Dr Oliver Mathematics

Other Normal Distributions

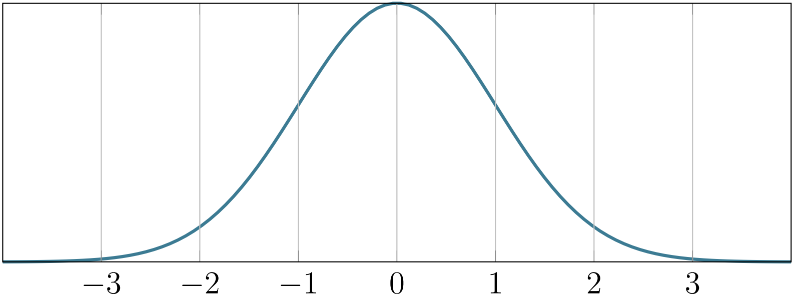
April 2014

The Equation of a Normal Distribution

Let

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}.$$

Then the function $f(z)$ has the following graph:



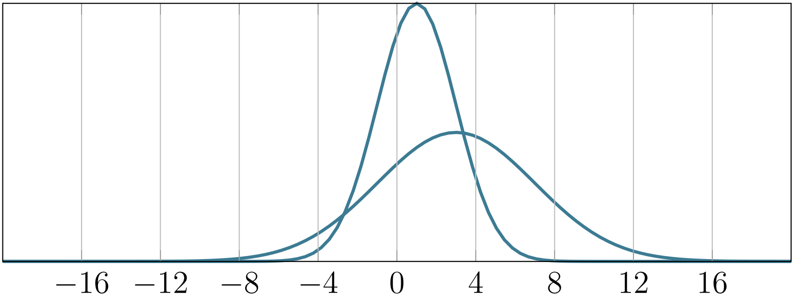
The Notation for a Normal Distribution

We use

$$X \sim \mathrm{N}(\mu, \sigma^2)$$

to represent the standard normal distribution that has a mean of μ and a standard deviation of σ.

The Equation of a Normal Distribution



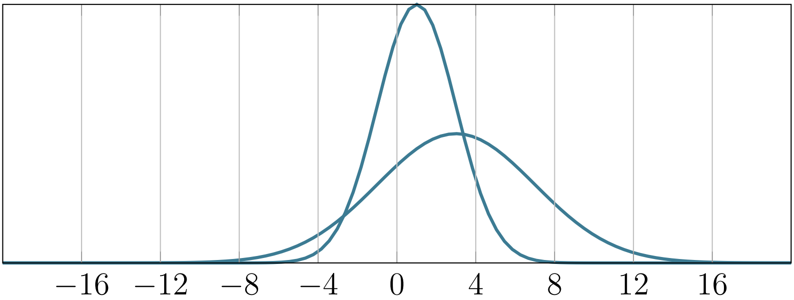
The above picture shows two different normal distributions:

$$X \sim \mathrm{N}(1, 2^2)$$

and

$$Y \sim \mathrm{N}(3, 4^2).$$

The Equation of a Normal Distribution



1. Each of $X \sim \mathrm{N}(1, 2^2)$ and $Y \sim \mathrm{N}(3, 4^2)$ peaks at its mean, just as $Z \sim \mathrm{N}(0, 1)$ peaked at 0.
2. The greater the standard deviation, the wider the graph of the normal distribution.
3. Since the area under each graph 1 is, the wider the graph, the lower the peak at the mean.

Key Results from Chapter 8

Recall from Chapter 8 that, for any random variable X and any constants a and b we have

$$\mathrm{E}(aX + b) = a\,\mathrm{E}(X) + b,$$

and

$$\mathrm{Var}(aX + b) = a^2\,\mathrm{Var}(X).$$

Standardisation

Suppose that $X \sim \mathrm{N}(\mu, \sigma^2)$, i.e., X has mean μ and variance σ^2. Then we can show that the random variable

$$\frac{X - \mu}{\sigma}$$

has mean 0 and variance 1, i.e., this is the standard normal distribution. This process is called standardisation and will allow us to convert any normal distribution to $Z \sim \mathrm{N}(0, 1)$ so that we can use our tables.

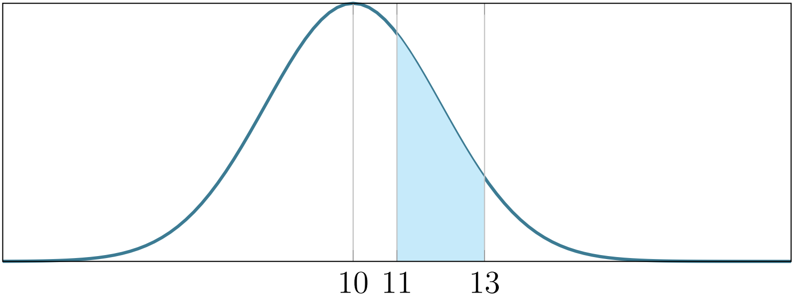
Standardisation: the mean is 0

$$
\begin{aligned}
\mathrm{E}\left(\frac{X-\mu}{\sigma}\right) &= \frac{1}{\sigma}\mathrm{E}(X-\mu) \\
&= \frac{1}{\sigma}\left[\mathrm{E}(X)-\mu\right] \\
&= \frac{1}{\sigma}\left[\mu-\mu\right] \\
&= 0.
\end{aligned}
$$

Standardisation: the variance is 1

$$
\begin{aligned}
\operatorname{Var}\left(\frac{X-\mu}{\sigma}\right) &= \frac{1}{\sigma^2}\operatorname{Var}(X-\mu) \\
&= \frac{1}{\sigma^2}\operatorname{Var}(X) \\
&= \frac{1}{\sigma^2} \times \sigma^2 \\
&= 1.
\end{aligned}
$$

Let $X \sim \mathrm{N}(10, 4)$. Find $\mathrm{P}(11 < X < 13)$



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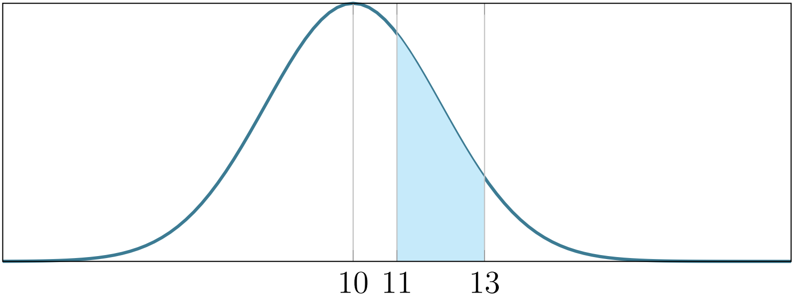

$$\mathrm{P}(11 < X < 13) = \mathrm{P}\left(\frac{11-10}{2} < Z < \frac{13-10}{2}\right)$$

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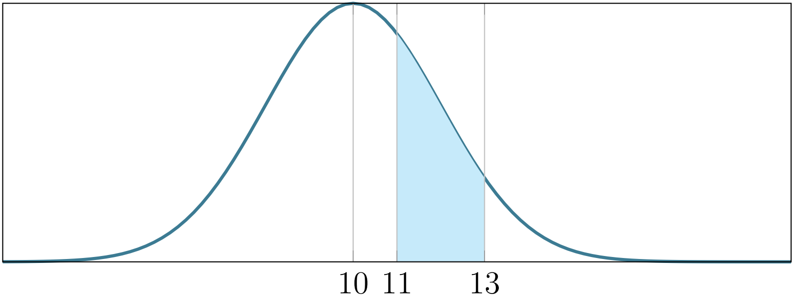

$$\mathrm{P}(11 < X < 13) = \mathrm{P}\left(\frac{11-10}{2} < Z < \frac{13-10}{2}\right)$$
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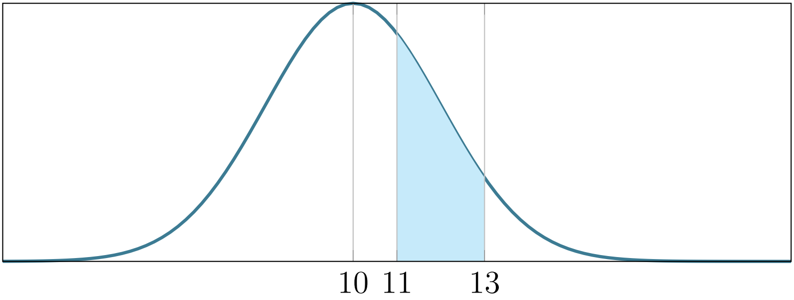

$$\begin{aligned}\mathrm{P}(11 < X < 13) &= \mathrm{P}\left(\frac{11-10}{2} < Z < \frac{13-10}{2}\right) \\ &= \mathrm{P}(0.5 < Z < 1.5) \\ &= \Phi(1.5) - \Phi(0.5)\end{aligned}$$

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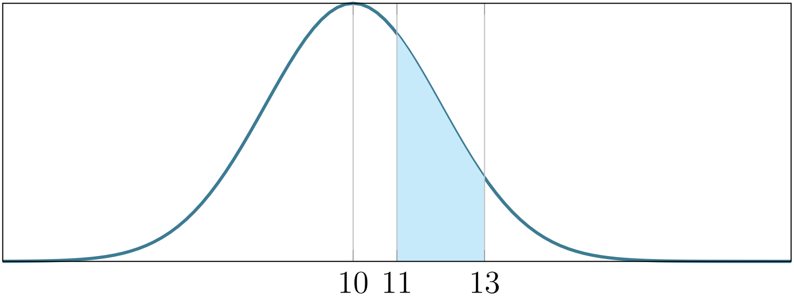

$$\begin{aligned} \mathrm{P}(11 < X < 13) &= \mathrm{P}\left(\frac{11-10}{2} < Z < \frac{13-10}{2}\right) \\ &= \mathrm{P}(0.5 < Z < 1.5) \\ &= \Phi(1.5) - \Phi(0.5) \\ &= 0.9932 - 0.6915 \end{aligned}$$

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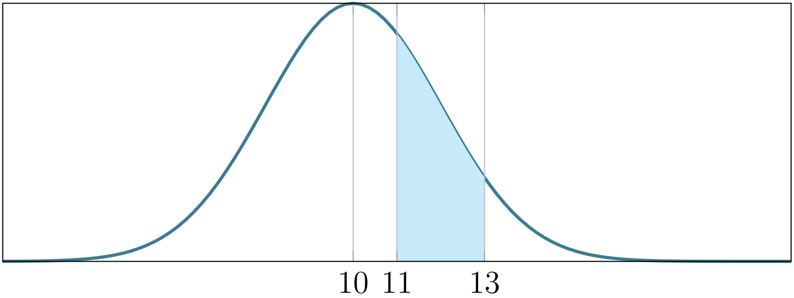

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\mathrm{P}(11 < X < 13) &= \mathrm{P}\left(\frac{11-10}{2} < Z < \frac{13-10}{2}\right) \\
&= \mathrm{P}(0.5 < Z < 1.5) \\
&= \Phi(1.5) - \Phi(0.5) \\
&= 0.9932 - 0.6915 \\
&= \underline{\underline{0.3017}}.
\end{aligned}$$

And over to you . . .

Exercise 9C (page 184)

Q1-10

- Show all of your working.
- Use the tables on pages 201 and 202.