

**Dr Oliver Mathematics**  
**Mathematics: Advanced Higher**  
**2022 Paper 1: Non-Calculator**  
**1 hour**

The total number of marks available is 36.

You must write down all the stages in your working.

1. (a) Given (3)

$$y = \frac{1 - 3x}{x^2 + 4},$$

find  $\frac{dy}{dx}$ .

Simplify your answer.

- (b) Given (2)

$$f(x) = \operatorname{cosec} 5x,$$

find  $f'(x)$ .

2. Use Gaussian elimination to solve the following system of equations: (4)

$$x - 2y + z = 4$$

$$2x + y - 3z = 3$$

$$x - 7y - 4z = 9.$$

3. Given that (3)

$$z_1 = 5 + 3i \text{ and } z_2 = 6 + 2i,$$

express  $z_1 \bar{z}_2$  in the form  $a + ib$ , where  $a$  and  $b$  are real numbers.

4. A curve is defined by the equation

$$y^3 + 4y = 2xy + 1.$$

- (a) Use implicit differentiation to find an expression for  $\frac{dy}{dx}$ . (3)

- (b) Find the gradient of the tangent to the curve when  $y = -1$ . (1)

- (c) Show that the curve has no stationary point. (2)

5. (a) Find, and simplify, the Maclaurin expansion for  $e^{-4x}$ , up to and including the term in  $x^3$ . (2)

- (b) Hence find the first four terms of the Maclaurin expansion of (2)

$$\frac{3 + 2x}{e^{4x}}.$$

6. (a) Consider the statement: (1)

For all odd numbers  $n$ ,  $n^2 + 4$  is prime.

Find a counterexample to show that the statement is false.

- (b) Prove directly that the difference between the cubes of any two consecutive integers is not divisible by 3. (3)

7. (a) Use the substitution  $u = y^2 + 1$ , or otherwise, to find the exact value of (4)

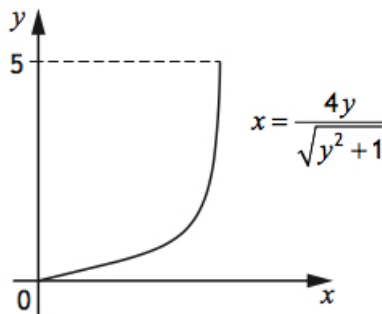
$$\int_0^5 \frac{4y}{\sqrt{y^2 + 1}} dy.$$

Student engineers are using a 3D printer to make a model.

Relative to a suitable set of axes, the cross-section of the model is **symmetrical about the  $y$ -axis** and is represented **in the first quadrant** by the curve with equation

$$x = \frac{4y}{\sqrt{y^2 + 1}}, \quad 0 \leq y \leq 5,$$

as shown in the diagram.



- (b) State the area of the cross-section. (1)

- (c) Express (1)

$$\frac{y^2}{y^2 + 1}$$

in the form

$$a + \frac{b}{y^2 + 1},$$

where  $a$  and  $b$  are real numbers.

The curve

$$x = \frac{4y}{\sqrt{y^2 + 1}}, \quad 0 \leq y \leq 5,$$

will be rotated through  $2\pi$  radians about the  $y$ -axis to make the model.

(d) Find the volume of the model.

(4)

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