Dr Oliver Mathematics
Mathematics
Logarithms Part 1
Past Examination Questions

This booklet consists of 24 questions across a variety of examination topics.
The total number of marks available is 150.
Note: there are some problems (such those involving a geometric series) that have not been included here.

1. Find, giving your answer to 3 significant figures where appropriate, the value of x for which

   (a) $3^x = 5$, (3)

   Solution

   $$3^x = 5 \Rightarrow x = \log_3 5 = 1.464\,973\,521 \text{ (FCD)} = \underline{\underline{1.46 \text{ (3 sf)}}}.$$

   (b) $\log_2(2x+1) - \log_2 x = 2$. (4)

   Solution

   $$\begin{aligned}\log_2(2x+1) - \log_2 x = 2 &\Rightarrow \log_2\left(\frac{2x+1}{x}\right) = 2\\ &\Rightarrow \frac{2x+1}{x} = 2^2\\ &\Rightarrow \frac{2x+1}{x} = 4\\ &\Rightarrow 2x+1 = 4x\\ &\Rightarrow 2x = 1\\ &\Rightarrow \underline{\underline{x = \tfrac{1}{2}}}.\end{aligned}$$

2. Solve

   (a) $5^x = 8$, giving your answer to 3 significant figures, (3)

   Solution

   $$x = \log_5 8 = 1.292\,029\,674 \text{ (FCD)} = \underline{\underline{1.29 \text{ (3 sf)}}}.$$

(b) $\log_2(x+1) - \log_2 x = \log_2 7$. (3)

Solution

$$\begin{aligned}
\log_2(x+1) - \log_2 x = \log_2 7 &\Rightarrow \log_2\left(\frac{x+1}{x}\right) = \log_2 7 \\
&\Rightarrow \frac{x+1}{x} = 7 \\
&\Rightarrow x + 1 = 7x \\
&\Rightarrow 6x = 1 \\
&\Rightarrow \underline{\underline{x = \tfrac{1}{6}}}.
\end{aligned}$$

3. The first term of a geometric series is 120. The common ratio is $\frac{3}{4}$. The sum of the first n terms of the series is greater than 300. Calculate the smallest possible value of n.

Solution

$$\begin{aligned}
\frac{120[1-(\frac{3}{4})^n]}{1-\frac{3}{4}} > 300 &\Rightarrow 120[1-(\tfrac{3}{4})^n] > 75 \\
&\Rightarrow 1-(\tfrac{3}{4})^n > \tfrac{5}{8} \\
&\Rightarrow \tfrac{3}{8} < (\tfrac{3}{4})^n \\
&\Rightarrow n > \frac{\log_2 \frac{3}{8}}{\log_2 \frac{3}{4}} \\
&\Rightarrow n > 3.409\,420\,084 \text{ (FCD)} \\
&\Rightarrow \underline{\underline{n = 4}}.
\end{aligned}$$

4. (a) Write down the value of $\log_6 36$. (1)

Solution

$$\log_6 36 = \log_6 6^2 = 2\log_6 6 = \underline{\underline{2}}.$$

(b) Express $2\log_a 3 + \log_a 11$ as a single logarithm to base a. (3)

Solution

$$2\log_a 3 + \log_a 11 = \log_a 3^2 + \log_a 11 = \log_a(9 \times 11) = \underline{\underline{\log_a 99}}.$$

5. Solve the equation $5^x = 17$, giving your answer to 3 significant figures. (3)

Solution

$$x = \log_5 17 = 1.760\,374\,428 \text{ (FCD)} = \underline{\underline{1.76 \text{ (3 sf)}}}.$$

6. (a) Find, to 3 significant figures, the value of x for which $8^x = 0.8$. (3)

Solution

$$x = \log_8 0.8 = -0.107\,309\,365 \text{ (FCD)} = \underline{\underline{-0.107 \text{ (3 sf)}}}.$$

(b) Solve the equation (3)

$$2\log_3 x - \log_3 7x = 1.$$

Solution

$$\begin{aligned} 2\log_3 x - \log_3 7x = 1 &\Rightarrow \log_3 x^2 - \log_3 7x = 1 \\ &\Rightarrow \log_3\left(\frac{x^2}{7x}\right) = 1 \\ &\Rightarrow \frac{x^2}{7x} = 3 \\ &\Rightarrow x^2 = 21x \\ &\Rightarrow x(x-21) = 0 \\ &\Rightarrow \underline{\underline{x = 21}} \text{ (because } x = 0 \text{ is not a solution).} \end{aligned}$$

7. Given that a and b are positive constants, solve the simultaneous equations (6)

$$a = 3b$$
$$\log_3 a + \log_3 b = 2.$$

Give your answers as exact numbers.

Solution

$$\begin{aligned}\log_3 a + \log_3 b = 2 &\Rightarrow \log_3 3b + \log_3 b = 2\\ &\Rightarrow \log_3 3b^2 = 2\\ &\Rightarrow 3b^2 = 3^2\\ &\Rightarrow 3b^2 = 9\\ &\Rightarrow b^2 = 3\\ &\Rightarrow \underline{\underline{b = \sqrt{3}}} \text{ (because } b = -\sqrt{3} \text{ is not a solution)}\\ &\Rightarrow \underline{\underline{a = 3\sqrt{3}}}.\end{aligned}$$

8. (a) Find, to 3 significant figures, the value of x for which $5^x = 7$. (2)

Solution

$$x = \log_5 7 = 1.209\,061\,955 \text{ (FCD)} = \underline{\underline{1.21 \text{ (3 sf)}}}.$$

(b) Solve the equation $5^{2x} - 12(5^x) + 35 = 0$. (4)

Solution

$$\begin{aligned}5^{2x} - 12(5^x) + 35 = 0 &\Rightarrow (5^x)^2 - 12(5^x) + 35 = 0\\ &\Rightarrow (5^x - 5)(5^x - 7) = 0\\ &\Rightarrow 5^x = 5 \text{ or } 5^x = 7\\ &\Rightarrow x = \log_5 5 \text{ or } x = \log_5 7\\ &\Rightarrow \underline{\underline{x = 1}} \text{ or } \underline{\underline{x = 1.209\,061\,955 \text{ (FCD)}}}.\end{aligned}$$

9. Given that $0 < x < 4$ and (6)

$$\log_5(4 - x) - 2\log_5 x = 1,$$

find the value of x.

Solution

$$\begin{aligned}\log_5(4-x) - 2\log_5 x = 1 &\Rightarrow \log_5(4-x) - \log_5 x^2 = 1 \\ &\Rightarrow \log_5\left(\frac{4-x}{x^2}\right) = 1 \\ &\Rightarrow \frac{4-x}{x^2} = 5 \\ &\Rightarrow 4 - x = 5x^2 \\ &\Rightarrow 5x^2 + x - 4 = 0 \\ &\Rightarrow (5x-4)(x+1) = 0 \\ &\Rightarrow \underline{\underline{x = \tfrac{4}{5}}} \text{ (because } x = -1 \text{ is not a solution)}\end{aligned}$$

10. (a) Find the value of y such that (2)

$$\log_2 y = -3.$$

Solution

$$\log_2 y = -3 \Rightarrow y = 2^{-3} = \underline{\underline{\tfrac{1}{8}}}.$$

(b) Find the value of x such that (5)

$$\frac{\log_2 32 + \log_2 16}{\log_2 x} = \log_2 x.$$

Solution

$$\begin{aligned}\frac{\log_2 32 + \log_2 16}{\log_2 x} = \log_2 x &\Rightarrow \log_2 512 = (\log_2 x)^2 \\ &\Rightarrow \log_2 2^9 = (\log_2 x)^2 \\ &\Rightarrow 9 = (\log_2 x)^2 \\ &\Rightarrow (\log_2 x)^2 - 9 = 0 \\ &\Rightarrow (\log_2 x - 3)(\log_2 x + 3) = 0 \\ &\Rightarrow \log_2 x = 3 \text{ or } \log_2 x = -3 \\ &\Rightarrow x = 2^3 \text{ or } x = 2^{-3} \\ &\Rightarrow \underline{\underline{x = 8}} \text{ or } \underline{\underline{x = \tfrac{1}{8}}}.\end{aligned}$$

11. (a) Find the positive value of x such that (2)

$$\log_x 64 = 2.$$

Solution

$$\log_x 64 = 2 \Rightarrow x^2 = 64 \Rightarrow \underline{\underline{x = 8}}.$$

(b) Solve for x (6)

$$\log_2(11 - 6x) = 2\log_2(x - 1) + 3.$$

Solution

$$\begin{aligned}
\log_2(11 - 6x) = 2\log_2(x-1) + 3 &\Rightarrow \log_2(11 - 6x) - \log_2(x-1)^2 = 3 \\
&\Rightarrow \log_2\left(\frac{11 - 6x}{(x-1)^2}\right) = 3 \\
&\Rightarrow \frac{11 - 6x}{(x-1)^2} = 2^3 \\
&\Rightarrow 11 - 6x = 8(x-1)^2 \\
&\Rightarrow 11 - 6x = 8(x^2 - 2x + 1) \\
&\Rightarrow 11 - 6x = 8x^2 - 16x + 8 \\
&\Rightarrow 8x^2 - 10x - 3 = 0 \\
&\Rightarrow (4x + 1)(2x - 3) = 0 \\
&\Rightarrow 2x - 3 = 0 \text{ (because } 4x + 1 = 0 \text{ is not a solution)} \\
&\Rightarrow \underline{\underline{x = 1\tfrac{1}{2}}}.
\end{aligned}$$

12. (a) Given that (5)

$$2\log_3(x - 5) - \log_3(2x - 13) = 1,$$

show that $x^2 - 16x + 64 = 0$.

Solution

$$
\begin{aligned}
2\log_3(x-5)-\log_3(2x-13)=1 &\Rightarrow \log_3(x-5)^2-\log_3(2x-13)=1\\
&\Rightarrow \log_3\left(\frac{(x-5)^2}{2x-13}\right)=1\\
&\Rightarrow \frac{(x-5)^2}{2x-13}=3\\
&\Rightarrow (x-5)^2=3(2x-13)\\
&\Rightarrow x^2-10x+25=6x-39\\
&\Rightarrow \underline{\underline{x^2-16x+64=0}}.
\end{aligned}
$$

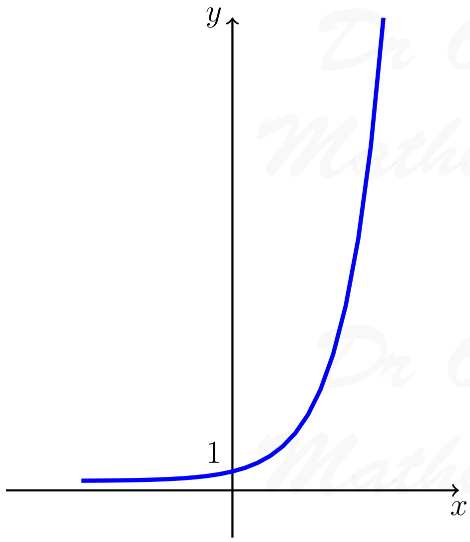
(b) Hence, or otherwise, solve $2\log_3(x-5)-\log_3(2x-13)=1$. (2)

Solution

$$x^2-16x+64=0 \Rightarrow (x-8)^2=0 \Rightarrow \underline{\underline{x=8}}.$$

13. (a) Sketch the graph of $y=7^x$, $x \in \mathbb{R}$, showing the coordinates of any points at which the graph crosses the axis. (2)

Solution



(b) Solve the equation (6)

$$7^{2x}-4(7^x)+3=0,$$

giving your answers to 2 decimal places where appropriate.

Solution

$$\begin{aligned}7^{2x} - 4(7^x) + 3 = 0 &\Rightarrow (7^x)^2 - 4(7^x) + 3 = 0\\ &\Rightarrow (7^x - 1)(7^x - 3) = 0\\ &\Rightarrow 7^x - 1 = 0 \text{ or } 7^x - 3 = 0\\ &\Rightarrow 7^x = 1 \text{ or } 7^x = 3\\ &\Rightarrow \underline{\underline{x = 0}} \text{ or } \underline{\underline{x = 0.56 \text{ (2 dp)}}}.\end{aligned}$$

14. Find, giving your answer to 3 significant figures, the value of x for which

(a) $5^x = 10$, (2)

Solution

$$x = \log_5 10 = 1.430\,676\,558 \text{ (FCD)} = \underline{\underline{1.43 \text{ (3 sf)}}}.$$

(b) $\log_3(x-2) = -1$. (2)

Solution

$$x - 2 = 3^{-1} \Rightarrow \underline{\underline{x = 2\tfrac{1}{3}}}.$$

15. Given that $y = 3x^2$,

(a) show that $\log_3 y = 1 + 2\log_3 x$. (3)

Solution

$$\begin{aligned}y = 3x^2 &\Rightarrow \log_3 y = \log_3(3x^2)\\ &\Rightarrow \log_3 y = \log_3 3 + \log_3(x^2)\\ &\Rightarrow \underline{\underline{\log_3 y = 1 + 2\log_3 x}}.\end{aligned}$$

(b) Hence, or otherwise, solve the equation (3)

$$1 + 2\log_3 x = \log_3(28x - 9).$$

Solution

$$\begin{aligned}1 + 2\log_3 x = \log_3(28x-9) &\Rightarrow 1 = \log_3(28x-9) - 2\log_3 x\\ &\Rightarrow 1 = \log_3\left(\frac{28x-9}{x^2}\right)\\ &\Rightarrow 3 = \frac{28x-9}{x^2}\\ &\Rightarrow 3x^2 = 28x - 9\\ &\Rightarrow 3x^2 - 28x + 9 = 0\\ &\Rightarrow (3x-1)(x-9) = 0\\ &\Rightarrow \underline{\underline{x = \tfrac{1}{3}}} \text{ or } \underline{\underline{x = 9}}.\end{aligned}$$

16. Find the values of x such that (5)

$$2\log_3 x - \log_3(x-2) = 2.$$

Solution

$$\begin{aligned}2\log_3 x - \log_3(x-2) = 2 &\Rightarrow \log_3\left(\frac{x^2}{x-2}\right) = 2\\ &\Rightarrow \frac{x^2}{x-2} = 3^2\\ &\Rightarrow x^2 = 9(x-2)\\ &\Rightarrow x^2 - 9x + 18 = 0\\ &\Rightarrow (x-3)(x-6) = 0\\ &\Rightarrow \underline{\underline{x = 3}} \text{ or } \underline{\underline{x = 6}}.\end{aligned}$$

17. Given that

$$2\log_2(x+15) - \log_2 x = 6,$$

(a) show that $x^2 - 34x + 225 = 0$. (5)

Solution

$$\begin{aligned} 2\log_2(x+15) - \log_2 x = 6 &\Rightarrow \log_2(x+15)^2 - \log_2 x = 6 \\ &\Rightarrow \log_2\left(\frac{(x+15)^2}{x}\right) = 6 \\ &\Rightarrow \frac{(x+15)^2}{x} = 2^6 \\ &\Rightarrow (x+15)^2 = 64x \\ &\Rightarrow x^2 + 30x + 225 = 64x \\ &\Rightarrow \underline{\underline{x^2 - 34x + 225 = 0}}. \end{aligned}$$

(b) Hence, or otherwise, solve $2\log_2(x+15) - \log_2 x = 6$. (2)

Solution

$$x^2 - 34x + 225 = 0 \Rightarrow (x-9)(x-25) = 0 \Rightarrow \underline{\underline{x = 9}} \text{ or } \underline{\underline{x = 25}}.$$

18. (a) Find the exact value of x for which (4)

$$\log_2(2x) = \log_2(5x+4) - 3.$$

Solution

$$\begin{aligned} \log_2(2x) = \log_2(5x+4) - 3 &\Rightarrow \log_2(2x) - \log_2(5x+4) = -3 \\ &\Rightarrow \log_2\left(\frac{2x}{5x+4}\right) = -3 \\ &\Rightarrow \frac{2x}{5x+4} = 2^{-3} \\ &\Rightarrow 2x = \tfrac{1}{8}(5x+4) \\ &\Rightarrow 2x = \tfrac{5}{8}x + \tfrac{1}{2} \\ &\Rightarrow \tfrac{11}{8}x = \tfrac{1}{2} \\ &\Rightarrow \underline{\underline{x = \tfrac{4}{11}}}. \end{aligned}$$

(b) Given that (3)

$$\log_a y + 3\log_a 2 = 5,$$

express y in terms of a. Give your answer in its simplest form.

Solution

$$\begin{aligned}\log_a y + 3\log_a 2 = 5 &\Rightarrow \log_a y + \log_a 2^3 = 5\\ &\Rightarrow \log_a(8y) = 5\\ &\Rightarrow 8y = a^5\\ &\Rightarrow \underline{\underline{y = \tfrac{1}{8}a^5}}.\end{aligned}$$

19. Given that $\log_3 x = a$, find in terms of a, giving each answer in its simplest form,

(a) $\log_3(9x)$, (2)

Solution

$$\log_3(9x) = \log_3 9 + \log_3 a = \log_3 3^2 + \log_3 a = 2\log_3 3 + \log_3 a = \underline{\underline{2 + a}}.$$

(b) $\log_3\left(\frac{x^5}{81}\right)$. (3)

Solution

$$\log_3\left(\frac{x^5}{81}\right) = \log_3 x^5 - \log_3 81 = 5\log_3 x - \log_3 3^4 = \underline{\underline{5a - 4}}.$$

(c) Solve, for x, (4)

$$\log_3(9x) + \log_3\left(\frac{x^5}{81}\right) = 3,$$

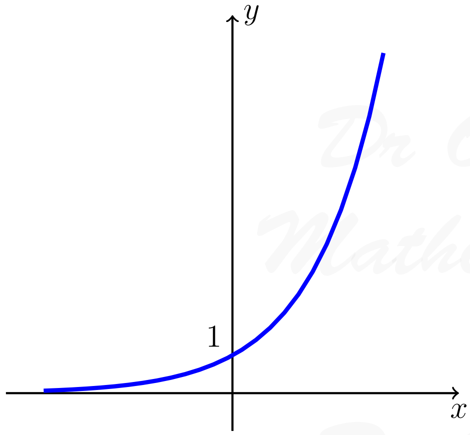
giving your answer to 4 significant figures.

Solution

$$\begin{aligned}\log_3(9x) + \log_3\left(\frac{x^5}{81}\right) = 3 &\Rightarrow (2 + a) + (5a - 4) = 3\\ &\Rightarrow 6a = 5\\ &\Rightarrow a = \tfrac{5}{6}\\ &\Rightarrow \log_3 x = \tfrac{5}{6}\\ &\Rightarrow x = 3^{\frac{5}{6}}\\ &\Rightarrow x = 2.498\,049\,533 \text{ (FCD)}\\ &\Rightarrow \underline{\underline{x = 2.498 \text{ (4 sf)}}}.\end{aligned}$$

20. (a) Sketch the graph of $y = 3^x$, $x \in \mathbb{R}$, showing the coordinates of any points at which the graph crosses the axis. (2)

Solution



(b) Solve the equation (6)

$$3^{2x} - 9(3^x) + 18 = 0,$$

giving your answers to 2 decimal places where appropriate.

Solution

$$\begin{aligned} 3^{2x} - 9(3^x) + 18 = 0 &\Rightarrow (3^x)^2 - 9(3^x) + 18 = 0 \\ &\Rightarrow (3^x - 3)(3^x - 6) = 0 \\ &\Rightarrow 3^x - 3 = 0 \text{ or } 3^x - 6 = 0 \\ &\Rightarrow 3^x = 3 \text{ or } 3^x = 6 \\ &\Rightarrow \underline{\underline{x = 1}} \text{ or } \underline{\underline{x = 1.63 \text{ (2 dp)}}}. \end{aligned}$$

21. (a) Solve (2)

$$5^y = 8,$$

giving your answer to 3 significant figures.

Solution

$$y = \log_5 8 = 1.292\,029\,674 \text{ (FCD)} = \underline{\underline{1.29 \text{ (3 sf)}}}.$$

(b) Use algebra to find the values for which (6)

$$\log_2(x + 15) - 4 = \tfrac{1}{2}\log_2 x.$$

Solution

$$\begin{aligned}\log_2(x+15)-4=\tfrac{1}{2}\log_2 x &\Rightarrow \log_2(x+15)-\tfrac{1}{2}\log_2 x = 4\\ &\Rightarrow \log_2(x+15)-\log_2 x^{\frac{1}{2}}=4\\ &\Rightarrow \log_2\left(\frac{x+15}{x^{\frac{1}{2}}}\right)=4\\ &\Rightarrow \frac{x+15}{x^{\frac{1}{2}}}=2^4\\ &\Rightarrow x+15=16x^{\frac{1}{2}}\\ &\Rightarrow x-16x^{\frac{1}{2}}+15=0\\ &\Rightarrow (x^{\frac{1}{2}}-1)(x^{\frac{1}{2}}-15)=0\\ &\Rightarrow x^{\frac{1}{2}}=1 \text{ or } x^{\frac{1}{2}}=15\\ &\Rightarrow \underline{\underline{x=1}} \text{ or } \underline{\underline{x=225}}.\end{aligned}$$

22. (a) Use logarithms to solve the equation (3)

$$8^{2x+1}=24,$$

giving your answer to 3 decimal places.

Solution

$$8^{2x+1}=24 \Rightarrow 2x+1=\log_8 24 \Rightarrow x=\frac{-1+\log_8 24}{2}=\underline{\underline{0.264\text{ (3 dp)}}}.$$

(b) Find the values of y such that (5)

$$\log_2(11y-3)-\log_2 3-2\log_2 y=1,\ y>\tfrac{3}{11}.$$

Solution

$$\begin{aligned} & \log_2(11y-3) - \log_2 3 - 2\log_2 y = 1 \\ \Rightarrow\quad & \log_2\left(\frac{11y-3}{3y^2}\right) = 1 \\ \Rightarrow\quad & \frac{11y-3}{3y^2} = 2 \\ \Rightarrow\quad & 11y - 3 = 6y^2 \\ \Rightarrow\quad & 6y^2 - 11y + 3 = 0 \\ \Rightarrow\quad & (3y-1)(2y-3) = 0 \\ \Rightarrow\quad & \underline{\underline{y = \tfrac{1}{3} \text{ or } 1\tfrac{1}{2}}}. \end{aligned}$$

23. (a) Given that (3)

$$\log_3(3b+1) - \log_3(a-2) = -1, a > 2,$$

express b in terms of a.

Solution

$$\begin{aligned} & \log_3(3b+1) - \log_3(a-2) = -1 \\ \Rightarrow\quad & \log_3\left(\frac{3b+1}{a-2}\right) = -1 \\ \Rightarrow\quad & \frac{3b+1}{a-2} = 3^{-1} \\ \Rightarrow\quad & 3b + 1 = \tfrac{1}{3}(a-2) \\ \Rightarrow\quad & 3b = \tfrac{1}{3}(a-2) - 1 = \tfrac{1}{3}a - \tfrac{5}{3} \\ \Rightarrow\quad & b = \tfrac{1}{9}a - \tfrac{5}{9} = \underline{\underline{\tfrac{1}{9}(a-5)}}. \end{aligned}$$

(b) Solve the equation (4)

$$2^{2x+5} - 7(2^x) = 0,$$

giving your answer to 2 decimal places.

Solution

$$\begin{aligned} & 2^{2x+5} - 7(2^x) = 0 \\ \Rightarrow\ & 2^x\, 2^{x+5} - 7(2^x) = 0 \\ \Rightarrow\ & 2^x(2^{x+5} - 7) = 0 \\ \Rightarrow\ & 2^x = 0 \text{ (no!) or } 2^{x+5} = 7 \\ \Rightarrow\ & x + 5 = \log_2 7 \\ \Rightarrow\ & x = \log_2 7 - 5 \\ \Rightarrow\ & x = -2.192\,645\,078 \text{ (FCD)} \\ \Rightarrow\ & \underline{\underline{x = -2.19 \text{ (2 dp)}}}. \end{aligned}$$

24.

$$2\log(x + a) = \log(16a^6), \text{ where } a \text{ is a positive constant.}$$

(a) Find x in terms of a, giving your answer in its simplest form. (3)

Solution

$$\begin{aligned} 2\log(x + a) = \log(16a^6) &\Rightarrow 2\log(x + a) = \log(4a^3)^2 \\ &\Rightarrow 2\log(x + a) = 2\log(4a^3) \\ &\Rightarrow x + a = 4a^3 \\ &\Rightarrow \underline{\underline{x = 4a^3 - a}}. \end{aligned}$$

$$\log_3(9y + b) - \log_3(2y - b) = 2, \text{ where } b \text{ is a positive constant.}$$

(b) Find y in terms of b, giving your answer in its simplest form. (4)

Solution

$$\begin{aligned} \log_3(9y + b) - \log_3(2y - b) = 2 &\Rightarrow \log_3\left(\frac{9y + b}{2y - b}\right) = 2 \\ &\Rightarrow \frac{9y + b}{2y - b} = 3^2 \\ &\Rightarrow 9y + b = 9(2y - b) \\ &\Rightarrow 9y + b = 18y - 9b \\ &\Rightarrow 10b = 9y \\ &\Rightarrow \underline{\underline{y = \tfrac{10}{9}b}}. \end{aligned}$$