Dr Oliver Mathematics
Mathematics
Indices
Past Examination Questions

This booklet consists of 35 questions across a variety of examination topics.
The total number of marks available is 130.

1. Simplify $(3+\sqrt{5})(3-\sqrt{5})$. (2)

Solution

$$9 - 3\sqrt{5} + 3\sqrt{5} - 5 = \underline{\underline{4}}.$$

2. Simplify $(\sqrt{7}+2)(\sqrt{7}-2)$. (2)

Solution

$$7 - 2\sqrt{7} + 2\sqrt{7} - 4 = \underline{\underline{3}}.$$

3. Write (2)

$$\sqrt{75} - \sqrt{27}$$

in the form $k\sqrt{x}$, where k and x are integers.

Solution

$$\begin{aligned}\sqrt{75} - \sqrt{27} &= \sqrt{25 \times 3} - \sqrt{9 \times 3} \\ &= \sqrt{25} \times \sqrt{3} - \sqrt{9} \times \sqrt{3} \\ &= 5\sqrt{3} - 3\sqrt{3} \\ &= \underline{\underline{2\sqrt{3}}}.\end{aligned}$$

4. Express 9^{3x+1} in the form 3^y, giving y in the form $ax+b$, where a and b are constant.

Solution

$$9^{3x+1} = (3^2)^{3x+1} = \underline{\underline{3^{6x+2}}};$$

hence, $\underline{\underline{y = 6x + 2}}$.

5. Express 8^{2x+3} in the form 2^y, stating y in terms of x.

Solution

$$8^{2x+3} = (2^3)^{2x+3} = \underline{\underline{2^{6x+9}}};$$

hence, $\underline{\underline{y = 6x + 9}}$.

6. Find the value of

   (a) $25^{\frac{1}{2}}$, (1)

   Solution

   $$25^{\frac{1}{2}} = \underline{\underline{5}}.$$

   (b) $25^{-\frac{3}{2}}$. (2)

   Solution

   $$25^{-\frac{3}{2}} = \frac{1}{25^{\frac{3}{2}}} = \frac{1}{(25^{\frac{1}{2}})^3} = \frac{1}{5^3} = \underline{\underline{\frac{1}{125}}}.$$

7. (a) Find the value of $8^{\frac{4}{3}}$. (2)

   Solution

   $$8^{\frac{4}{3}} = \left[8^{\frac{1}{3}}\right]^4 = 2^4 = \underline{\underline{16}}.$$

   (b) Simplify $\dfrac{15x^{\frac{4}{3}}}{3x}$. (2)

Solution

$$\frac{15x^{\frac{4}{3}}}{3x} = \underline{\underline{5x^{\frac{1}{3}}}}.$$

8. (a) Write down the value of $16^{\frac{1}{4}}$. (1)

Solution

$$16^{\frac{1}{4}} = \underline{\underline{2}}.$$

(b) Simplify $(16x^{12})^{\frac{3}{4}}$. (2)

Solution

$$\left(16x^{12}\right)^{\frac{3}{4}} = (16)^{\frac{3}{4}} \times \left(x^{12}\right)^{\frac{3}{4}} = \left(2^3\right) \times (x)^{12\times\frac{3}{4}} = \underline{\underline{8x^9}}.$$

9. (a) Write down the value of $16^{\frac{1}{2}}$. (1)

Solution

$$16^{\frac{1}{2}} = \underline{\underline{4}}.$$

(b) Find the value of $16^{-\frac{3}{2}}$. (2)

Solution

$$16^{-\frac{3}{2}} = \frac{1}{16^{\frac{3}{2}}} = \frac{1}{\left[16^{\frac{1}{2}}\right]^3} = \frac{1}{4^3} = \frac{1}{\underline{\underline{64}}}.$$

10. (a) Write down the value of $8^{\frac{1}{3}}$. (1)

Solution

$$8^{\frac{1}{3}} = \underline{\underline{2}}.$$

(b) Find the value of $8^{-\frac{2}{3}}$. (2)

Solution

$$8^{-\frac{2}{3}} = \frac{1}{8^{\frac{2}{3}}} = \frac{1}{\left(8^{\frac{1}{3}}\right)^2} = \frac{1}{2^2} = \underline{\underline{\frac{1}{4}}}.$$

11. (a) Write down the value of $125^{\frac{1}{3}}$. (1)

Solution

$$125^{\frac{1}{3}} = \underline{\underline{5}}.$$

(b) Find the value of $125^{-\frac{2}{3}}$. (2)

Solution

$$125^{-\frac{2}{3}} = \frac{1}{125^{\frac{2}{3}}} = \frac{1}{\left(125^{\frac{1}{3}}\right)^2} = \frac{1}{5^2} = \underline{\underline{\frac{1}{25}}}.$$

12. Given that $32\sqrt{2} = 2^a$, find the value of a. (3)

Solution

$$32\sqrt{2} = 2^5 \times 2^{\frac{1}{2}} = 2^{\frac{11}{2}},$$

hence, $\underline{\underline{a = 5\tfrac{1}{2}}}$.

13. Simplify (4)

$$\frac{5 - \sqrt{3}}{2 + \sqrt{3}},$$

giving your answer in the form $a + b\sqrt{3}$, where a and b are integers.

Solution

$$\begin{aligned}\frac{5-\sqrt{3}}{2+\sqrt{3}} &= \frac{5-\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} \\ &= \frac{10-5\sqrt{3}-2\sqrt{3}+3}{4+2\sqrt{3}-2\sqrt{3}-3} \\ &= \underline{\underline{13-7\sqrt{3}}};\end{aligned}$$

hence, $\underline{\underline{a=13}}$ and $\underline{\underline{b=-7}}$.

14. (a) Expand and simplify $(4+\sqrt{3})(4-\sqrt{3})$. (2)

Solution

$$(4+\sqrt{3})(4-\sqrt{3}) = 16 - 4\sqrt{3} + 4\sqrt{3} - 3 = \underline{\underline{13}}.$$

(b) Express $\dfrac{26}{4+\sqrt{3}}$ in the form $a + b\sqrt{5}$, where a and b are integers. (2)

Solution

$$\begin{aligned}\frac{26}{4+\sqrt{3}} &= \frac{26}{4+\sqrt{3}} \times \frac{4-\sqrt{3}}{4-\sqrt{3}} \\ &= \frac{26(4-\sqrt{3})}{13} \\ &= \underline{\underline{8-2\sqrt{3}}}.\end{aligned}$$

15. (a) Express $\sqrt{108}$ in the form $a\sqrt{3}$, where a is an integer. (2)

Solution

$$\sqrt{108} = \sqrt{36 \times 3} = \sqrt{36} \times \sqrt{3} = \underline{\underline{6\sqrt{3}}};$$

hence, $\underline{\underline{a=6}}$.

(b) Express $(2-\sqrt{3})^2$ in the form $b + c\sqrt{3}$, where b and c are integers to be found. (2)

Solution

$$(2-\sqrt{3})^2 = 4 - 2\sqrt{3} - 2\sqrt{3} + 3 = \underline{\underline{7-4\sqrt{3}}};$$

hence, $\underline{\underline{b=7}}$ and $\underline{\underline{c=-4}}$.

16. Simplify

(a) $(3\sqrt{7})^2$, (1)

Solution

$$(3\sqrt{7})^2 = 3^2 \times (\sqrt{7})^2 = \underline{\underline{63}}.$$

(b) $(8+\sqrt{5})(2-\sqrt{5})$. (3)

Solution

$$(8+\sqrt{5})(2-\sqrt{5}) = 16 + 2\sqrt{5} - 8\sqrt{5} - 5 = \underline{\underline{11 - 6\sqrt{5}}}.$$

17. (a) Find the value of $16^{-\frac{1}{4}}$. (2)

Solution

$$16^{-\frac{1}{4}} = \frac{1}{16^{\frac{1}{4}}} = \underline{\underline{\frac{1}{2}}}.$$

(b) Simplify $x\left(2x^{-\frac{1}{4}}\right)^4$. (2)

Solution

$$x\left(2x^{-\frac{1}{4}}\right)^4 = x(16x^{-1}) = \underline{\underline{16}}.$$

18. Simplify (4)

$$\frac{5-2\sqrt{3}}{\sqrt{3}-1}$$

in the form $p + q\sqrt{3}$, where p and q are rational numbers.

Solution

$$\begin{aligned}\frac{5-2\sqrt{3}}{\sqrt{3}-1} &= \frac{5-2\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} \\ &= \frac{5\sqrt{3}+5-6-2\sqrt{3}}{3-1} \\ &= \frac{3\sqrt{3}-1}{2} \\ &= \underline{\underline{-\frac{1}{2}+\frac{3}{2}\sqrt{3}}};\end{aligned}$$

hence, $\underline{\underline{p = -\frac{1}{2}}}$ and $\underline{\underline{q = \frac{3}{2}}}$.

19. Simplify (4)

$$\frac{7+\sqrt{5}}{\sqrt{5}-1}$$

in the form $a + b\sqrt{5}$, where a and b are integers.

Solution

$$\begin{aligned}\frac{7+\sqrt{5}}{\sqrt{5}-1} &= \frac{7+\sqrt{5}}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1} \\ &= \frac{7\sqrt{5}+5+7+\sqrt{5}}{5-1} \\ &= \frac{12+8\sqrt{5}}{4} \\ &= \underline{\underline{3+2\sqrt{5}}}.\end{aligned}$$

20. Express (4)

$$\frac{15}{\sqrt{3}} - \sqrt{27}$$

in the form $k\sqrt{3}$, where k is an integer.

Solution

$$\begin{aligned}\frac{15}{\sqrt{3}} - \sqrt{27} &= \frac{15}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} - \sqrt{9 \times 3} \\ &= \frac{15\sqrt{3}}{3} - \sqrt{9} \times \sqrt{3} \\ &= 5\sqrt{3} - 3\sqrt{3} \\ &= \underline{\underline{2\sqrt{3}}}.\end{aligned}$$

21. (a) Write down the value of $32^{\frac{1}{5}}$. (1)

Solution

$$32^{\frac{1}{5}} = \underline{\underline{2}}.$$

(b) Simplify fully $(32x^5)^{-\frac{2}{5}}$. (3)

Solution

$$(32x^5)^{-\frac{2}{5}} = 32^{-\frac{2}{5}} \times (x^5)^{-\frac{2}{5}} = \underline{\underline{\tfrac{1}{4}x^{-2}}}.$$

22. (a) Evaluate $81^{\frac{3}{2}}$. (2)

Solution

$$81^{\frac{3}{2}} = (81^{\frac{1}{2}})^3 = 9^3 = \underline{\underline{729}}.$$

(b) Simplify fully $x^2(4x^{-\frac{1}{2}})^2$. (2)

Solution

$$x^2(4x^{-\frac{1}{2}})^2 = x^2 \times 16x^{-1} = \underline{\underline{16x}}.$$

23. (a) Evaluate $32^{\frac{3}{5}}$, giving your answer as an integer. (2)

Solution

$$32^{\frac{3}{5}} = (32^{\frac{1}{5}})^3 = 2^3 = \underline{\underline{8}}.$$

(b) Simplify fully $\left(\frac{25x^4}{4}\right)^{-\frac{1}{2}}$. (2)

Solution

$$\left(\frac{25x^4}{4}\right)^{-\frac{1}{2}} = \left(\frac{4}{25x^4}\right)^{\frac{1}{2}} = \underline{\underline{\frac{2}{5x^2}}}.$$

24. Solve the equation (4)

$$10 + x\sqrt{8} = \frac{6x}{\sqrt{2}}.$$

Give your answer in the form $a\sqrt{b}$ where a and b are integers.

Solution

$$\begin{aligned} 10 + x\sqrt{8} = \frac{6x}{\sqrt{2}} &\Rightarrow 10\sqrt{2} + 4x = 6x \\ &\Rightarrow 10\sqrt{2} = 2x \\ &\Rightarrow \underline{\underline{x = 5\sqrt{2}}}. \end{aligned}$$

25. Solve

(a) $2^y = 8$, (1)

Solution

$$2^y = 8 \Rightarrow \underline{\underline{y = 3}}.$$

(b) $2^x \times 4^{x+1} = 8$. (4)

Solution

$$\begin{aligned} 2^x \times 4^{x+1} = 8 &\Rightarrow 2^x \times 2^{2x+2} = 8 \\ &\Rightarrow 2^{3x+2} = 8 \\ &\Rightarrow 3x + 2 = 3 \\ &\Rightarrow 3x = 1 \\ &\Rightarrow \underline{\underline{x = \tfrac{1}{3}}}. \end{aligned}$$

26. (a) Find the value of $8^{\frac{5}{3}}$. (2)

Solution

$$8^{\frac{5}{3}} = (8^{\frac{1}{3}})^5 = 2^5 = \underline{\underline{32}}.$$

(b) Simplify fully (3)

$$\frac{\left(2x^{\frac{1}{2}}\right)^3}{4x^2}.$$

Solution

$$\frac{\left(2x^{\frac{1}{2}}\right)^3}{4x^2} = \frac{8x^{\frac{3}{2}}}{4x^2} = \underline{\underline{2x^{-\frac{1}{2}}}}.$$

27. (a) Write $\sqrt{80}$ in the form $c\sqrt{5}$, where c is a positive constant. (1)

Solution

$$\sqrt{80} = \sqrt{16 \times 5} = \sqrt{16} \times \sqrt{5} = \underline{\underline{4\sqrt{5}}}.$$

A rectangle R has a length of $(1 + \sqrt{5})$ cm and an area of $\sqrt{80}$ cm^2.

(b) Calculate the width of R in cm. Express your answer in the form $p + q\sqrt{5}$, where p and q are integers to be found. (4)

Solution

$$\begin{aligned} \text{Width} &= \frac{\sqrt{80}}{1+\sqrt{5}} \\ &= \frac{4\sqrt{5}}{1+\sqrt{5}} \times \frac{1-\sqrt{5}}{1-\sqrt{5}} \\ &= \frac{4\sqrt{5} - 20}{1-5} \\ &= \frac{-4(5-\sqrt{5})}{-4} \\ &= \underline{\underline{5-\sqrt{5}}}. \end{aligned}$$

28. Show that

$$\frac{2}{\sqrt{12}-\sqrt{8}}$$

can be written in the form $\sqrt{a}+\sqrt{b}$, where a and b are integers.

Solution

$$\begin{aligned}\frac{2}{\sqrt{12}-\sqrt{8}} &= \frac{2}{\sqrt{4\times 3}-\sqrt{4\times 2}}\\ &= \frac{2}{\sqrt{4}\times\sqrt{3}-\sqrt{4}\times\sqrt{2}}\\ &= \frac{2}{2\sqrt{3}-2\sqrt{2}}\\ &= \frac{1}{\sqrt{3}-\sqrt{2}}\\ &= \frac{1}{\sqrt{3}-\sqrt{2}}\times\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}\\ &= \frac{\sqrt{3}+\sqrt{2}}{3-2}\\ &= \underline{\underline{\sqrt{3}+\sqrt{2}}};\end{aligned}$$

hence, $\underline{\underline{a=3}}$ and $\underline{\underline{b=2}}$.

29. Simplify

(a) $(2\sqrt{5})^2$, (1)

Solution

$$(2\sqrt{5})^2 = 4\times 5 = \underline{\underline{20}}.$$

(b) $\dfrac{\sqrt{2}}{2\sqrt{5}-3\sqrt{2}}$, giving your answer in the form $a+\sqrt{b}$, where a and b are integers. (4)

Solution

$$\begin{aligned}\frac{\sqrt{2}}{2\sqrt{5}-3\sqrt{2}} &= \frac{\sqrt{2}}{2\sqrt{5}-3\sqrt{2}} \times \frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}+3\sqrt{2}} \\ &= \frac{2\sqrt{10}+6}{20-18} \\ &= \frac{2(\sqrt{10}+3)}{2} \\ &= \underline{\underline{3+\sqrt{10}}};\end{aligned}$$

hence, $\underline{\underline{a = 3}}$ and $\underline{\underline{b = 10}}$.

30. Given that $y = 2^x$,

(a) express 4^x in terms of y. (1)

Solution

$$4^x = (2^2)^x = 2^{2x} = (2^x)^2 = \underline{\underline{y^2}}.$$

(b) Hence, or otherwise, solve (4)

$$8(4^x) - 9(2^x) + 1 = 0.$$

Solution

$$\begin{aligned}8(4^x) - 9(2^x) + 1 = 0 &\Rightarrow 8(2^{2x}) - 9(2^x) + 1 = 0 \\ &\Rightarrow [8(2^x) - 1]\,[(2^x) - 1] = 0 \\ &\Rightarrow [8(2^x) - 1] = 0 \text{ or } 2^x - 1 = 0 \\ &\Rightarrow 8(2^x) = 1 \text{ or } 2^x = 1 \\ &\Rightarrow 2^x = \tfrac{1}{8} \text{ or } 2^x = 1 \\ &\Rightarrow 2^x = 2^{-3} \text{ or } 2^x = 2^0 \\ &\Rightarrow \underline{\underline{x = -3}} \text{ or } \underline{\underline{x = 0}}.\end{aligned}$$

31. (a) Simplify (2)

$$\sqrt{50} - \sqrt{18},$$

giving your answer in the form $a\sqrt{2}$, where a is an integer.

Solution

$$\sqrt{50} - \sqrt{18} = \sqrt{25 \times 2} - \sqrt{9 \times 2} = 5\sqrt{2} - 3\sqrt{2} = \underline{\underline{2\sqrt{2}}};$$

hence, $\underline{\underline{a = 2}}$.

(b) Hence, or otherwise, simplify (3)

$$\frac{12\sqrt{3}}{\sqrt{50} - \sqrt{18}},$$

giving your answer in the form $b\sqrt{c}$, where b and c are integers.

Solution

$$\frac{12\sqrt{3}}{\sqrt{50} - \sqrt{18}} = \frac{12\sqrt{3}}{2\sqrt{2}} = \frac{12\sqrt{6}}{4} = \underline{\underline{3\sqrt{6}}};$$

hence, $\underline{\underline{b = 3}}$ and $\underline{\underline{c = 6}}$.

32. (a) Write $\sqrt{45}$ in the form $a\sqrt{5}$, where a is an integer. (1)

Solution

$$\sqrt{45} = \sqrt{9 \times 5} = \sqrt{9} \times \sqrt{5} = \underline{\underline{2\sqrt{5}}}.$$

(b) Express (5)

$$\frac{2(3 + \sqrt{5})}{3 - \sqrt{5}}$$

in the form $b + c\sqrt{5}$, where b and c are integers.

Solution

$$\begin{aligned}
\frac{2(3 + \sqrt{5})}{3 - \sqrt{5}} &= \frac{2(3 + \sqrt{5})}{3 - \sqrt{5}} \times \frac{3 + \sqrt{5}}{3 + \sqrt{5}} \\
&= \frac{2(3 + \sqrt{5})(3 + \sqrt{5})}{9 - 5} \\
&= \frac{9 + 3\sqrt{5} + 3\sqrt{5} + 5}{2} \\
&= \frac{14 + 6\sqrt{5}}{2} \\
&= \underline{\underline{7 + 3\sqrt{5}}};
\end{aligned}$$

hence, $\underline{\underline{b = 7}}$ and $\underline{\underline{c = 3}}$.

33. (a) Expand and simplify $(7+\sqrt{5})(3-\sqrt{5})$. (3)

Solution

$$(7+\sqrt{5})(7-\sqrt{5}) = 21 + 3\sqrt{5} - 7\sqrt{5} - 5 = \underline{\underline{16 - 4\sqrt{5}}}.$$

(b) Express (3)

$$\frac{7+\sqrt{5}}{3+\sqrt{5}}$$

in the form $a + b\sqrt{5}$, where a and b are integers.

Solution

$$\begin{aligned}\frac{7+\sqrt{5}}{3+\sqrt{5}} &= \frac{7+\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}} \\ &= \frac{21 + 3\sqrt{5} - 7\sqrt{5} - 5}{9-5} \\ &= \frac{16 - 4\sqrt{5}}{4} \\ &= \underline{\underline{4 - \sqrt{5}}};\end{aligned}$$

hence, $\underline{\underline{a = 4}}$ and $\underline{\underline{b = -1}}$.

34. (a) Simplify (2)

$$\sqrt{32} + \sqrt{18},$$

giving your answer in the form $a\sqrt{2}$, where a is as integer.

Solution

$$\begin{aligned}\sqrt{32} + \sqrt{18} &= \sqrt{16 \times 2} + \sqrt{9 \times 2} \\ &= \sqrt{16} \times \sqrt{2} + \sqrt{9} \times \sqrt{2} \\ &= 4\sqrt{2} + 3\sqrt{2} \\ &= \underline{\underline{7\sqrt{2}}}.\end{aligned}$$

(b) Simplify (4)

$$\frac{\sqrt{32} + \sqrt{18}}{3 + \sqrt{2}},$$

giving your answer in the form $b\sqrt{2} + c$, where b and c are integer.

Solution

$$\begin{aligned}\frac{\sqrt{32}+\sqrt{18}}{3+\sqrt{2}} &= \frac{7\sqrt{2}}{3+\sqrt{2}}\\ &= \frac{7\sqrt{2}}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}}\\ &= \frac{21\sqrt{2}-14}{9-2}\\ &= \frac{7(3\sqrt{2}-2)}{7}\\ &= \underline{\underline{3\sqrt{2}-2}}.\end{aligned}$$

35. (a) Express (3)

$$(5-\sqrt{8})(1+\sqrt{2})$$

in the form $a+b\sqrt{2}$, where a and b are integers.

Solution

$$\begin{aligned}(5-\sqrt{8})(1+\sqrt{2}) &= (5-\sqrt{4\times 2})(1+\sqrt{2})\\ &= (5-\sqrt{4}\times\sqrt{2})(1+\sqrt{2})\\ &= (5-2\sqrt{2})(1+\sqrt{2})\\ &= 5+5\sqrt{2}-2\sqrt{2}-4\\ &= \underline{\underline{1+3\sqrt{2}}}.\end{aligned}$$

(b) Express (3)

$$\sqrt{80}+\frac{30}{\sqrt{5}}$$

in the form $c\sqrt{5}$, where c is an integer.

Solution

$$
\begin{aligned}
\sqrt{80} + \frac{30}{\sqrt{5}} &= \sqrt{16 \times 5} + \frac{30}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} \\
&= \sqrt{16} \times \sqrt{5} + \frac{30\sqrt{5}}{5} \\
&= 4\sqrt{5} + 6\sqrt{5} \\
&= \underline{\underline{10\sqrt{5}}};
\end{aligned}
$$

hence, $\underline{\underline{c = 10}}$.