Dr Oliver Mathematics
Cambridge O Level Additional Mathematics
2008 June Paper 2: Calculator
2 hours

The total number of marks available is 80.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You must write down all the stages in your working.

1. The equation of a curve is given by

$$y = x^2 + ax + 3,$$

where a is a constant.

Given that this equation can also be written as

$$y = (x + 4)^2 + b,$$

find

(a) the value of a and of b, (2)

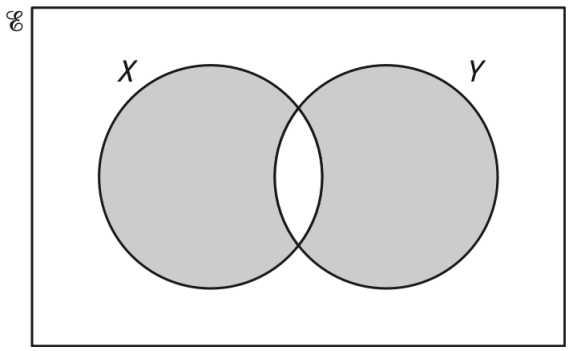
(b) the coordinates of the turning point of the curve. (1)

2. (a) Illustrate the following statements using a separate Venn diagram for each.

(i) $A \cap B = \varnothing$, (1)

(ii) $(C \cup D) \subset E$. (1)

(b) Express, in set notation, the set represented by the shaded region. (2)



3. Find the coordinates of the points where the straight line (5)

$$y = 2x - 3$$

intersects the curve

$$x^2 + y^2 + xy + x = 30.$$

4. (a) Sketch, on the same diagram, the graphs of (3)

$$y = x - 3 \text{ and } y = |2x - 9|.$$

(b) Solve the equation (2)

$$|2x - 9| = x - 3.$$

5. Find the coefficient of x^3 in the expansion of

(a) $(1 + 3x)^8$, (2)

(b) $(1 - 4x)(1 + 3x)^8$. (3)

6. (a) Given that (2)

$$\sin x = p,$$

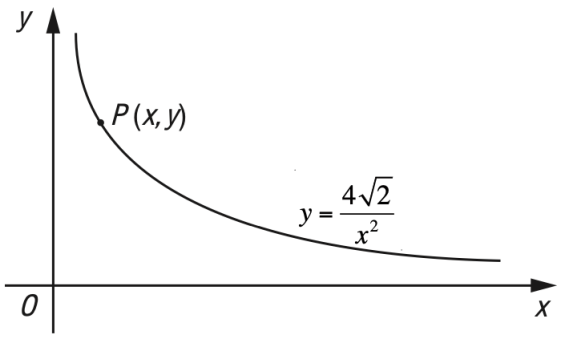
find an expression, in terms of p, for $\sec^2 x$.

(b) Prove that (4)

$$\sec A \operatorname{cosec} A - \cot A \equiv \tan A.$$

7. The diagram shows part of the curve

$$y = \frac{4\sqrt{2}}{x^2}.$$



The point $P(x, y)$ lies on this curve.

(a) Write down an expression, in terms of x, for $(OP)^2$. (1)

(b) Denoting $(OP)^2$ by S, find an expression for $\dfrac{\mathrm{d}S}{\mathrm{d}x}$. (2)

(c) Find the value of x for which S has a stationary value and the corresponding value of OP. (3)

8. Solve the equation

(a) $2^{2x+1} = 20$, (3)

(b) (4)

$$\frac{5^{4y-1}}{25^{y}} = \frac{125^{y+3}}{25^{2-y}}.$$

9. Given that

$$\mathbf{A} = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 & -5 \\ 0 & 2 \end{pmatrix}, \text{ and } \mathbf{C} = \begin{pmatrix} 4 \\ 1 \end{pmatrix},$$

calculate

(a) $\mathbf{AB}$, (2)

(b) $\mathbf{BC}$, (2)

(c) the matrix $\mathbf{X}$ such that $\mathbf{AX} = \mathbf{B}$. (4)

10. Find

(a) (i) (2)

$$\int \frac{12}{(2x-1)^4}\,\mathrm{d}x,$$

(ii) (3)

$$\int x(x-1)^2\,\mathrm{d}x.$$

(b) (i) Given that (3)

$$y = 2(x-5)\sqrt{x+4},$$

show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3(x+1)}{\sqrt{x+4}}.$$

(ii) Hence find (2)

$$\int \frac{x+1}{\sqrt{x+4}}\,\mathrm{d}x.$$

11. The function f is defined by

$$\mathrm{f}(x) = (x+1)^2 + 2, \text{ for } x \geqslant -1.$$

Find

(a) the range of f, (1)

(b) $f^2(1)$, (1)

(c) an expression for $f^{-1}(x)$. (3)

The function g is defined by

$$g(x) = \frac{20}{x+1}, \text{ for } x \geqslant 0.$$

Find

(d) $g^{-1}(2)$, (2)

(e) the value of x for which (4)

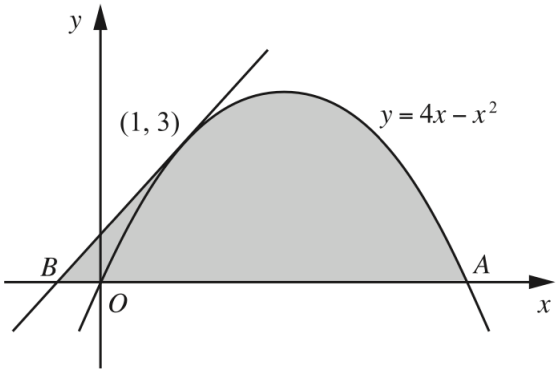
$$f\,g(x) = 38.$$

EITHER

12. The diagram shows the curve

$$y = 4x - x^2,$$

which crosses the x-axis at the origin O and the point A.



The tangent to the curve at the point $(1, 3)$ crosses the x-axis at the point B.

(a) Find the coordinates of A and of B. (5)

(b) Find the area of the shaded region. (5)

OR

13. The points $A(-2, 2)$, $B(4, 4)$, and $C(5, 2)$ are the vertices of a triangle. (10)

The perpendicular bisector of AB and the line through A parallel to BC intersect at the point D.

Find the area of the quadrilateral $ABCD$.