

Dr Oliver Mathematics
Advance Level Mathematics
Core Mathematics 2: Calculator
1 hour 30 minutes

The total number of marks available is 75.

You must write down all the stages in your working.

1. Figure 1 shows a sketch of part of the curve with equation

$$y = \frac{(x + 2)^{\frac{3}{2}}}{4}, \quad x \geq -2.$$

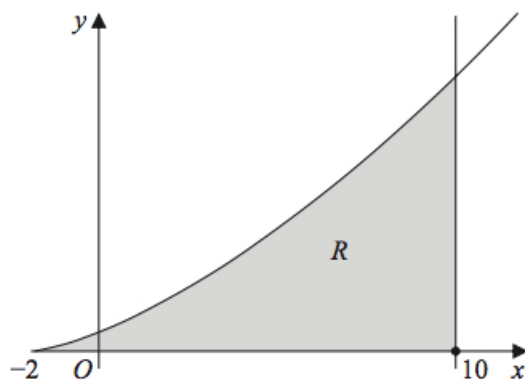


Figure 1: $y = \frac{(x + 2)^{\frac{3}{2}}}{4}$

The finite region R , shown shaded in Figure 1, is bounded by the curve, the x -axis and the line with equation $x = 10$.

The table below shows corresponding values of x and y for $y = \frac{(x + 2)^{\frac{3}{2}}}{4}$.

- (a) Complete the table, giving values of y corresponding to $x = 2$ and $x = 6$. (1)

x		-2	2	6	10
y		0			$6\sqrt{3}$

- (b) Use the trapezium rule, with all the values of y from the completed table, to find an approximate value for the area of R , giving your answer to 3 decimal places. (4)

2. (a) Find the first 4 terms, in ascending powers of x , of the binomial expansion of (4)

$$(2 + kx)^7,$$

where k is a non-zero constant. Give each term in its simplest form.

Given that the coefficient of x^3 in this expansion is 1 890,

- (b) find the value of k . (3)

3.

$$f(x) = 24x^3 + Ax^2 - 3x + B,$$

where A and B are constants.

When $f(x)$ is divided by $(2x - 1)$ the remainder is 30.

- (a) Show that $A + 4B = 114$. (2)

Given also that $(x + 1)$ is a factor of $f(x)$,

- (b) find another equation in A and B . (2)

- (c) Find the value of A and the value of B . (2)

- (d) Hence find a quadratic factor of $f(x)$. (2)

4. Figure 2 shows a flag $XYWZX$.

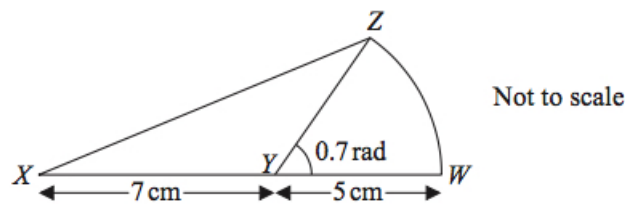


Figure 2: a flag

The flag consists of a triangle XYZ joined to a sector ZYW of a circle with radius 5 cm and centre Y .

The angle of the sector, angle ZYW , is 0.7 radians.

The points X , Y , and W lie on a straight line with $XY = 7$ cm and $YW = 5$ cm.

Find

- (a) the area of the sector ZYW in cm^2 , (2)

(b) the area of the flag, in cm^2 , to 2 decimal places, (3)

(c) the length of the perimeter, $XYWZX$, of the flag, in cm to 2 decimal places. (4)

5. The circle C has equation

$$x^2 + y^2 - 2x + 14y = 0.$$

Find

(a) the coordinates of the centre of C , (2)

(b) the exact value of the radius of C , (2)

(c) the y -coordinates of the points where the circle C crosses the y -axis. (2)

(d) Find an equation of the tangent to C at the point $(2, 0)$, giving your answer in the form $ax + by + c = 0$, where a , b , and c are integers. (4)

6. A geometric series with common ratio $r = -0.9$ has sum to infinity 10 000.

For this series,

(a) find the first term, (2)

(b) find the fifth term, (2)

(c) find the sum of the first twelve terms, giving this answer to the nearest integer. (3)

7. (a) Find the value of y for which (2)

$$1.01^{y-1} = 500.$$

Give your answer to 2 decimal places.

(b) Given that

$$2 \log_4(3x + 5) = \log_4(3x + 8) + 1, \quad x > -\frac{5}{3},$$

(i) show that (4)

$$9x^2 + 18x - 7 = 0.$$

(ii) Hence solve the equation (2)

$$2 \log_4(3x + 5) = \log_4(3x + 8) + 1, \quad x > -\frac{5}{3}.$$

8. *In this question solutions based entirely on graphical or numerical methods are not acceptable.*

(a) Solve for $0^\circ \leq x < 360^\circ$, (4)

$$4 \cos(x + 70)^\circ = 3,$$

giving your answers in degrees to one decimal place.

- (b) Find, for $0 \leq \theta < 2\pi$, all the solutions of (5)

$$6 \cos^2 \theta - 5 = 6 \sin^2 \theta + \sin \theta,$$

giving your answers in radians to 3 significant figures.

9. Figure 3 shows a sketch of part of the curve with equation

$$y = 7x^2(5 - 2\sqrt{x}), \quad x \geq 0.$$

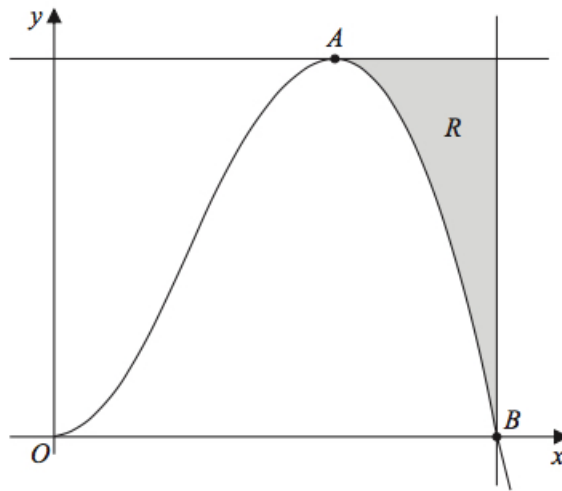


Figure 3: $y = 7x^2(5 - 2\sqrt{x})$

The curve has a turning point at the point A , where $x > 0$, as shown in Figure 3.

- (a) Using calculus, find the coordinates of the point A . (5)

The curve crosses the x -axis at the point B , as shown in Figure 3.

- (b) Use algebra to find the x -coordinate of the point B . (2)

The finite region R , shown shaded in Figure 3, is bounded by the curve, the line through A parallel to the x -axis and the line through B parallel to the y -axis.

- (c) Use integration to find the area of the region R , giving your answer to 2 decimal places. (5)