

# Dr Oliver Mathematics

## Worked Examples

### Proportion 1

**From:** O Level, 1957(!)

1. The cost, £ $y$ , of making a batch of articles depends on  $x$ , the number of articles in the batch.

$y$  is the sum of two numbers, one of which varies directly as  $x$  and the other inversely as  $x$ .

- When the number in the batch is 10, the cost is £28.
  - When the number in the batch is 20, the cost is £44.
- (a) Express  $y$  in terms of  $x$ .

#### Solution

Well: two variables!

$$y \propto \left(x + \frac{1}{x}\right) \Rightarrow y = kx + \frac{l}{x},$$

for some constants  $k$  and  $l$  respectively. Now,

$$x = 10, y = 28 \Rightarrow 28 = 10k + \frac{1}{10}l \quad (1)$$

and

$$x = 20, y = 44 \Rightarrow 44 = 20k + \frac{1}{20}l \quad (2).$$

Do  $2 \times (1)$ :

$$56 = 20k + \frac{1}{5}l \quad (3)$$

and do  $(3) - (2)$ :

$$\begin{aligned} 12 &= \frac{3}{20}l \Rightarrow l = 80 \\ &\Rightarrow 28 = 10k + \frac{1}{10}(80) \\ &\Rightarrow 28 = 10k + 8 \\ &\Rightarrow 10k = 20 \\ &\Rightarrow k = 2; \end{aligned}$$

hence,

$$\underline{\underline{y = 2x + \frac{80}{x}}}$$

(b) Find the number in the batch when the cost is £37.

**Solution**

$$37 = 2x + \frac{80}{x} \Rightarrow 37x = 2x^2 + 80$$
$$\Rightarrow 2x^2 - 37x + 80 = 0$$

$$\left. \begin{array}{l} \text{add to:} \quad \quad \quad -37 \\ \text{multiply to: } (+2) \times (+80) = +160 \end{array} \right\} -32, -5$$

$$\Rightarrow 2x^2 - 32x - 5x + 80 = 0$$
$$\Rightarrow 2x(x - 16) - 5(x - 16) = 0$$
$$\Rightarrow (2x - 5)(x - 16) = 0$$
$$\Rightarrow 2x - 5 = 0 \text{ or } x - 16 = 0$$
$$\Rightarrow x = 2\frac{1}{2} \text{ or } x = 16;$$

given that  $x \neq 2\frac{1}{2}$  — half an item! — so

$$\underline{\underline{x = 16 \text{ articles.}}}$$