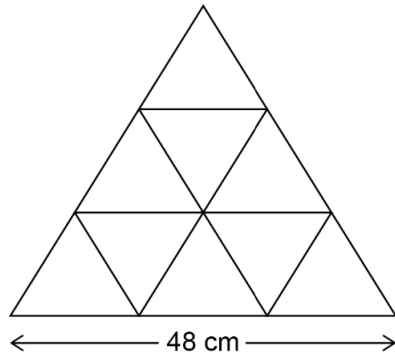
12. An isosceles triangle has base 16 cm and perpendicular height 15 cm.

(4)



Not drawn
accurately

Some of these triangles are used to make a large triangle.



Not drawn accurately

Work out the perimeter of the large triangle.

Solution

Well,

$$\frac{\text{base}}{2} = 8 \text{ cm}$$

and we can apply Pythagoras' theorem:

$$\begin{aligned} \text{opp}^2 + \text{adj}^2 &= \text{hyp}^2 \Rightarrow 15^2 + 8^2 = \text{hyp}^2 \\ &\Rightarrow 225 + 64 = \text{hyp}^2 \\ &\Rightarrow 289 = \text{hyp}^2 \\ &\Rightarrow \text{hyp} = 17. \end{aligned}$$

Finally,

$$\begin{aligned} \text{perimeter} &= 48 + 6(17) \\ &= \underline{\underline{150 \text{ cm}}}. \end{aligned}$$

13. 200 people recorded the time they spent on social media one day. The table shows the results.

| Time, t (mins) | Frequency | Midpoint |
|------------------|-----------|----------|
| $0 \leq t < 30$ | 24 | |
| $30 \leq t < 50$ | 76 | |
| $50 \leq t < 60$ | 52 | |
| $6 \leq t < 90$ | 48 | |
| Total= 200 | | |

(a) Work out an estimate of the mean time.

(3)

Solution

| Time, t (mins) | Frequency | Midpoint | Freq \times Midpoint |
|------------------|-------------|----------|------------------------|
| $0 \leq t < 30$ | 24 | 15 | $24 \times 15 = 360$ |
| $30 \leq t < 50$ | 76 | 40 | $76 \times 40 = 3040$ |
| $50 \leq t < 60$ | 52 | 55 | $52 \times 55 = 2860$ |
| $60 \leq t < 90$ | 48 | 75 | $48 \times 75 = 3600$ |
| | Total = 200 | | Total = 9860 |

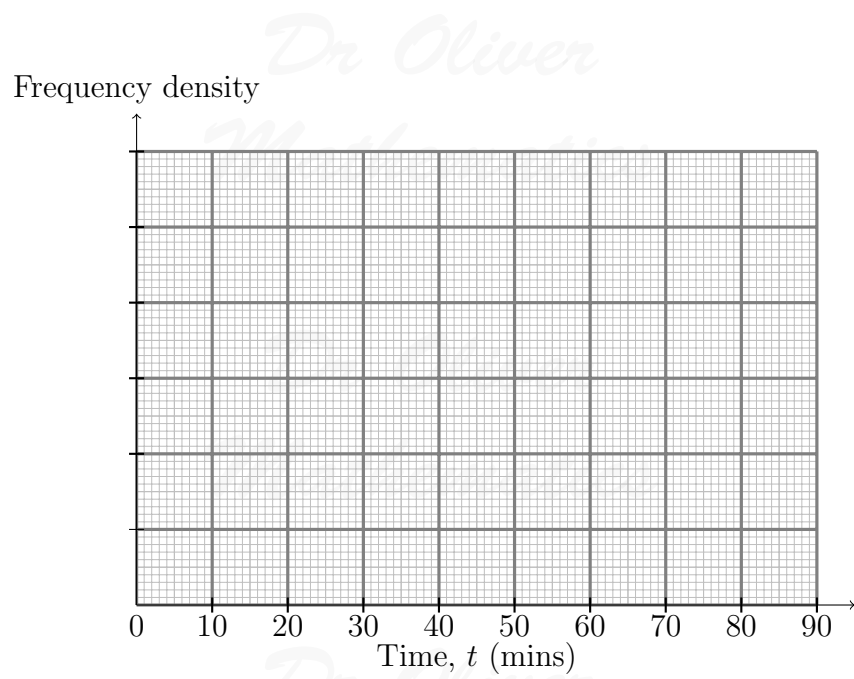
Now,

$$\begin{aligned} \text{mean time} &= \frac{\sum fx}{\sum x} \\ &\approx \frac{9860}{200} \\ &= \underline{\underline{49.3}}. \end{aligned}$$

(b) Draw a histogram to represent the results.

(4)

| Time, t (mins) | Frequency | Class width |
|------------------|-----------|-------------|
| $0 \leq t < 30$ | 24 | |
| $30 \leq t < 50$ | 76 | |
| $50 \leq t < 60$ | 52 | |
| $60 \leq t < 90$ | 48 | |



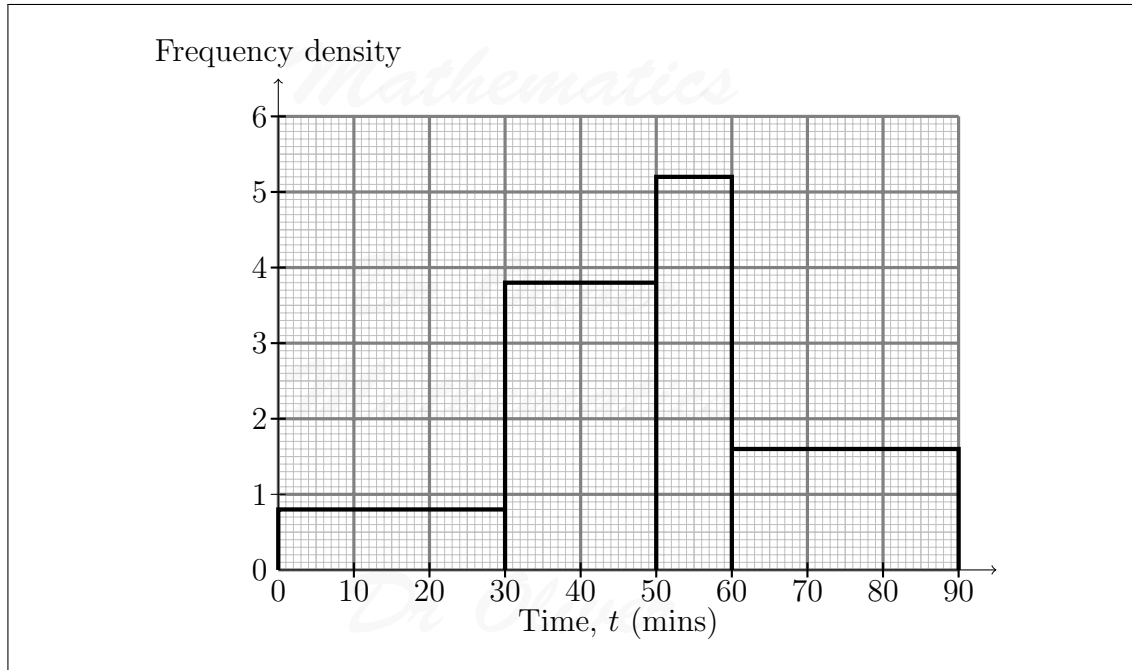
Solution

| Time, t (mins) | Frequency | Class width | Frequency density |
|------------------|-----------|-------------|-----------------------|
| $0 \leq t < 30$ | 24 | 30 | $\frac{24}{30} = 0.8$ |
| $30 \leq t < 50$ | 76 | 20 | $\frac{76}{20} = 3.8$ |
| $50 \leq t < 60$ | 52 | 10 | $\frac{52}{10} = 5.2$ |
| $60 \leq t < 90$ | 48 | 30 | $\frac{48}{30} = 1.6$ |

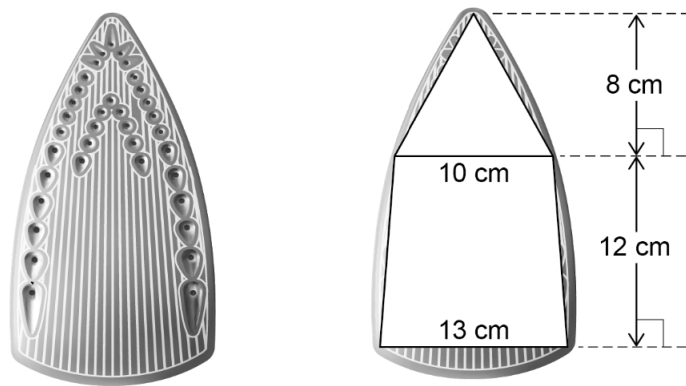
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14. Ralf has an iron.
He models the base as a triangle joined to a trapezium.



- (a) The iron applies a force of 25 newtons (N). (4)

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Work out the pressure using Ralf's model.

Solution

Well,

$$\begin{aligned} \text{area} &= \text{trapezium} + \text{triangle} \\ &= \left[\frac{1}{2} \times 12 \times (10 + 13)\right] + \left(\frac{1}{2} \times 10 \times 8\right) \\ &= 138 + 40 \\ &= 178 \end{aligned}$$

and

$$\begin{aligned} \text{pressure} &= \frac{25}{178} \\ &= 0.1404494382 \text{ (FCD)} \\ &= \underline{\underline{0.140 \text{ Pa (3 sf)}}}. \end{aligned}$$

- (b) Is the actual pressure greater than, equal to, or less than your answer to part (a)?
Tick **one** box. (2)

greater than

equal to

less than

Give a reason for your answer.

Solution

Less than: he has not included all the base.

15. Rearrange (1)

$$y = \sqrt{w^3}$$

to make w the subject. Circle your answer.

$$w = y^6 \quad w = \sqrt[3]{y^2} \quad w = \sqrt{y^3} \quad w = y^5$$

Solution

$$\begin{aligned} y = \sqrt{w^3} &\Rightarrow y^2 = w^3 \\ &\Rightarrow w = \sqrt[3]{y^2} \end{aligned}$$

so

$$w = y^6 \quad \underline{w = \sqrt[3]{y^2}} \quad w = \sqrt{y^3} \quad w = y^5$$

16. (a) Show that

$$a\% \text{ of } b = b\% \text{ of } a.$$

(1)

Solution

Well,

$$\begin{aligned} a\% \text{ of } b &= \frac{a}{100} \times b \\ &= \frac{b}{100} \times a \\ &= \underline{b\% \text{ of } a.} \end{aligned}$$

(b) Rosie says, “160% of 40 = 140% of 60 because $a\% \text{ of } b = b\% \text{ of } a$.”

(1)

Is she correct?

Tick a box.

Yes

No

Give a reason for your answer.

Solution

No:

$$160\% \text{ of } 40 = 64$$

whilst

$$140\% \text{ of } 60 = 84.$$

17. A packet contains 80 sweets.

The flavour of each sweet is lemon, orange, or apple.

A sweet is taken at random.

- (a) $P(\text{lemon or orange}) \leq 0.85$. (2)

Work out the minimum possible number of **apple** sweets in the packet.

Solution

$$1 - 0.85 = 0.15$$

and

$$\text{minimum possible} = 0.15 \times 80 = \underline{\underline{12}}.$$

- (b) $P(\text{lemon or apple}) < 0.71$. (2)

There are 31 lemon sweets.

Work out the maximum possible number of **apple** sweets in the packet.

Solution

Well,

$$P(\text{lemon or apple}) < 0.71 \Rightarrow \text{lemon or apple} < 0.71 \times 80$$

$$\Rightarrow \text{lemon or apple} < 56.8$$

$$\Rightarrow \text{apple} < 56.8 - 31$$

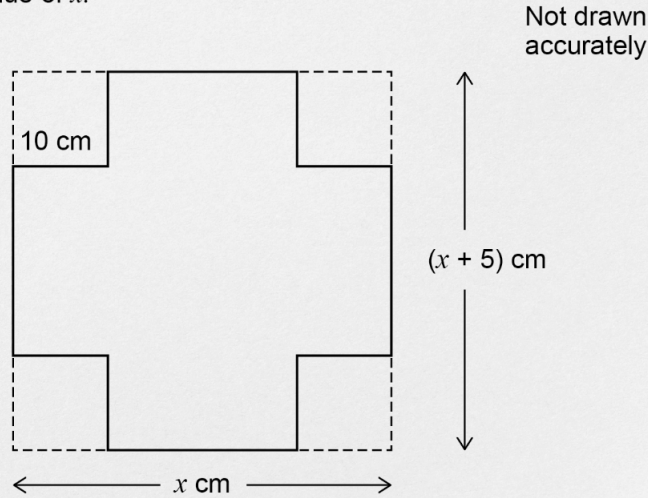
$$\Rightarrow \text{apple} < 25.8$$

so

$$\text{maximum possible} = \underline{\underline{25}}.$$

18. Kate has the following question for homework.

The net of a box is made by cutting four squares from a piece of cardboard
 The cardboard is a rectangle with width x cm and length $(x + 5)$ cm
 Each square has side length 10 cm
 The area of the net is 1000 cm^2
 Work out the value of x .



(a) Show that Kate can form the equation

(3)

$$x^2 + 5x - 1400 = 0.$$

Solution

$$x(x + 5) - (4 \times 10 \times 10) = 1000 \Rightarrow x^2 + 5x - 400 = 1000$$

$$\Rightarrow \underline{\underline{x^2 + 5x - 1400 = 0}},$$

as required.

Kate correctly factorises the equation to get

$$(x + 40)(x - 35) = 0.$$

Her answer to the homework question is

$$x = -40 \text{ or } x = 35.$$

(b) Is her answer correct?

(1)

Tick a box.

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Yes

No

Give a reason for your answer.

Solution

No: she cannot have a negative answer because distances are non-negative.

19. Circle the word that describes the graph

(1)

$$y = \sin x.$$

periodic exponential cubic quadratic

Solution

periodic exponential cubic quadratic

20. (7, 28) is a point on the graph $y = f(x)$.

(1)

Circle the point which must be on the graph $y = f(x) + 2$.

(7, 26) (7, 30) (5, 28) (9, 28)

Solution

(7, 26) (7, 30) (5, 28) (9, 28)

21. n is the middle integer of three consecutive positive integers.

(4)

- The three integers are multiplied to give a product.
- n is then added to the product.

Prove that the result is a cube number.

Solution

Well, the number is

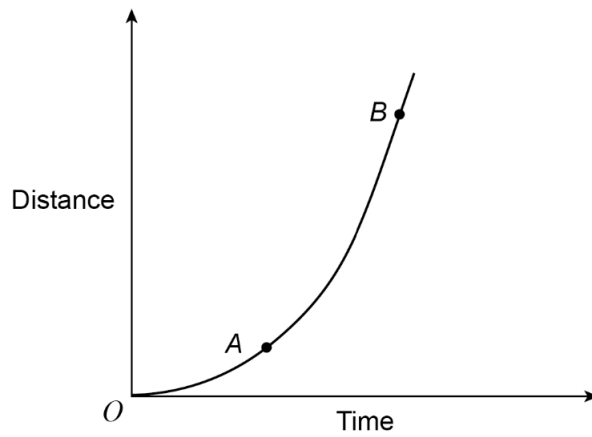
$$(n - 1)n(n + 1) + n = n[(n - 1)(n + 1) + 1]$$

difference of two squares:

$$\begin{aligned} &= n[(n^2 - 1) + 1] \\ &= n(n^2) \\ &= \underline{\underline{n^3}}. \end{aligned}$$

22. Here is a sketch of a distance-time graph.

(1)



Which of these represents the average speed between A and B?
Tick **one** box.

The gradient of the tangent at A

The gradient of the tangent at B

The gradient of the chord from A to B

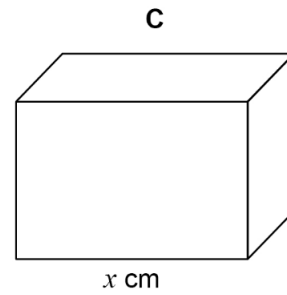
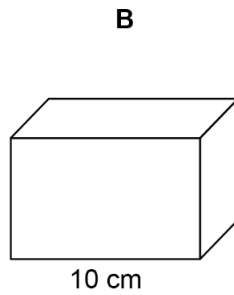
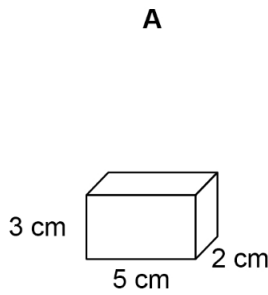
The gradient of the chord from O to B

Solution

The gradient of the chord from A to B .

23. Here are three similar cuboids: A , B , and C .

- A has length 5 cm, width 2 cm, and height 3 cm.
- B has length 10 cm.
- C has length x cm



(a) The total surface area of A is 62 cm^2 .

Tim wants to work out the total surface area of B .

Here is his working.

(1)

| |
|--|
| $10 \div 5 = 2$ |
| $62 \times 2 = 124$ |
| Total surface area of $B = 124 \text{ cm}^2$ |

Make one criticism of Tim's method.

Solution

In the second line, he has not use of 2^2 :

$$62 \times 2^2 = \underline{248}$$

$$\text{Volume of } A \times \frac{125}{8} = \text{volume of } C.$$

(b) Work out the value of x .

(3)

Solution

Well,

$$\begin{aligned} \text{volume of } A &= 2 \times 3 \times 5 \\ &= 30 \end{aligned}$$

and

$$\begin{aligned} \text{volume of } C &= \text{volume of } A \times \frac{125}{8} \\ &= 30 \times \frac{125}{8} \\ &= \underline{468.75 \text{ cm}^3}. \end{aligned}$$

24. Here are two inequalities.

(3)

$$-2 \leq x \leq 3$$

$$9 \leq x + y \leq 11.$$

x and y are integers.

Work out the greatest possible value of

$$y - x.$$

Solution

Well,

$$\begin{aligned} -2 \leq x \leq 3 &\Rightarrow -3 \leq -x \leq 2 \\ &\Rightarrow -6 \leq -2x \leq 4. \end{aligned}$$

Now,

$$y - x = (y + x) - 2x$$

and

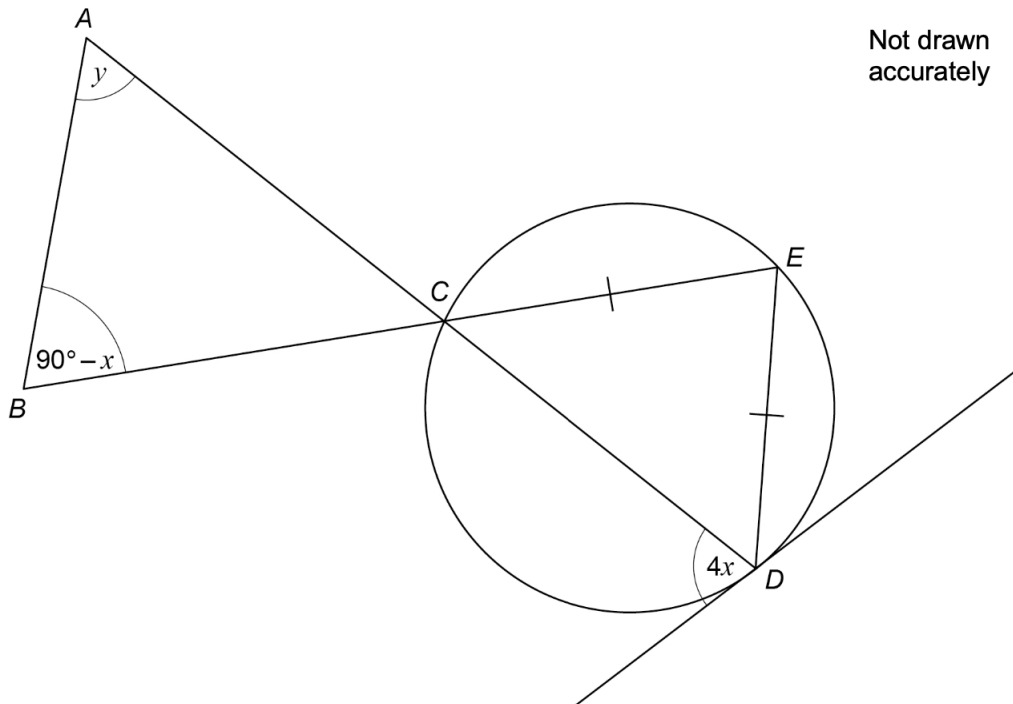
$$9 - 6 \leq (y + x) - 2x \leq 4 + 11 \Rightarrow 3 \leq y - x \leq 15$$

so

$$\underline{\underline{y - x \leq 15.}}$$

25. C , D , and E are points on a circle.
 $CE = DE$.
The tangent at D is shown.
 ACD and BCE are straight lines.

(4)



Prove that

$$y = 3x.$$

Solution

Well, $\angle ACB = 180 - (90 - x) - y = (90 + x - y)^\circ$ (completing the triangle)

$\angle DCE = (90 + x - y)^\circ$ (vertically opposite angles)

$\angle CED = \frac{1}{2}[180 - 2(90 + x - y)] = (2y - 2x)^\circ$ (vertically opposite angles)

Finally, by the alternating segment theorem,

$$\begin{aligned} 2y - 2x = 4x &\Rightarrow 2y = 6x \\ &\Rightarrow \underline{\underline{y = 3x}}, \end{aligned}$$

as required.

26. P , Q , and R have positive values.

(5)

- P is directly proportional to the square of Q .
- When $P = 1.25$, $Q = 0.5$.
- Q is inversely proportional to R .
- When $Q = 0.5$, $R = 6$.

Work out the value of R when $P = 0.8$.

Solution

Well,

$$P \propto Q^2 \Rightarrow P = kQ^2,$$

where k is some constant. Now,

$$\begin{aligned} P = 1.25, Q = 0.5 &\Rightarrow 1.25 = k(0.5^2) \\ &\Rightarrow k = 5 \end{aligned}$$

so

$$P = 5Q^2.$$

Next,

$$Q \propto \frac{1}{R} \Rightarrow Q = \frac{l}{R},$$

where l is some constant. Now,

$$\begin{aligned}Q = 0.5, R = 6 &\Rightarrow 0.5 = \frac{l}{6} \\ &\Rightarrow l = 3\end{aligned}$$

so

$$Q = \frac{3}{R}.$$

Finally,

$$\begin{aligned}P = 0.8 &\Rightarrow 0.8 = 5Q^2 \\ &\Rightarrow Q^2 = 0.16 \\ &\Rightarrow Q = 0.4\end{aligned}$$

and

$$\begin{aligned}\Rightarrow 0.4 &= \frac{3}{R} \\ \Rightarrow R &= \frac{3}{0.4} \\ \Rightarrow R &= \underline{\underline{7\frac{1}{2}}}.\end{aligned}$$

27.

$$x_{n+1} = \sqrt[3]{3x_n + 7}.$$

(3)

Use a starting value of $x_1 = 2$ to work out a solution to

$$x = \sqrt[3]{3x + 7}.$$

Give your answer to 3 decimal places.

Solution

$$\begin{aligned}x_1 = 2 &\Rightarrow x_2 = 2.351\ 334\ 688 \text{ (FCD)} \\ &\Rightarrow x_3 = 2.413\ 237\ 275 \text{ (FCD)} \\ &\Rightarrow x_4 = 2.423\ 820\ 198 \text{ (FCD)} \\ &\Rightarrow x_5 = 2.425\ 620\ 238 \text{ (FCD)} \\ &\Rightarrow x_6 = 2.425\ 926\ 14 \text{ (FCD)};\end{aligned}$$

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as $x_5 = x_6$ agree to 3 decimal places,

$$\underline{\underline{x = 2.426 \text{ (3 dp)}}}.$$

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