

Dr Oliver Mathematics

Four Ratios on a Straight Line

In this note, we will explore four ratios that lie in order on a straight line.

1. The points A , B , C , D , and E lie in order on a straight line.

$$AB : BE = 1 : 4.$$

$$AC : CE = 1 : 1.$$

$$AD : CD = 3 : 2.$$

Work out $AB : BC : CD : DE$.

Solution

$$\begin{aligned} AB : BE = 1 : 4 &\Rightarrow 4AB = BE \\ &\Rightarrow 4AB = BC + CD + DE \quad (1), \end{aligned}$$

$$\begin{aligned} AC : CE = 1 : 1 &\Rightarrow AC = CE \\ &\Rightarrow AB + BC = CD + DE \\ &\Rightarrow AB = CD + DE - BC \\ &\Rightarrow 4AB = 4CD + 4DE - 4BC \quad (2), \end{aligned}$$

and

$$\begin{aligned} AD : DE = 3 : 2 &\Rightarrow 2AD = 3DE \\ &\Rightarrow 2AB + 2BC + 2CD = 3DE \\ &\Rightarrow 2AB = 3DE - 2BC - 2CD \\ &\Rightarrow 4AB = 6DE - 4BC - 4CD \quad (3). \end{aligned}$$

Subtract (1) – (2):

$$\begin{aligned} 0 &= (BC + CD + DE) - (4CD + 4DE - 4BC) \\ &\Rightarrow 5BC = 3CD + 3DE \quad (4). \end{aligned}$$

Subtract (1) – (3):

$$\begin{aligned} 0 &= (BC + CD + DE) - (6DE - 4BC - 4CD) \\ &\Rightarrow 5BC = -5CD + 5DE \quad (5). \end{aligned}$$

Subtract (4) – (5):

$$\begin{aligned} 0 &= (3CD + 3DE) - (-5CD + 5DE) \\ &\Rightarrow 8CD = 2DE \\ &\Rightarrow 4CD = DE \\ &\Rightarrow \boxed{CD : DE = 1 : 4.} \end{aligned}$$

Next,

$$\begin{aligned}5BC &= 3CD + 3DE \\ \Rightarrow 5BC &= 3CD + 12CD \\ \Rightarrow BC &= 3CD \\ \Rightarrow \boxed{BC : CD = 3 : 1.}\end{aligned}$$

Next,

$$\begin{aligned}4AB &= BC + CD + DE \\ \Rightarrow 4AB &= BC + 5CD \\ \Rightarrow 4AB &= BC + \frac{5}{3}BC \\ \Rightarrow 4AB &= \frac{8}{3}BC \\ \Rightarrow 12AB &= 8BC \\ \Rightarrow 3AB &= 2BC \\ \Rightarrow \boxed{AB : BC = 2 : 3.}\end{aligned}$$

Finally,

$$AB : BC : CD : DE = \underline{\underline{2 : 3 : 1 : 4.}}$$