Dr Oliver Mathematics
Worked Examples
Frustum 1

From: IGCSE 2018 June 1H, Q16

1. A frustum is made by removing a small cone from a large cone.
The cones are mathematically similar.

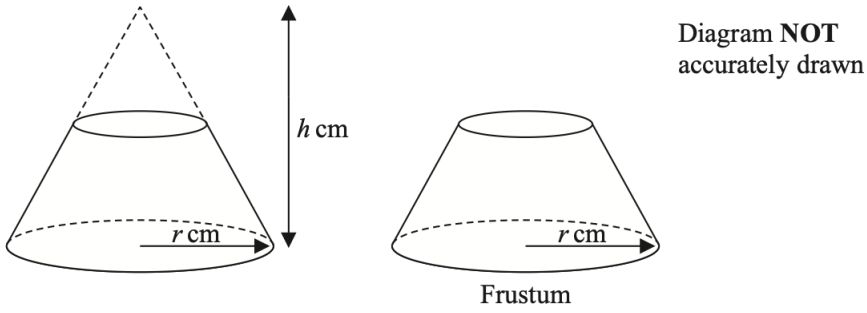


Diagram **NOT** accurately drawn

The large cone has base radius r cm and height h cm.

Given that

$$\frac{\text{volume of frustum}}{\text{volume of large cone}} = \frac{98}{125},$$

find an expression, in terms of h, for the height of the frustum.

Solution

Imagine if we had a little cone sitting atop the frustum, giving us one large cone:

$$\text{little cone} + \text{frustum} = \text{large cone}.$$

Now,

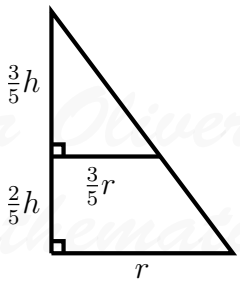
$$\begin{aligned} & \frac{\text{volume of frustum}}{\text{volume of large cone}} = \frac{98}{125} \\ \Rightarrow \quad & \frac{\text{volume of little cone}}{\text{volume of large cone}} = 1 - \frac{98}{125} \\ \Rightarrow \quad & \frac{\text{volume of little cone}}{\text{volume of large cone}} = \frac{27}{125}. \end{aligned}$$

Next, VSF (volume scale factor) is

$$\frac{27}{125} = \frac{3^3}{5^5}$$

which means the LSF (length scale factor) is $\frac{3}{5}$.

So, we have a picture:



Hence, the height of the frustum is $\underline{\underline{\frac{2}{5}h}}$.