Dr Oliver Mathematics
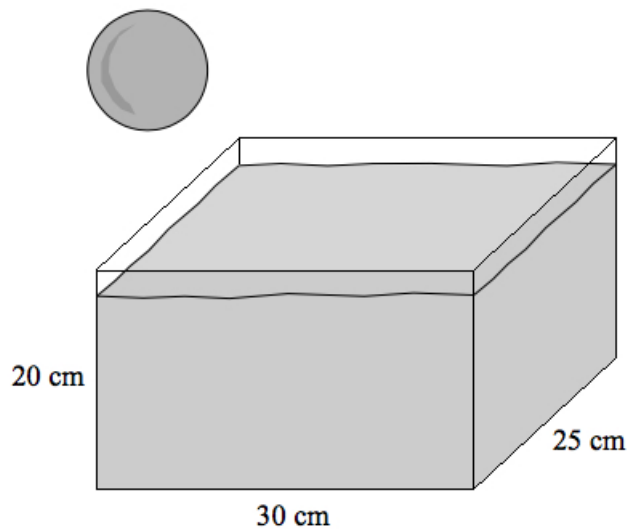# Lower and Upper Bounds: Part 1

1. Karl is setting up a fish tank.   (6)
   The tank is in the shape of a cuboid measuring 30 cm by 25 cm by 20 cm.
   These measurements are accurate.

Karl has filled the tank with water to a depth of 18 cm, correct to the nearest centimetre.

Karl has an ornament in the shape of a sphere.
The radius of the sphere is 6 cm, correct to the nearest centimetre.

Karl puts the ornament into the tank.

Find out if there is any risk of the water overflowing.

Solution
$$17.5 \text{ cm} \leqslant \text{depth} < 18.5 \text{ cm}$$
and
$$5.5 \text{ cm} \leqslant \text{radius} < 6.5 \text{ cm}.$$
Now, the volume of the tank when it is full:
$$\begin{aligned} \text{volume} &= 30 \times 25 \times 20 \\ &= 15\,000 \text{ cm}^3. \end{aligned}$$

Next,

$$\text{upper bound} = (30 \times 25 \times 18.5) + (\tfrac{4}{3} \times \pi \times 6.5^3)$$
$$= 13\,875 + 1\,150.346\,51 \text{ (FCD)}$$
$$= 15\,025.346\,51 \text{ (FCD)}$$
$$> 15\,000;$$

hence, there is a risk of the water <u>overflowing</u>.