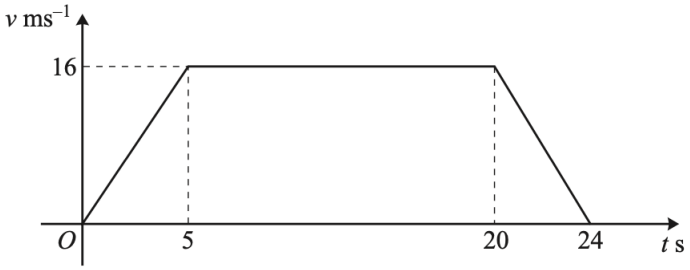
Dr Oliver Mathematics
Cambridge O Level Additional Mathematics
2013 June Paper 2 Variant 1: Calculator
2 hours

The total number of marks available is 80.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You must write down all the stages in your working.

1. Prove that (4)
$$\left(\frac{1+\sin\theta}{\cos\theta}\right)^2+\left(\frac{1-\sin\theta}{\cos\theta}\right)^2 \equiv 2+4\tan^2\theta.$$

2. The velocity-time graph represents the motion of a particle moving in a straight line.



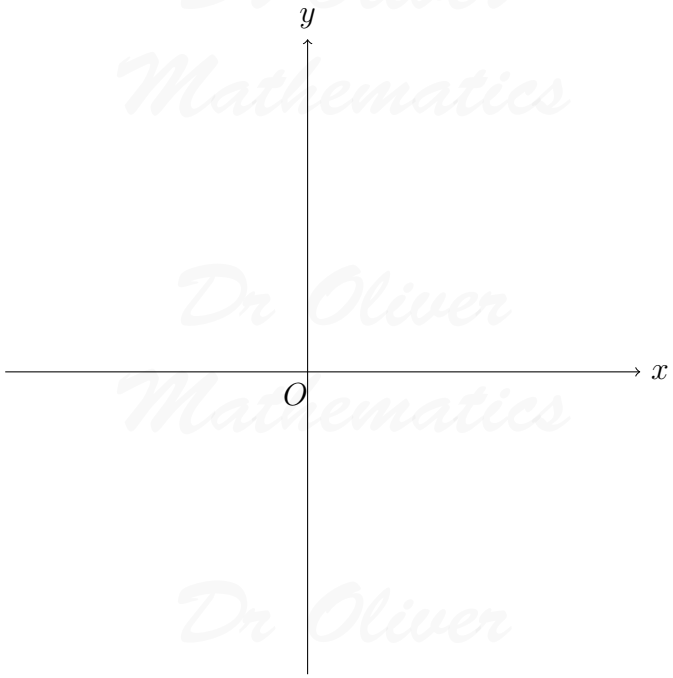
   (a) Find the acceleration during the first 5 seconds. (1)

   (b) Find the length of time for which the particle is travelling with constant velocity. (1)

   (c) Find the total distance travelled by the particle. (3)

3. Variables x and y are related by the equation (6)
$$y = 10 - 4\sin^2 x, \text{ where } 0 \leqslant x \leqslant \tfrac{1}{2}\pi.$$
   Given that x is increasing at a rate of 0.2 radians per second, find the corresponding rate of change of y when $y = 8$.

4. (a) Sketch the graph of (3)
$$y = |4x - 2|$$
   on the axes below, showing the coordinates of the points where the graph meets the axes.



(b) Solve the equation (3)

$$|4x - 2| = x.$$

5. A piece of wire of length 96 cm is formed into the rectangular shape $PQRSTU$ shown in the diagram.



It is given that $PQ = TU = SR = x$ cm.
It may be assumed that PQ and TU coincide and that TS and QR have the same length.

(a) Show that the area, A cm^2, enclosed by the wire is given by (2)

$$A = \frac{96 - 3x^2}{2}.$$

(b) Given that x can vary, find the stationary value of A and determine the nature of this stationary value. (4)

6. Find the equation of the normal to the curve (6)

$$y = \frac{x^2 + 8}{x - 2}$$

at the point on the curve where $x = 4$.

7. (a) Find the first four terms in the expansion of (3)

$$(2 + x)^6$$

in ascending powers of x.

(b) Hence find the coefficient of x^3 in the expansion of (4)

$$(1 + 3x)(1 - x)(2 + x)^6.$$

8. The line (7)

$$y = 2x - 8$$

cuts the curve

$$2x^2 + y^2 - 5xy + 32 = 0$$

at the points A and B.

Find the length of the line AB.

9. It is given that $x \in \mathbb{R}$ and that

- $\mathscr{E} = \{x : -5 < x < 12\}$,
- $S = \{x : 5x + 24 > x^2\}$, and
- $T = \{x : 2x + 7 > 15\}$.

Find the values of x such that

(a) $x \in S$, (3)

(b) $x \in S \cup T$, (2)

(c) $x \in (S \cap T)'$. (3)

10. A plane, whose speed in still air is 240 kmh^{-1}, flies directly from A to B, where B is 500 km from A on a bearing of 032°.

There is a constant wind of 50 kmh^{-1} blowing from the west.

(a) Find the bearing on which the plane is steered. (4)

(b) Find, to the nearest minute, the time taken for the flight. (4)

11. A one-one function f is defined by

$$\mathrm{f}(x) = (x-1)^2 - 5 \text{ for } x \geqslant k.$$

(a) State the least value that k can take. (1)

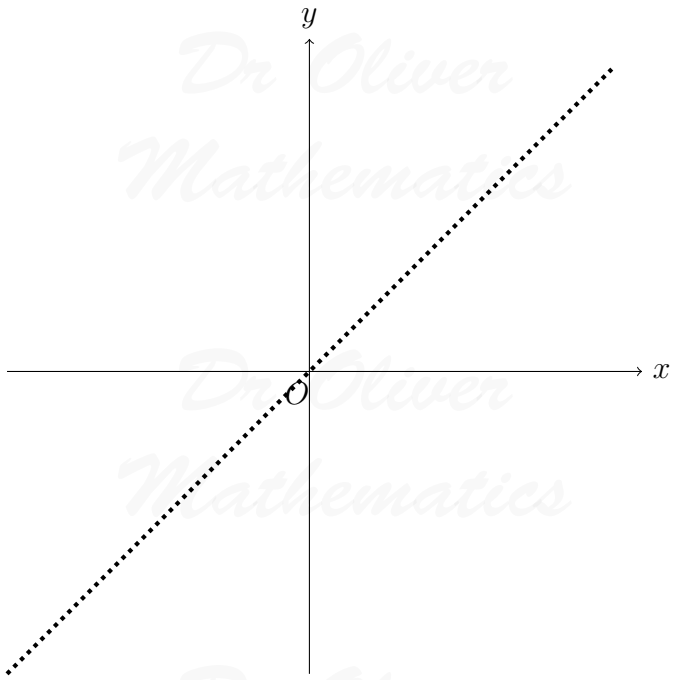
For this least value of k,

(b) write down the range of f, (1)

(c) find $\mathrm{f}^{-1}(x)$, (2)

(d) sketch and label, on the axes below, the graph of (2)

$$y = \mathrm{f}(x) \text{ and of } y = \mathrm{f}^{-1}(x).$$



(e) find the value of x for which (2)

$$\mathrm{f}(x) = \mathrm{f}^{-1}(x).$$

12. The function

$$\mathrm{f}(x) = x^3 + x^2 + ax + b$$

is divisible by $(x-3)$ and leaves a remainder of 20 when divided by $(x+1)$.

(a) Show that $b = 6$ and find the value of a. (4)

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(b) Using your value of a and taking b as 6, find the non-integer roots of the equation (5)

$$\mathrm{f}(x) = 0$$

in the form

$$p \pm \sqrt{q},$$

where p and q are integers.

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