

Dr Oliver Mathematics
GCSE Mathematics
2014 November Paper 2H: Calculator
1 hour 45 minutes

The total number of marks available is 100.
You must write down all the stages in your working.

1. Here are the ingredients needed to make 10 pancakes.

Pancakes	
Ingredients to make 10 pancakes	
300 ml	of milk
120 g	of flour
2	eggs

Matthew makes 30 pancakes.

- (a) Work out how much flour he uses.

(2)

Solution

$$3 \times 120 = \underline{\underline{360 \text{ g}}}$$

Tara makes some pancakes.
She uses 750 ml of milk.

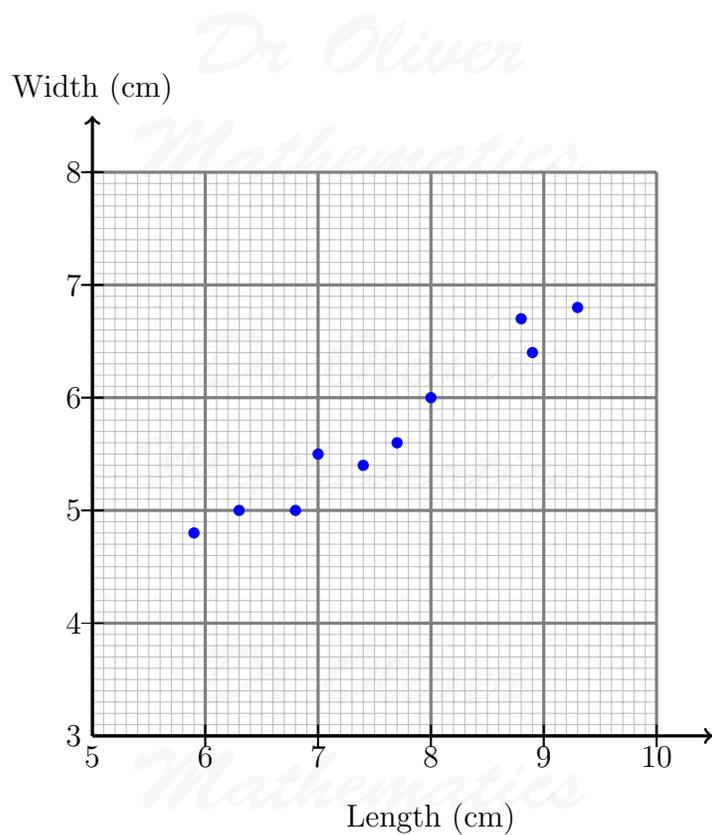
- (b) Work out how many pancakes she makes.

(2)

Solution

$$\frac{750}{300} \times 10 = \underline{\underline{25 \text{ pancakes}}}$$

2. The scatter graph shows some information about ten pine cones from the same tree.
It shows the length and the width of each pine cone.



- (a) Describe the relationship between the length and the width of a pine cone. (1)

Solution

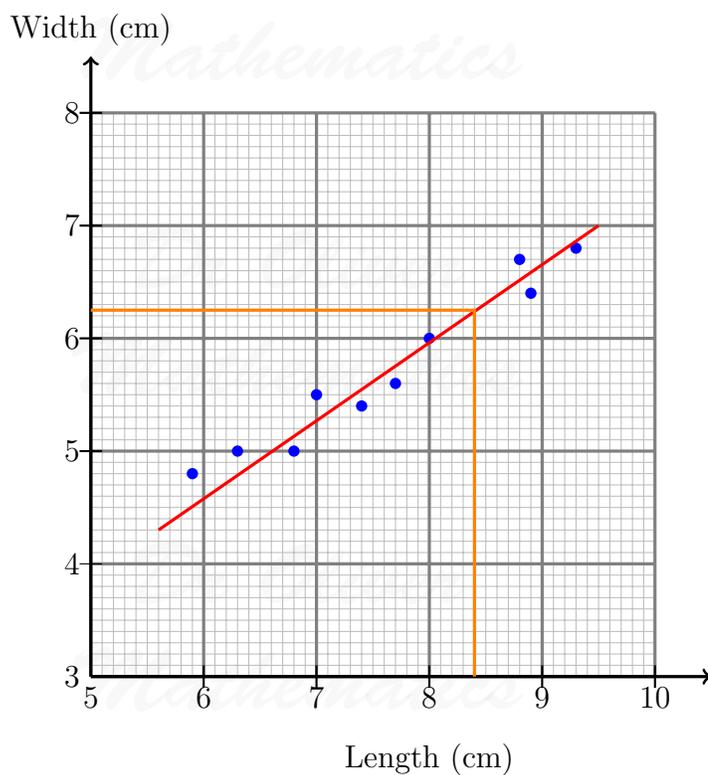
Positive correlation, i.e., as the length of pine grow increases, the width also increases.

Another pine cone from this tree has a length of 8.4 cm.

- (b) Estimate the width of this pine cone. (2)

Solution

Draw a line of best fit:



Correct read-off: approximately 6.25 cm.

3. $f = 3g + 7h$.

(a) Work out the value of f when $g = -5$ and $h = 2$.

(2)

Solution

$$\begin{aligned} f &= [3 \times (-5)] + [7 \times 2] \\ &= -15 + 14 \\ &= \underline{\underline{-1}}. \end{aligned}$$

(b) Factorise

$$3x + 6.$$

(1)

Solution

$$3x + 6 = \underline{\underline{3(x + 2)}}.$$

(c) Expand and simplify

$$5(y - 2) + 2(y - 3).$$

(2)

Solution

$$\begin{aligned} 5(y - 2) + 2(y - 3) &= 5y - 10 + 2y - 6 \\ &= \underline{\underline{7y - 16}}. \end{aligned}$$

(d) Simplify

$$m^5 \times m^3.$$

(1)

Solution

$$m^5 \times m^3 = m^{5+3} = \underline{\underline{m^8}}.$$

(e) Simplify

$$\frac{p^6}{p^2}.$$

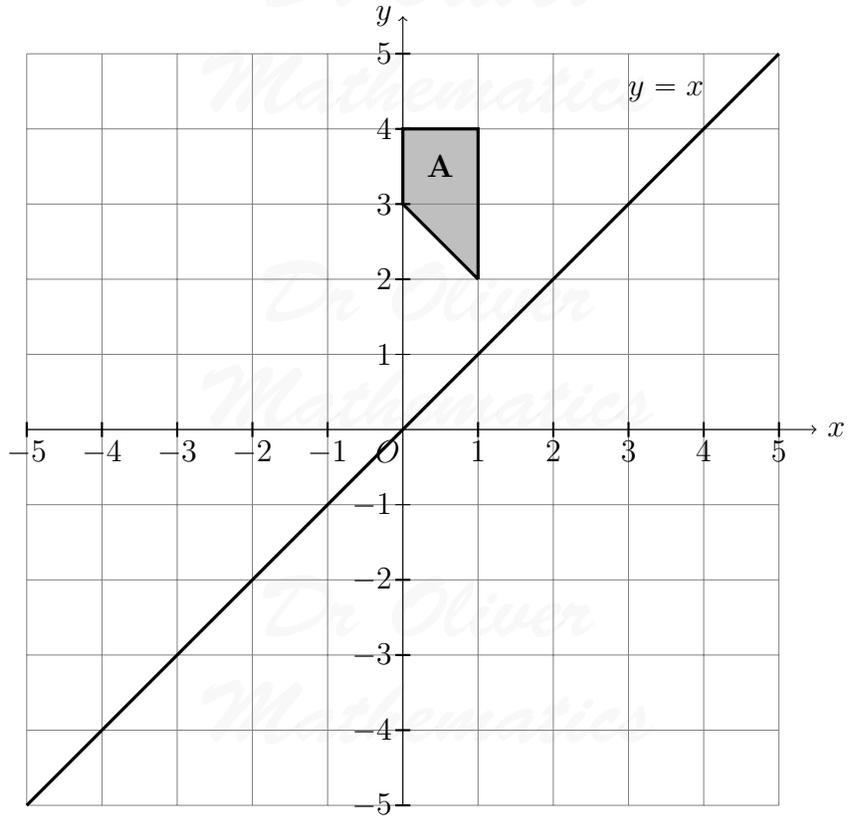
(1)

Solution

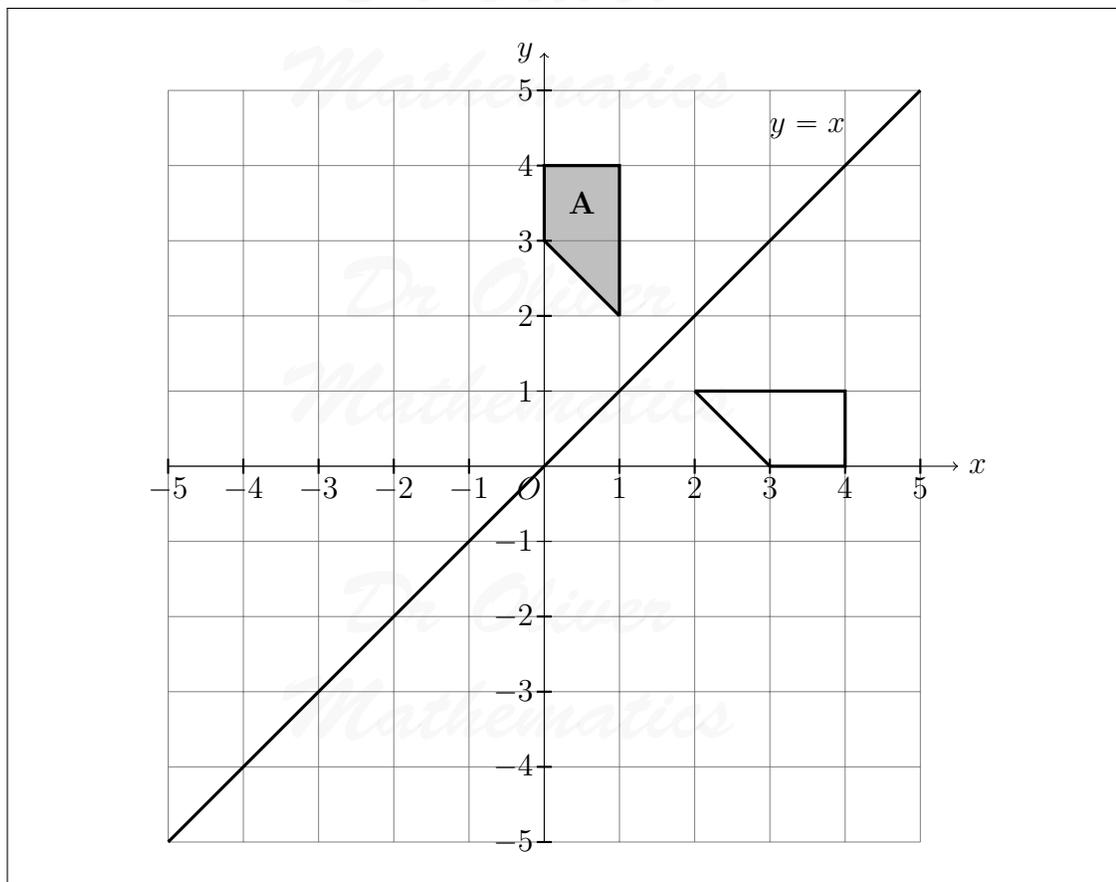
$$\frac{p^6}{p^2} = p^{6-2} = \underline{\underline{p^4}}.$$

4. (a) On the grid, reflect shape **A** in the line $y = x$.

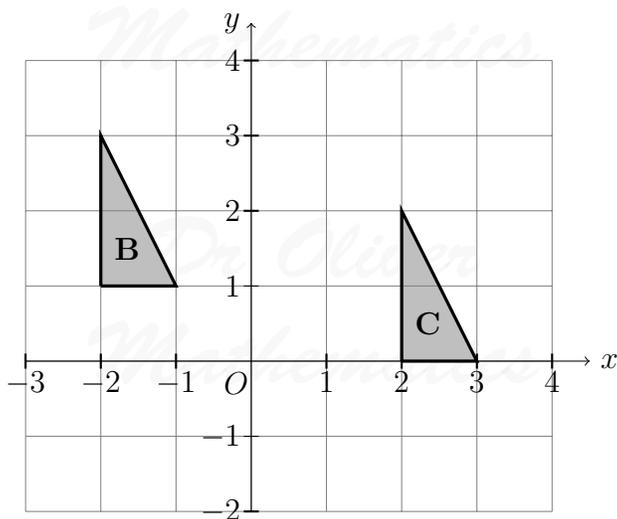
(2)



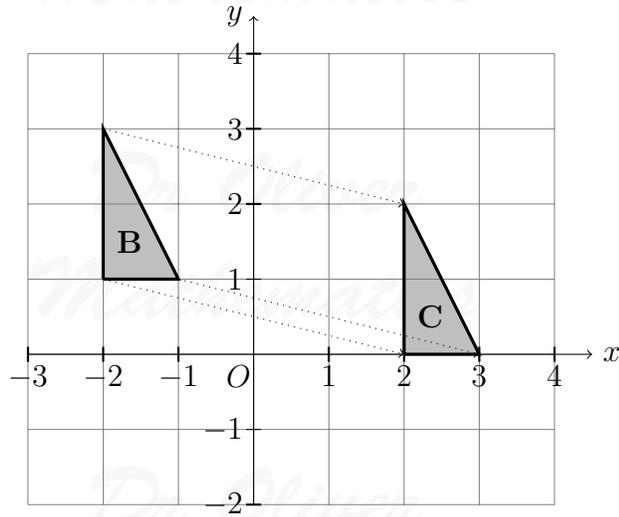
Solution



(b) Describe fully the single transformation that maps triangle **B** onto triangle **C**. (2)



Solution



Translation by $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$.

5. There are some green counters, some yellow counters, some blue counters, and some red counters in a bag.

The table shows the probabilities that a counter taken at random from the bag will be green or yellow or red.

Colour	Green	Yellow	Blue	Red
Probability	0.16	0.4		0.24

Mary takes at random a counter from the bag.

- (a) Work out the probability that the counter will be blue.

(2)

Solution

Let the probability that the counter will be blue be x . Now,

$$\begin{aligned} 0.16 + 0.4 + x + 0.24 &= 1 \Rightarrow 0.8 + x = 1 \\ &\Rightarrow \underline{x = 0.2}. \end{aligned}$$

Mary puts the counter back into the bag.

There are 125 counters in the bag.

- (b) Work out the number of green counters in the bag. (2)

Solution

$$125 \times 0.16 = \underline{\underline{20}}.$$

6. Margaret is on holiday in France. (3)

She buys an English newspaper.

The cost of the newspaper is 5 euros.

In England, the cost of the same newspaper is £2.50.

The exchange rate is £1 = 1.16 euros.

Work out the difference between the cost of the newspaper in France and the cost of the newspaper in England.

Solution

In pounds, the newspaper is

$$\frac{5}{1.16} = \text{£}4.310\dots$$

and the difference is

$$4.31 - 2.5 = \underline{\underline{\text{£}1.81}}.$$

7. Here are the first five terms of an arithmetic sequence.

2 6 10 14 18

- (a) Write down an expression, in terms of n , for the n th term of this sequence. (2)

Solution

Let the

$$nth \text{ term} = an + b.$$

$$\begin{array}{cccc} 2 & 6 & 10 & 14 \\ 4 & & 4 & 4 \\ a + b & 2a + b & 3a + b & 4a + b \\ a & a & a & \end{array}$$

We compare terms:

$$a = 4$$

and

$$\begin{aligned} a + b = 2 &\Rightarrow 4 + b = 2 \\ &\Rightarrow b = -2. \end{aligned}$$

Hence,

$$nth \text{ term} = \underline{\underline{4n - 2}}.$$

(b) Is 86 a term in the sequence?

You must give a reason for your answer.

(1)

Solution

$$\begin{aligned} 4n - 2 = 86 &\Rightarrow 4n = 88 \\ &\Rightarrow n = 22; \end{aligned}$$

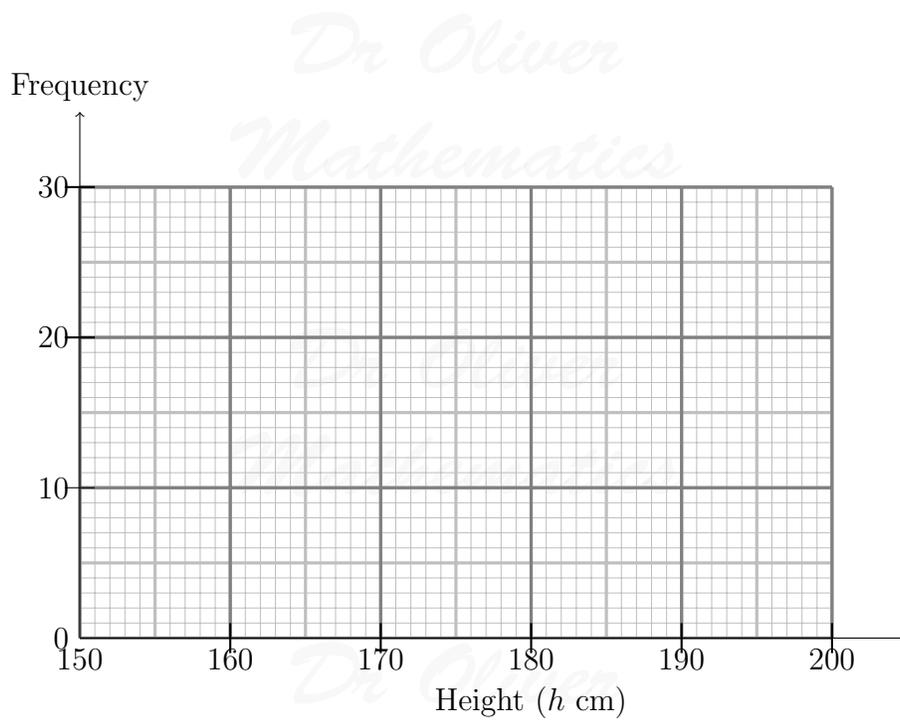
so, yes, it is 22nd term.

8. The frequency table gives information about the heights of some people.

(2)

Height (h cm)	Frequency
$160 < h \leq 165$	2
$165 < h \leq 170$	5
$170 < h \leq 175$	10
$175 < h \leq 180$	21
$180 < h \leq 185$	16
$185 < h \leq 190$	4

Draw a frequency polygon for this information.

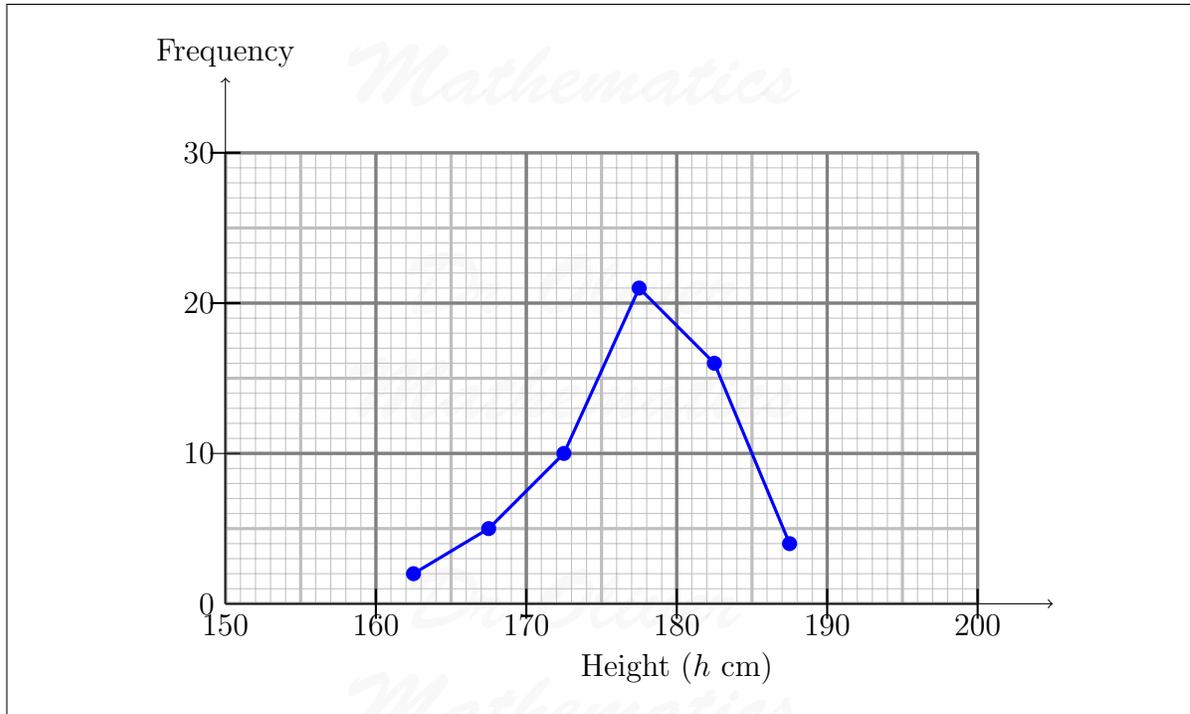


Solution

Height (h cm)	Midpoint	Frequency
$160 < h \leq 165$	162.5	2
$165 < h \leq 170$	167.5	5
$170 < h \leq 175$	172.5	10
$175 < h \leq 180$	177.5	21
$180 < h \leq 185$	182.5	16
$185 < h \leq 190$	187.5	4

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9. The table gives some information about student attendance at a school on Friday. (3)

Year	Number of students		
	Present	Absent	Total
Year 7	192	16	208
Year 8	219	22	241
Year 9	234	28	262
Year 10	233	28	261
Year 11	214	24	238

The school has a target of 94% of students being present each day.

Did the school meet its target on Friday?

Solution

Year	Number of students		
	Present	Absent	Total
Year 7	192	16	208
Year 8	219	22	241
Year 9	234	28	262
Year 10	233	28	261
Year 11	214	24	238
Total	1 092	118	1 210

On Friday, the school had

$$\frac{1\,092}{1\,210} \times 100\% = 90.247\dots\%$$

so the school did not meet its target.

10. The equation

$$x^3 - 2x = 125$$

(4)

has a solution between 5 and 6.

Use a trial and improvement method to find this solution.

Give your answer correct to 1 decimal place.

You must show all your working.

Solution

You must be in TABLE mode; on my calculator (Casio fx-991) it is Mode 3.

F(X)= and you type in $X^3 + 2X$; then you press [=].

Start? and you enter 5; then you press [=].

End? and you enter 6; then you press [=].

Step? and enter 0.05 – 1 decimal place divided by 2; then you press [=].

x	$f(x)$	Comment
5.1	122.45	too low
5.15	126.29	too high

Clearly,

$$5.1 < x < 5.15$$

and the answer is

$$\underline{\underline{x = 5.1 \text{ (1 dp)}}}$$

11. Saphia is organising a conference.
People at the conference will sit at circular tables.

(4)



Each table has a diameter of 140 cm.
Each person needs 60 cm around the circumference of the table.

There are 12 of these tables in the conference room.
A total of 90 people will be at the conference.

Are there enough tables in the conference room?

Solution

Well, the circumference is

$$\pi \times 140 = 439.822\,971\,5 \text{ cm (FCD)}$$

and

$$\frac{439.822\dots}{60} = 7.330\dots;$$

so, we get 7 chairs around the table. Now, there are spaces for

$$12 \times 7 = 84;$$

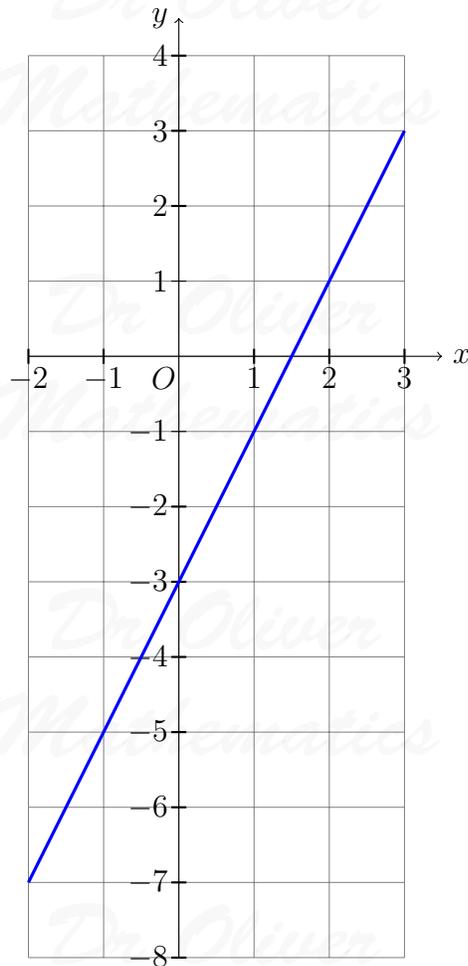
so, no, there are not enough tables in the conference room.

12. Draw the graph of $y = 2x - 3$ for values of x from -2 to 3 .

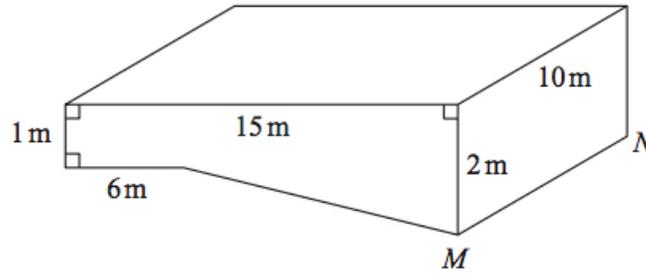
(4)

Solution

x	-2	-1	0	1	2	3
y	-7	-5	-3	-1	1	3



13. The diagram shows a swimming pool in the shape of a prism.



The swimming pool is empty.

The swimming pool is filled with water at a constant rate of 50 litres per minute.

- (a) Work out how long it will take for the swimming pool to be completely full of water. (5)
 Give your answer in hours.
 ($1 \text{ m}^3 = 1000 \text{ litres}$)

Solution

Now,

$$\begin{aligned} \text{cross-sectional area} &= (1 \times 15) + \left[\frac{1}{2} \times (15 - 6) \times (2 - 1)\right] \\ &= 15 + 4.5 \\ &= 19.5 \text{ m}^2 \end{aligned}$$

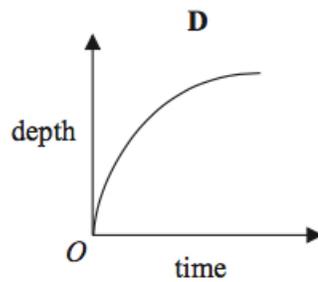
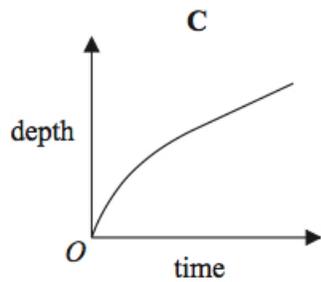
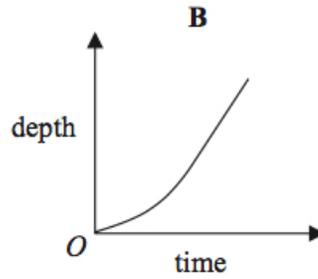
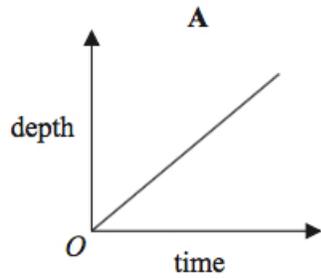
and

$$\begin{aligned} \text{volume} &= 19.5 \times 10 \\ &= 195 \text{ m}^3. \end{aligned}$$

Hence, the time for the swimming pool to be completely full of water is

$$\begin{aligned} \frac{195 \text{ m}^3}{50 \text{ litres a minute}} &= \frac{195 \text{ m}^3}{0.05 \text{ m}^3 \text{ a minute}} \\ &= 3900 \text{ minutes} \\ &= \underline{\underline{65 \text{ hours}}}. \end{aligned}$$

Here are four graphs.



- (b) Write down the letter of the graph that best shows how the depth of the water in the pool above the line MN changes with time as the pool is filled. (1)

Solution

C.

14. Peter has £20 000 to invest in a savings account for 2 years. (4)

He finds information about two savings accounts.

<p>Bonus Saver</p> <p>Compound interest</p> <p>4% for the first year then 1.5% each year</p>

<p>Fixed Rate</p> <p>Compound interest</p> <p>2.5% each year</p>

Peter wants to have as much money as possible in his savings account at the end of 2 years.

Which of these savings accounts should he choose?

Solution

Bonus Saver:

$$20\,000 \times 1.04 \times 1.015 = 21\,112.$$

Fixed Rate:

$$20\,000 \times 1.025^2 = 21\,012.5.$$

Peter should invest in the Bonus Saver.

15. A cinema sells adult tickets and child tickets.

(4)

The total cost of 3 adult tickets and 1 child ticket is £30.

The total cost of 1 adult ticket and 3 child tickets is £22.

Work out the cost of an adult ticket and the cost of a child ticket.

Solution

Let a and c be the costs of an adult tickets and a child ticket respectively. Then

$$3a + c = 30 \quad (1)$$

$$a + 3c = 22 \quad (2)$$

Now, $3 \times (2)$:

$$3a + c = 30 \quad (1)$$

$$3a + 9c = 66 \quad (3)$$

Subtract:

$$8c = 36 \Rightarrow \underline{c = £4.50}$$

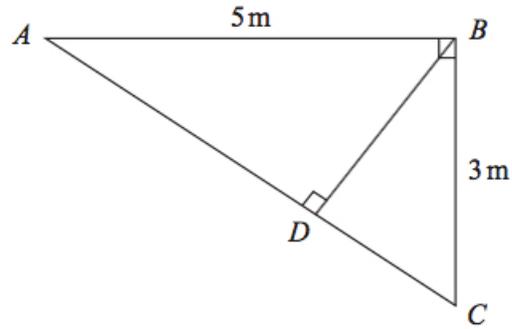
$$\Rightarrow 3a + 4.5 = 30$$

$$\Rightarrow 3a = 25.5$$

$$\Rightarrow \underline{a = £8.50}.$$

16. The diagram represents a metal frame.

(5)



The frame is made from four metal bars: AB , AC , BC , and BD .

Angle $ABC = \text{angle } ADB = 90^\circ$.

$AB = 5 \text{ m}$.

$BC = 3 \text{ m}$.

Work out the total length of the four metal bars of the frame.

Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned} AC &= \sqrt{AB^2 + BC^2} \\ &= \sqrt{5^2 + 3^2} \\ &= 5.830951895 \text{ (FCD)}. \end{aligned}$$

Now,

$$\sin ACB = \frac{5}{5.830\dots} \text{ and } \sin BCD = \frac{BD}{3}$$

and, yet,

$$\angle ACB = \angle BCD$$

are the same:

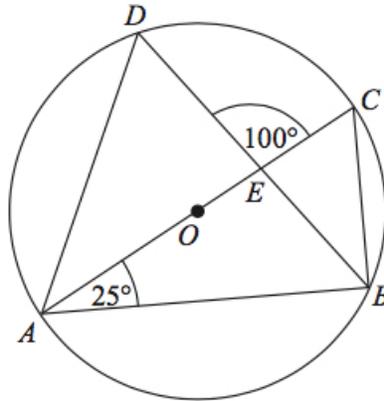
$$\begin{aligned} \frac{5}{5.830\dots} &= \frac{BD}{3} \Rightarrow BD = \frac{15}{5.830\dots} \\ &= 2.572478777 \text{ (FCD)}. \end{aligned}$$

Finally,

$$\begin{aligned} \text{total length} &= 5 + 3 + 5.830\dots + 2.572\dots \text{ (FCD)} \\ &= 16.40343067 \text{ (FCD)} \\ &= \underline{\underline{16.4 \text{ m (3 sf)}}}. \end{aligned}$$

17. A , B , C , and D are points on the circumference of a circle, centre O .

(4)



AC is a diameter of the circle.
 AC and BD intersect at E .

Angle $CAB = 25^\circ$.
 Angle $DEC = 100^\circ$.

Work out the size of angle DAC .
 You must show all your working.

Solution

$\angle AEB = 100^\circ$ (vertically opposite angles)
 $\angle ABE = 180 - 25 - 100 = 55^\circ$ (completing the triangle)
 $\angle EBC = 90 - 55 = 35^\circ$ (angle in a semi-circle)
 $\angle DAC = \underline{\underline{35^\circ}}$ (angles in the same segment)

18. The table gives some information about the lengths of time, in hours, that some adults watched TV last week.

Length of time (t hours)	Frequency
$0 \leq t < 10$	8
$10 \leq t < 15$	15
$15 \leq t < 20$	11
$20 \leq t < 30$	10
$30 \leq t < 50$	6

(a) Work out an estimate for the mean length of time.

(4)

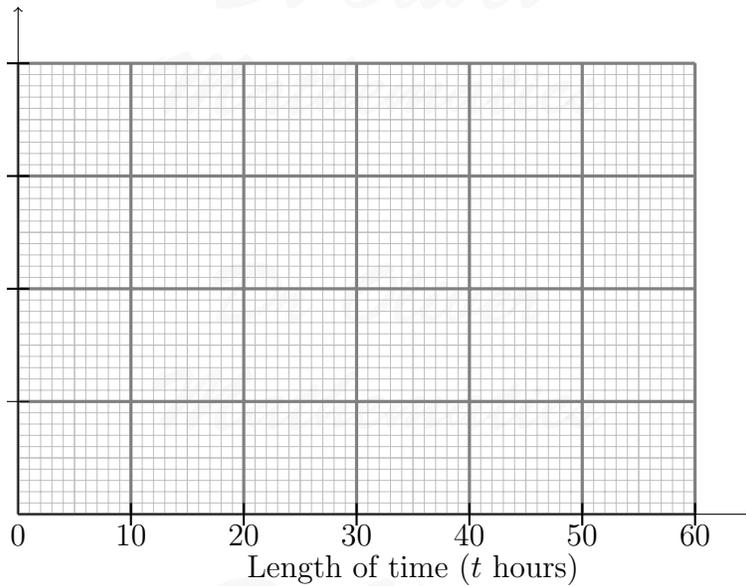
Solution

Length of time (t hours)	Frequency	Midpoint	Freq \times Midpoint
$0 \leq t < 10$	8	5	$8 \times 5 = 40$
$10 \leq t < 15$	15	12.5	$15 \times 12.5 = 187.5$
$15 \leq t < 20$	11	17.5	$11 \times 17.5 = 192.5$
$20 \leq t < 30$	10	25	$10 \times 25 = 250$
$30 \leq t < 50$	6	40	$6 \times 40 = 240$
Total	50		910

$$\begin{aligned} \text{Mean} &\approx \frac{\sum fx}{\sum f} \\ &= \frac{910}{50} \\ &= \underline{\underline{18.2 \text{ minutes.}}} \end{aligned}$$

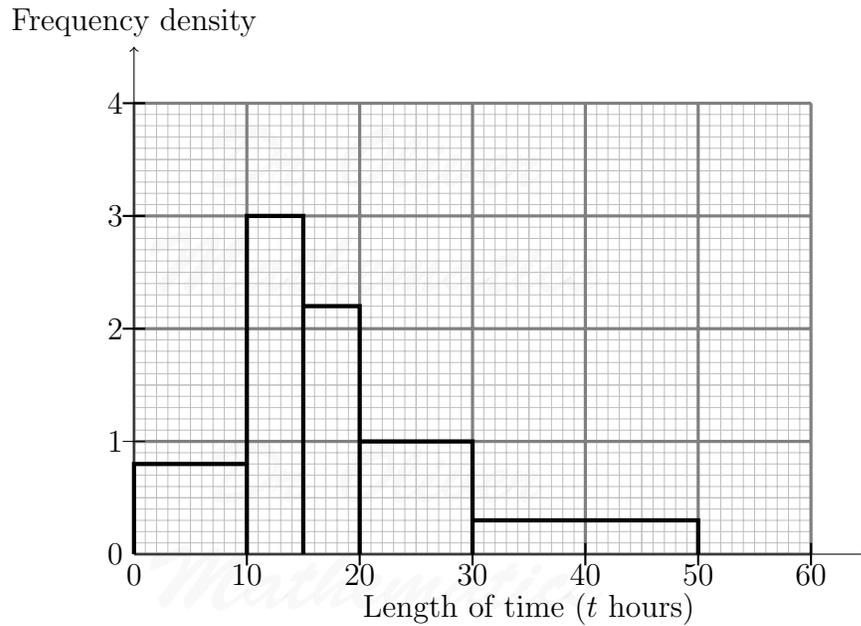
(b) Draw a histogram for the information in the table.

(3)



Solution

Length of time (t hours)	Frequency	Width	Frequency Density
$0 \leq t < 10$	8	10	$\frac{8}{10} = 0.8$
$10 \leq t < 15$	15	5	$\frac{15}{5} = 3$
$15 \leq t < 20$	11	5	$\frac{11}{5} = 2.2$
$20 \leq t < 30$	10	10	$\frac{10}{10} = 1$
$30 \leq t < 50$	6	20	$\frac{6}{20} = 0.3$



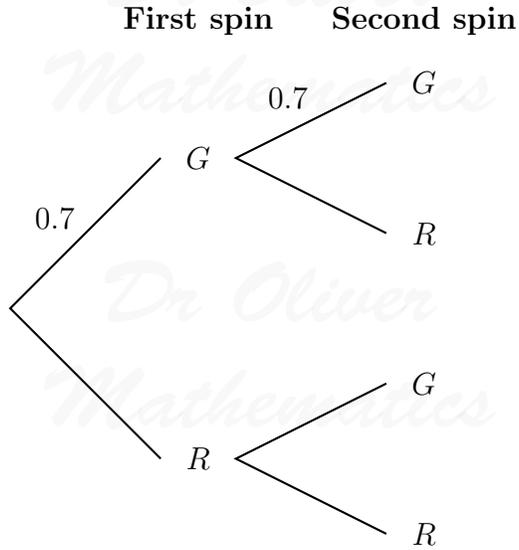
19. Louise makes a spinner.

The spinner can land on green or on red.
 The probability that the spinner will land on green is 0.7.

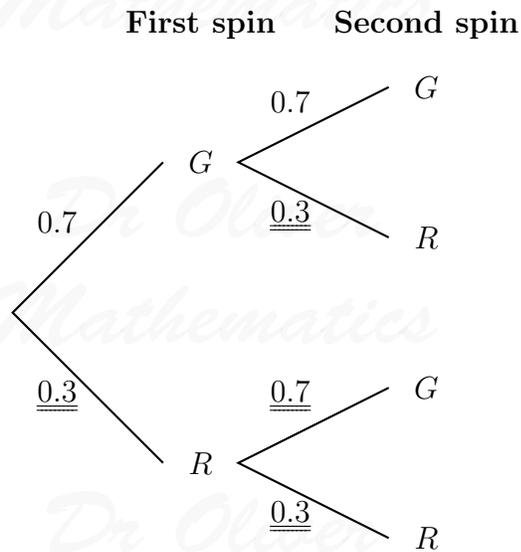
Louise spins the spinner twice.

(a) Complete the probability tree diagram.

(2)



Solution



(b) Work out the probability that the spinner lands on two different colours.

(3)

Solution

$$\begin{aligned}
 P(\text{two different colours}) &= P(GR) + P(RG) \\
 &= (0.7 \times 0.3) + (0.3 \times 0.7) \\
 &= 0.21 + 0.21 \\
 &= \underline{\underline{0.42}}.
 \end{aligned}$$

20. Solve

$$3x^2 - 5x - 1 = 0.$$

(3)

Give your solutions correct to 3 significant figures.

Solution

$a = 3$, $b = -5$, and $c = -1$:

$$\begin{aligned}
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{5 \pm \sqrt{(-5)^2 - 4 \times 3 \times (-1)}}{2 \times 3} \\
 &= \frac{5 \pm \sqrt{37}}{6} \\
 &= -0.180\ 460\ 421\ 7 \text{ or } 1.847\ 127\ 088 \text{ (FCD)} \\
 &= \underline{\underline{-0.180 \text{ or } 1.85 \text{ (3 sf)}}}.
 \end{aligned}$$

21. ABC is a triangle.

(5)

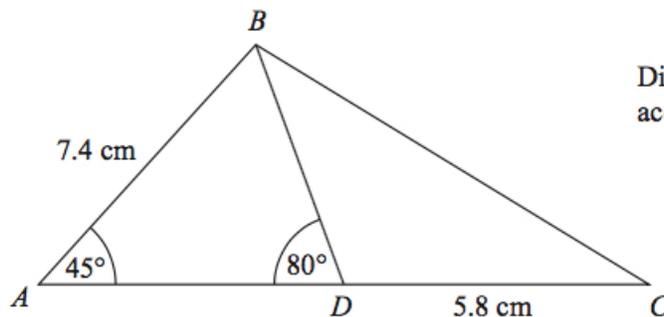


Diagram **NOT**
accurately drawn

D is a point on AC .
Angle $BAD = 45^\circ$.

Angle $ADB = 80^\circ$.
 $AB = 7.4$ cm.
 $DC = 5.8$ cm.

Work out the length of BC .
 Give your answer correct to 3 significant figures.

Solution

$$\frac{BD}{\sin 45^\circ} = \frac{7.4}{\sin 80^\circ} \Rightarrow BD = \frac{7.4 \sin 45^\circ}{\sin 80^\circ}$$

$$\Rightarrow BD = 5.313\ 311\ 319 \text{ (FCD).}$$

Now,

$$\angle BDC = 180 - 80 = 100^\circ \text{ (supplementary angles)}$$

and so

$$BC = \sqrt{BD^2 + CD^2 - 2 \cdot BD \cdot CD \cdot \cos BDC}$$

$$= \sqrt{5.313\dots^2 + 5.8^2 - 2 \times 5.313\dots \times 5.8 \times \cos 100^\circ}$$

$$= 8.519\ 036\ 352 \text{ (FCD)}$$

$$= \underline{\underline{8.52 \text{ cm (3 sf)}}}.$$

22. (a) Simplify fully

$$\frac{2x^2 - 5x + 3}{x^2 + 5x - 6}$$

(3)

Solution

$$\left. \begin{array}{l} \text{add to:} \quad -5 \\ \text{multiply to: } (+2) \times (+3) = +6 \end{array} \right\} -2, -3$$

$$2x^2 - 5x + 3 = 2x^2 - 2x - 3x + 3$$

$$= 2x(x - 1) - 3(x - 1)$$

$$= (2x - 3)(x - 1).$$

$$\left. \begin{array}{l} \text{add to:} \quad +5 \\ \text{multiply to: } -6 \end{array} \right\} +6, -1$$

$$x^2 + 5x - 6 = (x + 6)(x - 1).$$

Hence,

$$\begin{aligned}\frac{2x^2 - 5x + 3}{x^2 + 5x - 6} &= \frac{(2x - 3)(x - 1)}{(x + 6)(x - 1)} \\ &= \frac{2x - 3}{\underline{\underline{x + 6}}}\end{aligned}$$

(b) Make m the subject of

$$\frac{m}{v} - \frac{t}{b} = \frac{m - t}{R}.$$

(4)

Solution

$\times bvR$:

$$\begin{aligned}\frac{m}{v} - \frac{t}{b} &= \frac{m - t}{R} \\ \Rightarrow mbR - tvR &= bv(m - t) \\ \Rightarrow mbR - tvR &= mbv - btv \\ \Rightarrow mbR - mbv &= tvR - btv \\ \Rightarrow mb(R - v) &= tv(R - b) \\ \Rightarrow m &= \frac{tv(R - b)}{\underline{\underline{b(R - v)}}}\end{aligned}$$

23. A road is 4530 m long, correct to the nearest 10 metres.

Kirsty drove along the road in 205 seconds, correct to the nearest 5 seconds.

(5)

The average speed limit for the road is 80 km/h.

Could Kirsty's average speed have been greater than 80 km/h?

You must show your working.

Solution

$$4525 \leq \text{metres} < 4535$$

and

$$202.5 \leq \text{seconds} < 207.5.$$

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Now,

$$\begin{aligned}\text{upper bound} &= \frac{4\,535 \text{ metres}}{202.5 \text{ seconds}} \\ &= \frac{4.535 \text{ km}}{3\frac{3}{8} \text{ minutes}} \\ &= \frac{4.535 \text{ km}}{\frac{9}{160} \text{ hours}} \\ &= 80\frac{28}{45} \text{ km/h}\end{aligned}$$

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so the average speed could have been higher.

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