Dr Oliver Mathematics
Applied Mathematics: Mechanics or Statistics
Section B
2007 Paper
1 hour

The total number of marks available is 32.
You must write down all the stages in your working.

1. Find the exact value of (5)

$$\int_0^{\frac{1}{6}\pi} x \sin 3x \, \mathrm{d}x.$$

2. Use the binomial theorem to expand (4)

$$\left(x^3 - \frac{2}{x}\right)^4$$

and simplify your answer.

3. A curve is defined parametrically by (5)

$$x = \frac{t}{t^2+1} \text{ and } y = \frac{t-1}{t^2+1}.$$

Obtain $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ as a function of t.

4. (a) For the matrix (3)

$$\mathbf{A} = \begin{pmatrix} \lambda & 2 \\ 2 & \lambda - 3 \end{pmatrix},$$

find the values of λ such that the matrix is singular.

(b) Write down the matrix $\mathbf{A}^{-1}$ when $\lambda = 3$. (1)

5. Obtain the solution of the differential equation (5)

$$x\frac{\mathrm{d}y}{\mathrm{d}x} - y = x^2\mathrm{e}^x,$$

for which $y = 2$ when $x = 1$.

6. (a) Express (4)

$$\frac{8}{x(x+2)(x+4)}$$

in partial fractions.

(b) Calculate the area under the curve (5)

$$y = \frac{8}{x^3 + 6x^2 + 8x}$$

between $x = 1$ and $x = 2$.
Express your answer in the form $\ln \frac{a}{b}$, where a and b are positive integers.