Dr Oliver Mathematics
AQA GCSE Mathematics
2018 June Paper 1: Non-Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. Work out (1)

$$\sqrt[3]{64 \times 1\,000}.$$

Circle your answer.

40 80 400 4 000

Solution

$$\begin{aligned}\sqrt[3]{64 \times 1\,000} &= \sqrt[3]{64} \times \sqrt[3]{1\,000} \\ &= 4 \times 10 \\ &= 40\end{aligned}$$

so

$\underline{\underline{40}}$ 80 400 4 000

2. The vector (1)

$$\begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

translates A to B.

Circle the vector that translates B to A.

$$\begin{pmatrix} -2 \\ 3 \end{pmatrix} \quad \begin{pmatrix} -3 \\ 2 \end{pmatrix} \quad \begin{pmatrix} 3 \\ -2 \end{pmatrix} \quad \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

Solution

$$\begin{pmatrix} -2 \\ 3 \end{pmatrix} \quad \begin{pmatrix} -3 \\ 2 \end{pmatrix} \quad \begin{pmatrix} 3 \\ -2 \end{pmatrix} \quad \underline{\underline{\begin{pmatrix} 2 \\ -3 \end{pmatrix}}}$$

3. Circle the expression that is equivalent to (1)

$$3a - a \times 4a + 2a.$$

$$8a^2 + 2a \quad 12a^2 \quad 5a - 4a^2 \quad 3a^6a^2$$

Solution

BODMAS:

$$\begin{aligned} 3a - a \times 4a + 2a &= 3a - (a \times 4a) + 2a \\ &= 3a - 4a^2 + 2a \\ &= 5a - 4a^2 \end{aligned}$$

so

$$8a^2 + 2a \quad 12a^2 \quad \underline{\underline{5a - 4a^2}} \quad 3a^6a^2$$

4. Circle the number that is closest in value to (1)

$$\frac{9.8}{0.0195}.$$

$$5 \quad 50 \quad 500 \quad 5000$$

Solution

Use 1 significant figure:

$$\begin{aligned} \frac{9.8}{0.0195} &\approx \frac{10}{0.02} \\ &= 500 \end{aligned}$$

so

$$5 \quad 50 \quad \underline{\underline{500}} \quad 5000$$

5. Solve (2)

$$5(x + 3) < 60.$$

Solution

$$5(x+3) < 60 \Rightarrow x+3 < 12$$
$$\Rightarrow \underline{\underline{x < 9}}.$$

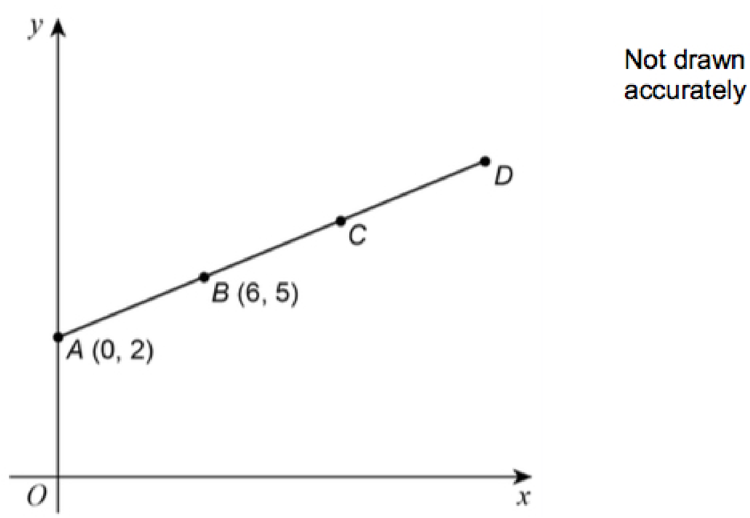
6. The height of Zak is 1.86 metres. (3)
The height of Fred is 1.6 metres.
Write the height of Zak as a fraction of the height of Fred.
Give your answer in its simplest form.

Solution

Well,

$$\frac{1.86 \text{ m}}{1.6 \text{ m}} = \frac{1.86}{1.6} = \frac{186}{160} = \frac{93}{80} = \underline{\underline{1\tfrac{13}{80}}}.$$

7. $A(0,2)$ and $B(6,5)$ are points on the straight line $ABCD$. (3)



$AB = BC = CD$.

Work out the coordinates of D.

Solution

Well,

$$\begin{aligned}\overrightarrow{AB} &= \begin{pmatrix} 6 \\ 5 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \end{pmatrix} \\ &= \begin{pmatrix} 6 \\ 3 \end{pmatrix}\end{aligned}$$

and

$$\begin{aligned}\overrightarrow{AD} &= 3\overrightarrow{AB} \\ &= 3\begin{pmatrix} 6 \\ 3 \end{pmatrix} \\ &= \begin{pmatrix} 18 \\ 9 \end{pmatrix};\end{aligned}$$

so

$$\begin{aligned}\overrightarrow{OD} &= \overrightarrow{OA} + \overrightarrow{AD} \\ &= \begin{pmatrix} 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 18 \\ 9 \end{pmatrix} \\ &= \begin{pmatrix} 18 \\ 11 \end{pmatrix}.\end{aligned}$$

Hence, the coordinates of D are $\underline{\underline{(18, 11)}}$.

8. A coin is thrown 50 times.
   It lands on heads 31 times.

   (a) Write down the relative frequency it lands on heads. (1)

   Solution

   $$\frac{31}{50} = \underline{\underline{0.62}}.$$

   Raj says, "The coin is biased towards heads."

(b) Use the data to give a reason why he might be correct. (1)

Solution
E.g., relative frequency (probability?) is more than 0.5.

9. The range of a set of numbers is $15\frac{1}{4}$. (3)

The smallest number is $-2\frac{7}{8}$.

Work out the largest number.

Solution
Well, the largest number is

$$\begin{aligned} 15\tfrac{1}{4} + (-2\tfrac{7}{8}) &= 15\tfrac{1}{4} - 2\tfrac{7}{8} \\ &= 13\tfrac{2}{8} - \tfrac{7}{8} \\ &= 13 - \tfrac{5}{8} \\ &= \underline{\underline{12\tfrac{3}{8}}}. \end{aligned}$$

10. y is inversely proportional to x. (2)

x	12	6	
y		4	8

Complete the table.

Solution
Well,

$$y \propto \frac{1}{x} \Rightarrow y = \frac{k}{x},$$

where k is some number. Now,

$$\begin{aligned} x = 6, y = 4 \Rightarrow 4 &= \frac{k}{6} \\ \Rightarrow k &= 24 \end{aligned}$$

and so

$$y = \frac{24}{x}.$$

So

x	12	6	$\underline{\underline{3}}$
y	$\underline{\underline{2}}$	4	8

11. A large rectangle is made by joining three identical small rectangles as shown. (4)

Not drawn accurately

The perimeter of one small rectangle is 15 cm.

Work out the perimeter of the large rectangle.

Solution

Let l cm the length (on the right, as you look at it) one rectangle. Then

$$h = 2l$$

and

$$\begin{aligned} l + 2l + l + 2l = 15 &\Rightarrow 6l = 15 \\ &\Rightarrow l = 2\tfrac{1}{2}. \end{aligned}$$

Now,

$$\begin{aligned} \text{perimeter of the large rectangle} &= l + l + 2l + l + 2l + l + 2l \\ &= 10l \\ &= 10(2.5) \\ &= \underline{\underline{25 \text{ cm}}}. \end{aligned}$$

12. Put these numbers in order from smallest to largest. (2)

$$8 \times 10^{-4} \quad 4 \times 10^{-2} \quad 6 \times 10^{-4} \quad 0.07$$

Solution

Well,

$$\begin{aligned} 8 \times 10^{-4} &= 0.000\,8 \\ 4 \times 10^{-2} &= 0.04 \\ 6 \times 10^{-4} &= 0.000\,6 \\ &0.07 \end{aligned}$$

so, from smallest to largest:

$$\underline{\underline{6 \times 10^{-4},\ 8 \times 10^{-4},\ 0.04,\ 0.07}}.$$

13. Circle the volume that is the same as 15 cm^3: (1)

$$15\,000 \text{ mm}^3 \quad 1.5 \text{ mm}^3 \quad 0.001\,5 \text{ mm}^3 \quad 150 \text{ mm}^3$$

Solution

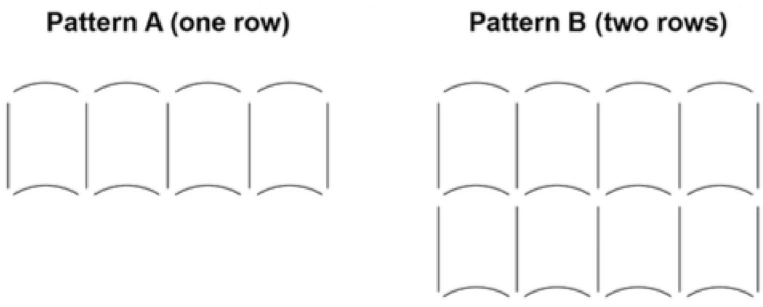
Well,

$$\begin{aligned} 15 \text{ cm}^3 &= 15 \times 1 \text{ cm}^3 \\ &= 15 \times 1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm} \\ &= 15 \times 10 \text{ mm} \times 10 \text{ mm} \times 10 \text{ mm} \\ &= 15 \times 1\,000 \text{ mm}^3 \\ &= 15\,000 \text{ mm}^3 \end{aligned}$$

so

$$\underline{\underline{15\,000 \text{ mm}^3}} \quad 1.5 \text{ mm}^3 \quad 0.001\,5 \text{ mm}^3 \quad 150 \text{ mm}^3$$

14. Patterns are made using straight lines and arcs.



More rows are added to **Pattern B** so that

$$\text{number of straight lines} : \text{number of arcs} = 10 : 9.$$

(a) How many rows are added? (2)

Solution

What is added is, to each line, is $5 : 4$ so let x be the number of rows needed. Now,

$$\begin{aligned} 5 + 5x : 8 + 4x = 10 : 9 &\Rightarrow \frac{5 + 5x}{8 + 4x} = \frac{10}{9} \\ &\Rightarrow 9(5 + 5x) = 10(8 + 4x) \\ &\Rightarrow 45 + 45x = 80 + 40x \\ &\Rightarrow 5x = 35 \\ &\Rightarrow x = 7; \end{aligned}$$

so, it is the sixth row.

A different pattern is made using 20 straight lines and 16 arcs.

- The straight lines and arcs are made from metal.
- 20 straight lines cost £12.
- Cost of one straight line : cost of one arc = 2 : 3.

(b) Work out the **total** cost of the metal in the pattern. (3)

Solution

Well,

$$\begin{aligned} \text{20 straight lines cost } £12 &\Leftrightarrow \text{1 straight line costs } £0.60 \\ &\Leftrightarrow \text{2 straight lines cost } £1.20 \\ &\Leftrightarrow \text{3 arcs cost } \left(£1.20 \times \frac{3}{2}\right) \\ &\Leftrightarrow \text{3 arcs cost } £1.80 \\ &\Leftrightarrow \text{1 arc costs } £0.90. \end{aligned}$$

Hence,

$$\begin{aligned} \text{total cost} &= (20 \times 0.60) + (16 \times 0.90) \\ &= 12 + 14.40 \\ &= \underline{\underline{£26.40}}. \end{aligned}$$

15. A biased dice is thrown. (3)
Here are the probabilities of each score.

Score	1	2	3	4	5	5
Probability	0.25	0.05	0.15	0.05	0.3	0.2

The dice is thrown 200 times.

Work out the expected number of times the score will be odd.

Solution

$$\begin{aligned} \text{Expected number} &= (200 \times 0.25) + (200 \times 0.15) + (200 \times 0.3) \\ &= 50 + 30 + 60 \\ &= \underline{\underline{140}}. \end{aligned}$$

16. The value of y is 20% more than the value of x. (1)

Circle the ratio $x : y$.

$$5 : 6 \quad 6 : 5 \quad 4 : 5 \quad 5 : 4.$$

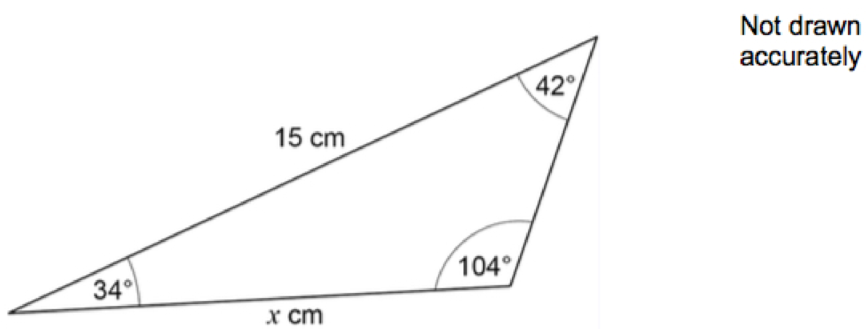
Solution

$$\begin{aligned} x : y &= x : 1.2x \\ &= 1 : 1.2 \\ &= 5 : 6 \end{aligned}$$

so

$$\underline{\underline{5 : 6}} \quad 6 : 5 \quad 4 : 5 \quad 5 : 4.$$

17. Here is a triangle. (1)



Circle the correct equation.

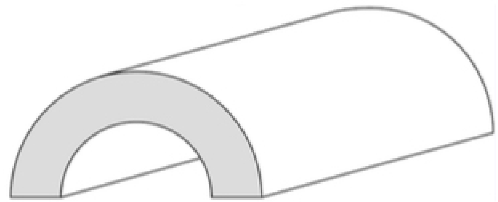
$$\frac{\sin x}{42} = \frac{\sin 15^\circ}{104} \quad \frac{x}{\sin 42^\circ} = \frac{15}{\sin 104^\circ} \quad \frac{\sin x}{34} = \frac{\sin 15^\circ}{104} \quad \frac{x}{\sin 42^\circ} = \frac{15}{\sin 34^\circ}$$

Solution

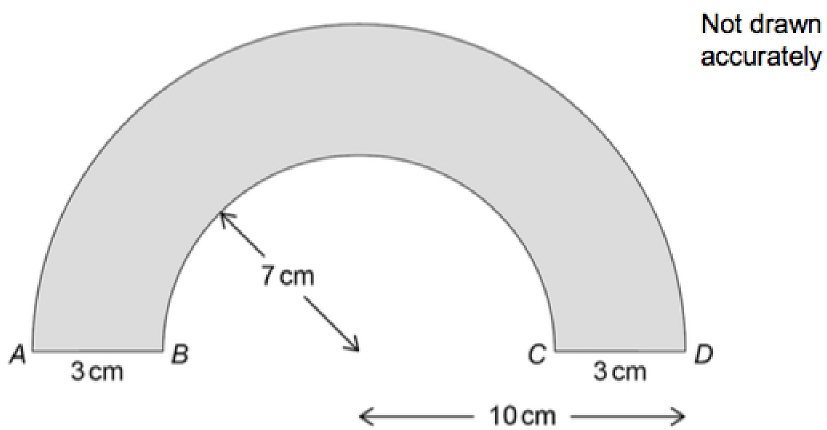
Sine rule:

$$\frac{\sin x}{42} = \frac{\sin 15^\circ}{104} \quad \underline{\underline{\frac{x}{\sin 42^\circ} = \frac{15}{\sin 104^\circ}}} \quad \frac{\sin x}{34} = \frac{\sin 15^\circ}{104} \quad \frac{x}{\sin 42^\circ} = \frac{15}{\sin 34^\circ}$$

18. Here is a tunnel for a toy train. (5)



The diagram below shows the cross section of the tunnel.



- AD is a semicircular arc of radius 10 cm.
- BC is a semicircular arc of radius 7 cm.
- The length of the tunnel is 30 cm.

Work out the total area of all **six** faces of the tunnel.
Give your answer in terms of π.

Solution

The six areas are

- a 'semi'-cylinder, radius 10 cm and length 30 cm:

$$\begin{aligned} \text{area}_1 &= \tfrac{1}{2} \times 2 \times \pi \times 10 \times 30 \\ &= 300\pi; \end{aligned}$$

- a 'semi'-cylinder, radius 7 cm and length 30 cm:

$$\begin{aligned} \text{area}_2 &= \tfrac{1}{2} \times 2 \times \pi \times 7 \times 30 \\ &= 210\pi; \end{aligned}$$

- two semi-circular area like in second picture:

$$\begin{aligned}\text{area}_3 &= \text{big circle} - \text{small circle}\\ &= (\tfrac{1}{2} \times \pi \times 10^2) - (\tfrac{1}{2} \times \pi \times 7^2)\\ &= 50\pi - \tfrac{49}{2}\pi\\ &= \tfrac{51}{2}\pi;\end{aligned}$$

and

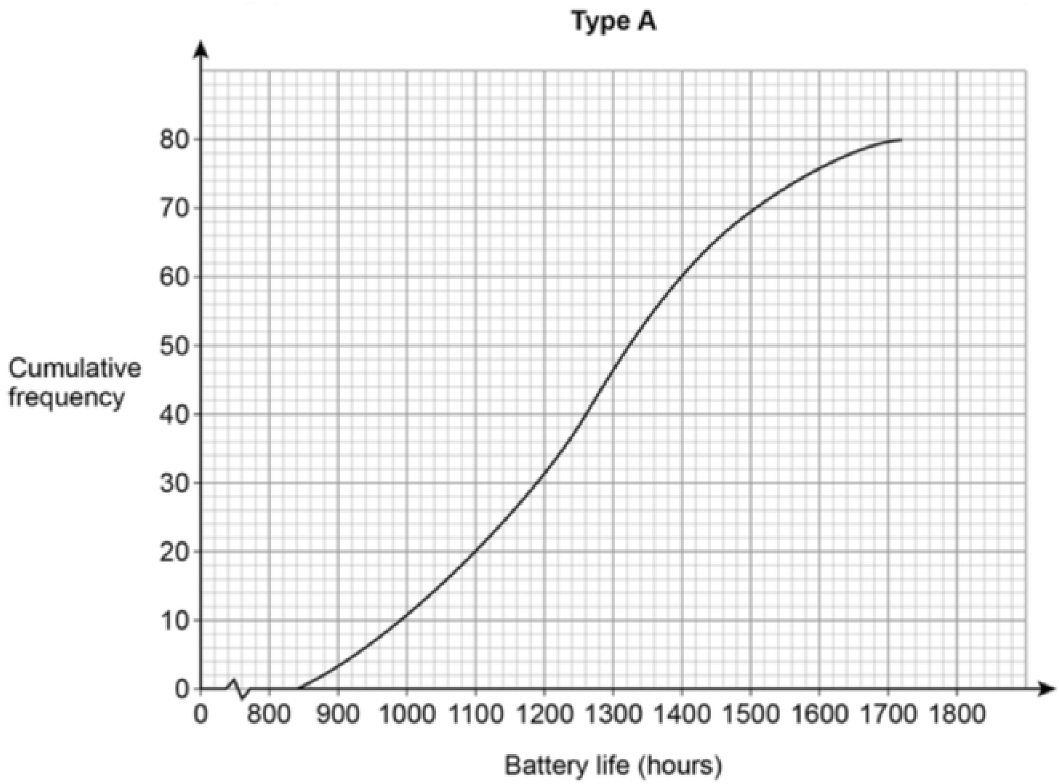
- two areas of 3 cm by 30 cm:

$$\begin{aligned}\text{area}_4 &= 3 \times 30\\ &= 90.\end{aligned}$$

Hence,

$$\begin{aligned}\text{total area} &= \text{area}_1 + \text{area}_2 + 2\,\text{area}_3 + 2\,\text{area}_4\\ &= 300\pi + 210\pi + 2(\tfrac{51}{2}\pi) + 2(90)\\ &= 510\pi + 51\pi + 180\\ &= \underline{\underline{(561\pi + 180)\ \text{cm}^2}}.\end{aligned}$$

19. Type A batteries and type B batteries were tested.
The cumulative frequency diagram shows information about the battery life of type A.



(a) Estimate the interquartile range for type A. (2)

Solution

$$\begin{aligned} \text{IQR} &= \text{UQ} - \text{LQ} \\ &= 1\,400 - 1\,100 \\ &= \underline{\underline{300 \text{ hours}}}. \end{aligned}$$

(b) Estimate the number of type A batteries that had a battery life of more than 1 600 hours. (1)

Solution

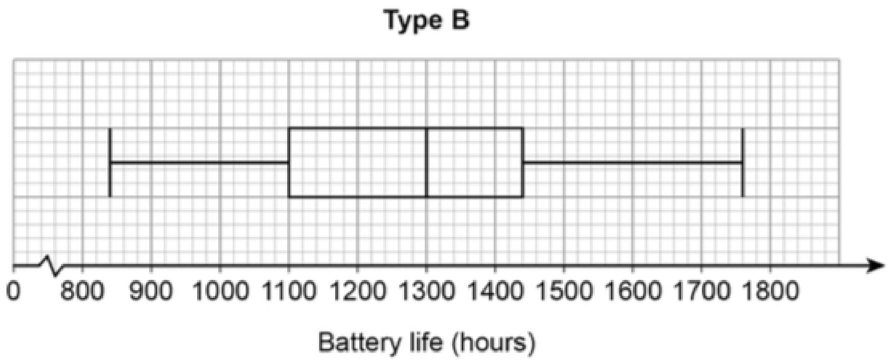
Well,

$$\text{battery life} = 1\,600 \Rightarrow \text{cumulative frequency} = 76$$

and so

$$80 - 76 = \underline{\underline{4 \text{ batteries}}}.$$

The box plot shows information about the battery life of type B.



(c) On average, which type had the greater battery life? (2)
Tick a box.

Using data from **both** diagrams, state how you chose your answer.

Solution
Type A batteries have a median of 1 260 and Type B batteries have a median of 1 300 so Type B batteries.

20. A linear sequence starts (4)

$$a + 2b \quad a + 6b \quad a + 10b \quad \ldots\ldots\ldots \quad \ldots\ldots\ldots$$

- The 2nd term has value 8.
- The 5th term has value 44.

Work out the values of a and b.

Solution
Well, the 5th is

$$a + 2b + 4(4b) = a + 18b$$

and

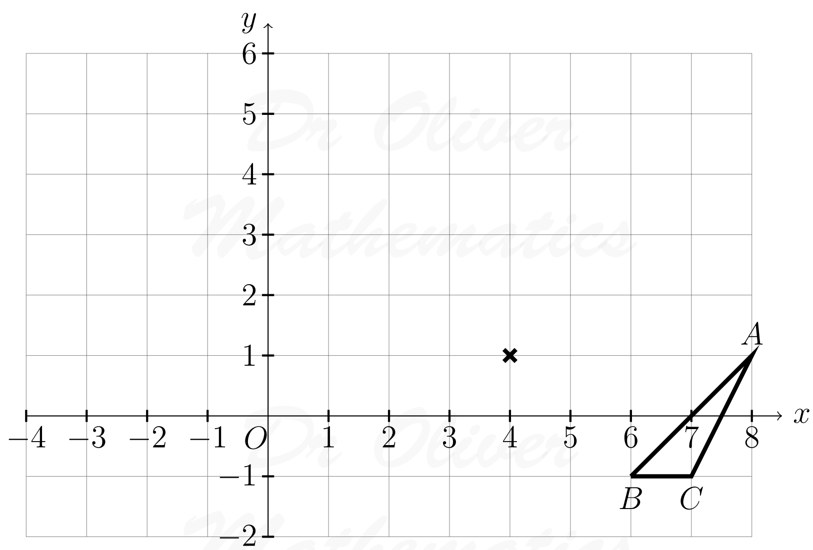
$$a + 6b = 8 \quad (1)$$
$$a + 18b = 44 \quad (2).$$

Do (2) − (1):

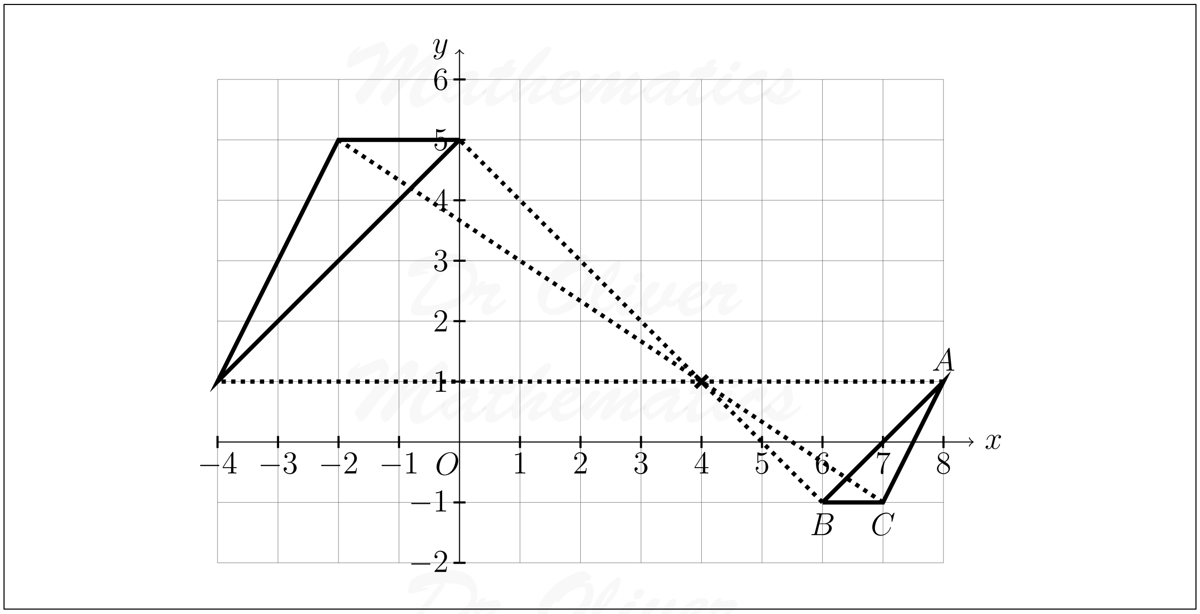
$$12b = 36 \Rightarrow \underline{\underline{b = 3}}$$

$$\Rightarrow a + 6(3) = 8$$
$$\Rightarrow a + 18 = 8$$
$$\Rightarrow \underline{\underline{a = -10}}.$$

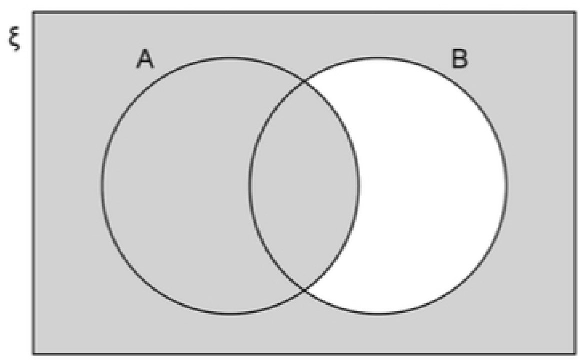
21. Enlarge triangle ABC by scale factor -2, centre $(4, 1)$. (2)



Solution



22. Which of these represents the shaded region? (1)



Circle your answer.

$A \cap B'$ B' $A \cup B'$ $A' \cup B'$

Solution

$A \cap B'$ B' $\underline{\underline{A \cup B'}}$ $A' \cup B'$

23. A shopkeeper compares the income from sales of a laptop in March and April. (3)

Price	$\frac{1}{5}$ more than March
Number sold	$\frac{1}{4}$ less than March

By what fraction does the income from these sales decrease in April?

Solution

Let x be the number of computers that the shopkeeper sold in March. Then

$$\begin{aligned}(1+\tfrac{1}{5})x \times (1-\tfrac{1}{4}) &= \tfrac{6}{5}x \times \tfrac{3}{4} \\ &= \tfrac{18}{20}x \\ &= \tfrac{9}{10}x;\end{aligned}$$

hence, the shopkeeper sales have fallen by one-tenth.

24. (a) Work out the value of (3)

$$2^{14} \div (2^9)^2.$$

Give your answer as a fraction in its simplest form.

Solution

$$\begin{aligned}2^{14} \div (2^9)^2 &= \frac{2^{14}}{(2^9)^2} \\ &= \frac{2^{14}}{2^{18}} \\ &= \frac{1}{2^4} \\ &= \tfrac{1}{16}.\end{aligned}$$

(b) Work out the value of (2)

$$25^{\frac{3}{2}}.$$

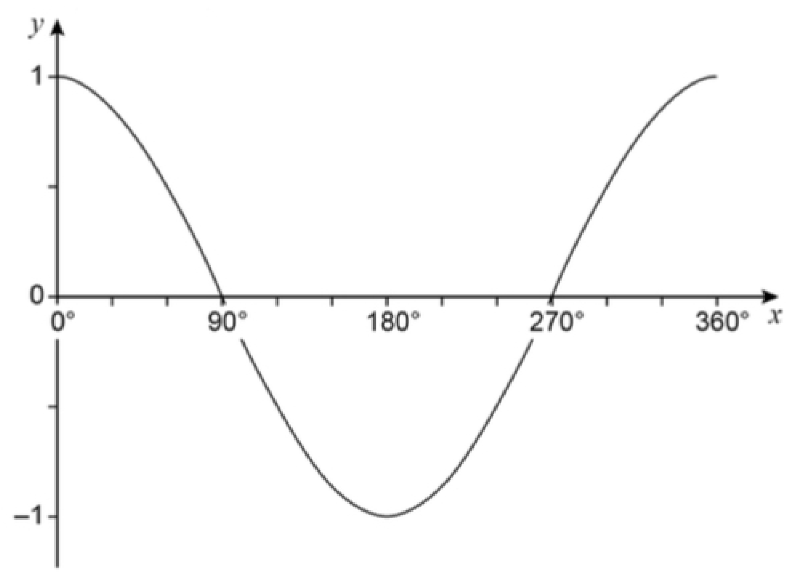
Solution

$$25^{\frac{3}{2}} = (25^{\frac{1}{2}})^3 = 5^3 = \underline{\underline{125}}.$$

25. Here is a sketch of the graph of

$$y = \cos x$$

for values of x from 0° to 360°.



(a) (1)

$$\cos x = \cos 60^\circ.$$

Work out the value of x when $90^\circ \leqslant x \leqslant 360^\circ$.

Solution

$\cos(360 - 60) = \underline{\underline{\cos 300^\circ}}$.

(b) (1)

$$\cos x = -\cos 60^\circ.$$

Work out the value of x when $180^\circ \leqslant x \leqslant 360^\circ$.

Solution

$\underline{\underline{\cos 240^\circ}}$.

26. • b is two-thirds of c. (3)

• $5a = 4c$.

Work out the ratio

$$a : b : c.$$

Give your answer in its simplest form where a, b, and c are integers.

Solution

Well,

$$b = \tfrac{2}{3}c$$

and

$$5a = 4c \Rightarrow a = \tfrac{4}{5}c.$$

Hence,

$$\begin{aligned} a : b : c &= \tfrac{4}{5}c : \tfrac{2}{3}c : c \\ &= \tfrac{4}{5} : \tfrac{2}{3} : 1 \\ &= \underline{\underline{12 : 10 : 15}} \end{aligned}$$

if we multiply each term by 15.

27. Jo wants to work out the solutions of

$$x^2 + 3x - 5 = 0.$$

She says, "The solutions **cannot** be worked out because $x^2 + 3x - 5$ does **not** factorise to $(x + a)(x + b)$ where a and b are integers."

Is Jo correct?
Tick a box.

☐ **Yes** ☐ **No**

(a) Give a reason for your answer. (1)

Solution

Well, $a = 1$, $b = 3$, and $c = -5$ and

$$\begin{aligned} \text{discriminant} &= 3^2 - 4 \times 1 \times (-5) \\ &= 9 + 20 \\ &= 29; \end{aligned}$$

hence, the answer is $\underline{\underline{\text{No}}}$ (real, distinct solutions).

(b) **Without** expanding any brackets, show how to work out the exact solutions of (3)

$$9(x+3)^2 = 4.$$

Give the solutions.

Solution

$$\begin{aligned} 9(x+3)^2 = 4 &\Rightarrow (x+3)^2 = \tfrac{4}{9} \\ &\Rightarrow x + 3 = \pm\tfrac{2}{3} \\ &\Rightarrow x = -3 \pm \tfrac{2}{3} \\ &\Rightarrow \underline{\underline{x = -3\tfrac{2}{3} \text{ or } x = -2\tfrac{1}{3}}}. \end{aligned}$$

28. Simplify (3)

$$\sqrt{80} + \sqrt{2\tfrac{2}{9}}$$

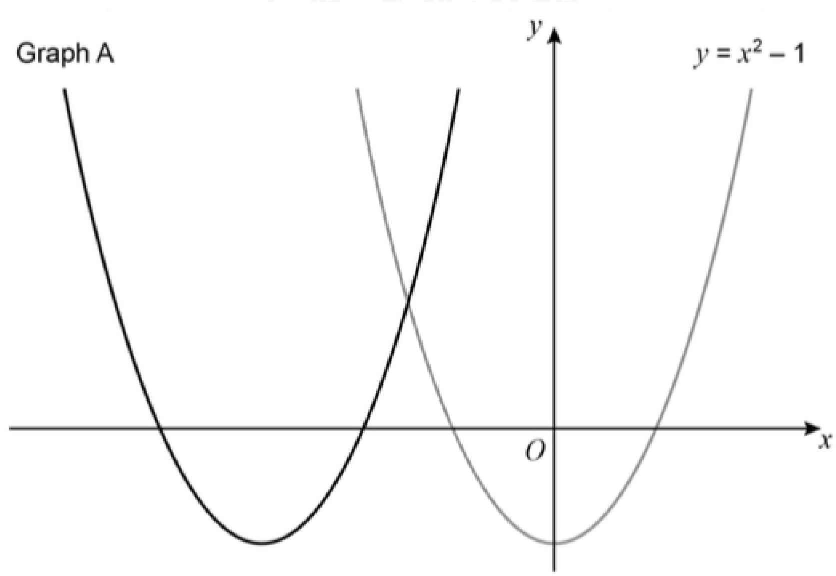
Give your answer in the form $\dfrac{a\sqrt{5}}{b}$ where a and b are integers.

Solution

$$\begin{aligned}
\sqrt{80} + \sqrt{2\tfrac{2}{9}} &= \sqrt{16 \times 5} + \sqrt{\tfrac{20}{9}} \\
&= \sqrt{16 \times 5} + \frac{\sqrt{20}}{\sqrt{9}} \\
&= \sqrt{16} \times \sqrt{5} + \frac{\sqrt{4 \times 5}}{3} \\
&= 4\sqrt{5} + \frac{\sqrt{4} \times \sqrt{5}}{3} \\
&= 4\sqrt{5} + \frac{2\sqrt{5}}{3} \\
&= \frac{12\sqrt{5}}{3} + \frac{2\sqrt{5}}{3} \\
&= \frac{12\sqrt{5} + 2\sqrt{5}}{3} \\
&= \underline{\underline{\frac{14\sqrt{5}}{3}}};
\end{aligned}$$

so, $\underline{\underline{a = 14}}$ and $\underline{\underline{b = 3}}$.

29. Here are sketches of two graphs.



The graph of

$$y = x^2 - 1$$

is translated 3 units to the left to give graph A.

(a) The equation of graph A can be written in the form (3)

$$y = x^2 + bx + c.$$

Work out the values of b and c.

Solution

Well, graph A is

$$y = (x + 3)^2 - 1$$

$\times$	x	$+3$
x	x^2	$+3x$
$+3$	$+3x$	$+9$

$$= (x^2 + 6x + 9) - 1$$
$$= \underline{\underline{x^2 + 6x + 8}};$$

hence, $\underline{\underline{b = 6}}$ and $\underline{\underline{c = 8}}$.

The graph of

$$y = x^2 - 1$$

is reflected in the x-axis to give graph B.

(b) Work out the equation of graph B. (1)

Solution

$\underline{\underline{y = 1 - x^2}}$.

30. Show that the value of (3)

$$\cos 30^\circ \times \tan 60^\circ + \sin 30^\circ$$

is an integer.

Solution

$$
\begin{aligned}
(\cos 30^\circ \times \tan 60^\circ) + \sin 30^\circ &= \left(\frac{\sqrt{3}}{2} \times \sqrt{3}\right) + \tfrac{1}{2} \\
&= \tfrac{3}{2} + \tfrac{1}{2} \\
&= \underline{\underline{2}}.
\end{aligned}
$$