Dr Oliver Mathematics
Mathematics: Higher
2026 Paper 2: Calculator
1 hour 30 minutes

The total number of marks available is 65.
You must write down all the stages in your working.

1. The equation (3)

$$x^2 + kx + (k+3) = 0$$

has equal roots.

Find, algebraically, the values of k.

Solution

Well,

$$\begin{aligned} b^2 - 4ac = 0 &\Rightarrow k^2 - 4(1)(k+3) = 0 \\ &\Rightarrow k^2 - 4k - 12 = 0 \end{aligned}$$

$$\left.\begin{array}{ll} \text{add to:} & -4 \\ \text{multiply to:} & -12 \end{array}\right\} +2,\ -6$$

$$\begin{aligned} &\Rightarrow (k+2)(k-6) = 0 \\ &\Rightarrow k+2 = 0 \text{ or } k-6 = 0 \\ &\Rightarrow \underline{\underline{k = -2 \text{ or } k = 6}}. \end{aligned}$$

2. Vectors $\mathbf{u}$ and $\mathbf{v}$ are defined as

$$\mathbf{u} = \begin{pmatrix} 3 \\ 2 \\ 6 \end{pmatrix} \text{ and } \mathbf{v} = \begin{pmatrix} -5 \\ -2 \\ 4 \end{pmatrix}.$$

(a) Find $\mathbf{u}.\mathbf{v}$. (1)

Solution
Well,

$$\begin{aligned}\mathbf{u}.\mathbf{v} &= -15 - 4 + 24 \\ &= \underline{\underline{5}}.\end{aligned}$$

(b) Calculate the acute angle between $\mathbf{u}$ and $\mathbf{v}$. (4)

Solution
Now,

$$\begin{aligned}|\mathbf{u}| &= \sqrt{3^2 + 2^2 + 6^2} \\ &= 7\end{aligned}$$

and

$$\begin{aligned}|\mathbf{v}| &= \sqrt{(-5)^2 + (-2)^2 + 4^2} \\ &= 3\sqrt{5}.\end{aligned}$$

Let θ be the acute angle between $\mathbf{u}$ and $\mathbf{v}$. Then,

$$\begin{aligned}\cos\theta = \frac{\mathbf{u}.\mathbf{v}}{|\mathbf{u}||\mathbf{v}|} &\Rightarrow \cos\theta = \frac{5}{7 \times 3\sqrt{5}} \\ &\Rightarrow \theta = 83.887\,590\,6 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{\theta = 83.9^\circ \text{ (1 dp)}}}.\end{aligned}$$

3. A function, g, is defined on $\mathbb{R}$, the set of real numbers, by (3)

$$\mathrm{g}(x) = \tfrac{1}{5}x + 6.$$

Find the inverse function, $\mathrm{g}^{-1}(x)$.

Solution
Now,

$$\begin{aligned}y = \tfrac{1}{5}x + 6 &\Rightarrow y - 6 = \tfrac{1}{5}x \\ &\Rightarrow 5(y-6) = x,\end{aligned}$$

and so

$$\mathrm{g}^{-1}(x) = \underline{\underline{5(x-6)}}.$$

4. A circle has centre $(-2, 5)$. (2)

   The point $(3, 7)$ lies on the circle.

   Find the equation of the circle.

Solution

Well, the equation of the circle is

$$\begin{aligned} & [x-(-2)]^2 + (y-5)^2 = [3-(-2)]^2 + (7-5)^2 \\ \Rightarrow\quad & (x+2)^2 + (y-5)^2 = 5^2 + 2^2 \\ \Rightarrow\quad & \underline{\underline{(x+2)^2 + (y-5)^2 = 29}}. \end{aligned}$$

5. A sequence is defined by the recurrence relation

$$u_{n+1} = 1.125\,u_n + 5,\ u_0 = 2.$$

   (a) Calculate the value of u_1. (1)

Solution

Well,

$$\begin{aligned} u_1 &= 1.125\,u_0 + 5 \\ &= 1.125(2) + 5 \\ &= \underline{\underline{7.25}}. \end{aligned}$$

   (b) Explain why this sequence does not approach a limit as $n \to \infty$. (1)

Solution

E.g., $|1.125| > 1$;
is it because of 1.125 that multiplies u_n: it ensures that it will go off to infinity.

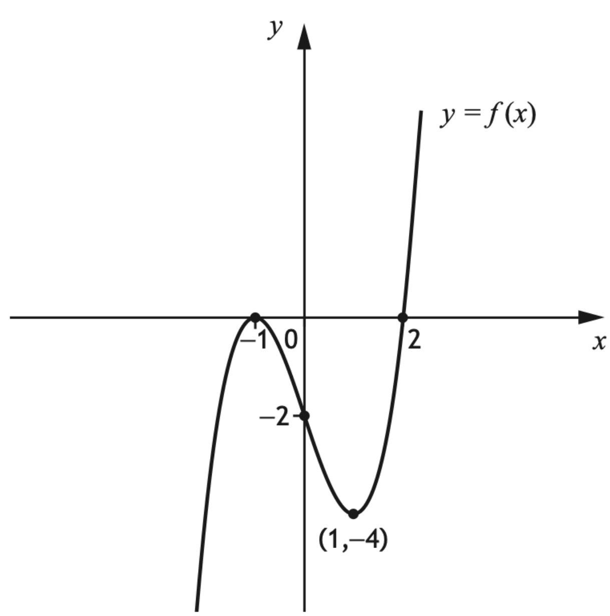
(c) Determine the smallest value of n for which $u_n > 30$. (2)

Solution
Now,

$$\begin{aligned} u_2 &= 1.125\,u_1 + 5 \\ &= 1.125(7.25) + 5 \\ &= 13\tfrac{5}{32}; \\ u_3 &= 1.125(13\tfrac{5}{32}) + 5 \\ &= 19\tfrac{205}{256}; \\ u_4 &= 1.125(19\tfrac{205}{256}) + 5 \\ &= 27.275\ldots; \\ u_5 &= 1.125(27.275\ldots) + 5 \\ &= 35.685\ldots; \end{aligned}$$

hence, $\underline{\underline{n = 5}}$.

6. The diagram shows the graph of a cubic function $y = \mathrm{f}(x)$ with stationary points at $(-1, 0)$ and $(1, -4)$. (3)



On the diagram in your answer booklet, sketch the graph of

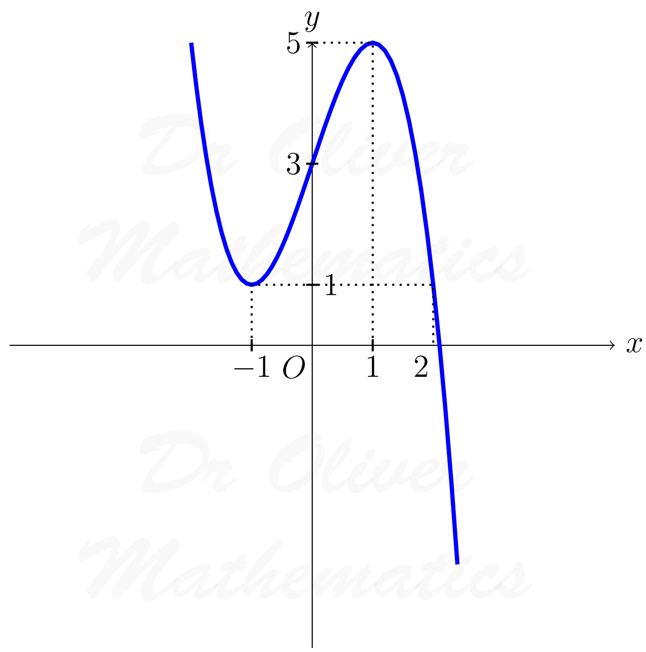
$$y = -\,\mathrm{f}(x) + 1.$$

Solution

Well, $y = -\mathrm{f}(x)$ has a reflection in the x-axis which means it goes through the points $(-1, 0)$, $(0, 2)$, $(1, 4)$, and $(2, 0)$.

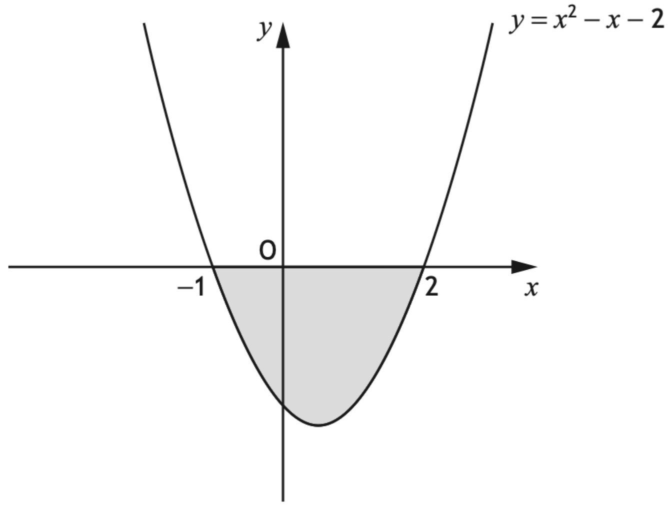
Now, $y = -\mathrm{f}(x) + 1$ has a translation one unit up which means it goes through the points $(-1, 1)$, $(0, 3)$, $(1, 5)$, and $(2, 1)$.

Hence, here is the graph:



7. The diagram shows the curve with equation (4)

$$y = x^2 - x - 2.$$



Calculate the shaded area.

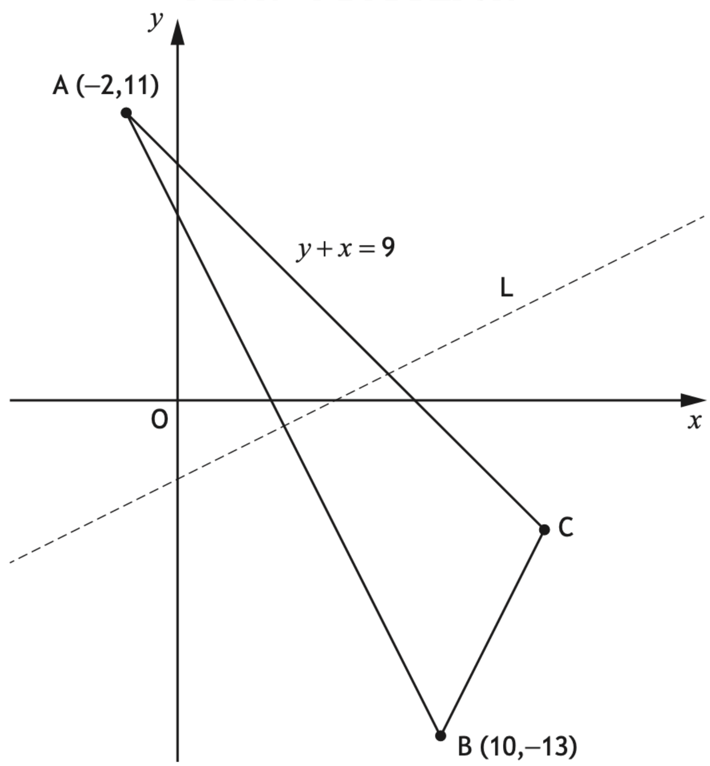
Solution

Well,

$$\begin{aligned}\int_{-1}^{2} (x^2 - x - 2)\,\mathrm{d}x &= \left[\tfrac{1}{3}x^3 - \tfrac{1}{2}x^2 - 2x\right]_{x=-1}^{2} \\ &= \left(\tfrac{8}{3} - 2 - 4\right) - \left(-\tfrac{1}{3} - \tfrac{1}{2} + 2\right) \\ &= -4\tfrac{1}{2};\end{aligned}$$

hence, the shaded area is $\underline{\underline{4\tfrac{1}{2}}}$.

8. A and B are the points $(-2, 11)$ and $(10, -13)$.



(a) Find the equation of L, the perpendicular bisector of AB. (4)

Solution

Let M be the midpoint of AB. Then,

$$\left(\frac{-2+10}{2}, \frac{11+(-13)}{2}\right) = M(4, -1).$$

Now,

$$\begin{aligned} m_{AB} &= \frac{-13-11}{10-(-2)} \\ &= \frac{-24}{12} \\ &= -2, \end{aligned}$$

and

$$m_{\text{normal}} = \tfrac{1}{2}.$$

Hence, the equation of L, the perpendicular bisector of AB is

$$\begin{aligned} y + 1 = \tfrac{1}{2}(x-4) &\Rightarrow y + 1 = \tfrac{1}{2}x - 2 \\ &\Rightarrow \underline{\underline{y = \tfrac{1}{2}x - 3}}. \end{aligned}$$

The equation of AC is

$$y + x = 9.$$

(b) Calculate the size of the obtuse angle between L and AC. (2)

Solution

Well,

$$y + x = 9 \Rightarrow y = -x + 9.$$

Now, the angle between AC and the x-axis is 135° (why?) and the angle between L and the x-axis is

$$\tan^{-1} \tfrac{1}{2} = 26.565\,005\,118^\circ \text{ (FCD)}.$$

Finally,

$$\begin{aligned} \text{obtuse angle} &= 135 - 26.565\ldots \\ &= 108.434\,948\,8 \text{ (FCD)} \\ &= \underline{\underline{108^\circ \text{ (3 sf)}}}. \end{aligned}$$

9. Determine the range of values of x for which the function (4)

$$\mathrm{f}(x) = 4x^3 - 12x^2 + 7$$

strictly decreasing.

Justify your answer.

Solution

Well,

$$\mathrm{f}(x) = 4x^3 - 12x^2 + 7 \Rightarrow \mathrm{f}'(x) = 12x^2 - 24x$$

and

$$\begin{aligned} \mathrm{f}'(x) < 0 &\Rightarrow 12x^2 - 24x < 0 \\ &\Rightarrow 12x(x-2) < 0. \end{aligned}$$

We need a ‘table of signs’:

	$x<0$	$x=0$	$0<x<2$	$x=2$	$x>2$
$12x$	$-$	0	$+$	$+$	$+$
$(x-2)$	$-$	$-$	$-$	0	$+$
$12x(x-2)$	$+$	0	$-$	0	$+$

Hence, the range of values of x are

$$\underline{\underline{0 < x < 2}}.$$

10. Solve the equation (5)

$$3\cos 2x^\circ + 10\cos x^\circ = 1, \text{ where } for 0 \leqslant x < 360.$$

Solution

Well,

$$\begin{aligned} 3\cos 2x^\circ + 10\cos x^\circ = 1 &\Rightarrow 3(2\cos^2 x^\circ - 1) + 10\cos x^\circ = 1 \\ &\Rightarrow 6\cos^2 x^\circ - 3 + 10\cos x^\circ - 1 = 0 \\ &\Rightarrow 6\cos^2 x^\circ + 10\cos x^\circ - 4 = 0 \\ &\Rightarrow 2(3\cos^2 x^\circ + 5\cos x^\circ - 2) = 0 \end{aligned}$$

$$\left.\begin{array}{ll} \text{add to:} & +5 \\ \text{multiply to:} & (+3)\times(-2) = -6 \end{array}\right\} +6,\ -1$$

e.g.,

$$\begin{aligned} &\Rightarrow 2[3\cos^2 x^\circ + 6\cos x^\circ - \cos x^\circ - 2] = 0 \\ &\Rightarrow 2[3\cos x^\circ(\cos x^\circ + 2) - (\cos x^\circ + 2)] = 0 \\ &\Rightarrow 2(3\cos x^\circ - 1)(\cos x^\circ + 2) = 0 \\ &\Rightarrow 3\cos x^\circ - 1 = 0 \end{aligned}$$

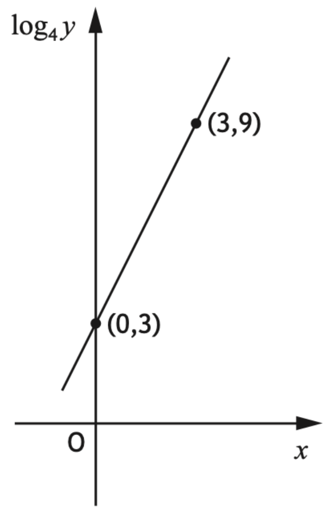
because $\cos x^\circ \neq -2$:

$$\begin{aligned} &\Rightarrow \cos x^\circ = \tfrac{1}{3} \\ &\Rightarrow x = 70.528\,779\,37,\ 289.471\,220\,6 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{x = 70.5,\ 289 \text{ (3 sf)}}}. \end{aligned}$$

11. Two variables x and y are connected by the equation (5)

$$y = ab^x.$$

The graph of $\log_4 y$ against x is a straight line as shown.



Find the values of a and b.

Solution

Well,

$$\begin{aligned} m &= \frac{9-3}{3-0} \\ &= \frac{6}{3} \\ &= 2, \end{aligned}$$

so we have the equation

$$\begin{aligned} \log_4 y - 3 = 2(x-0) &\Rightarrow \log_4 y = 2x + 3 \\ &\Rightarrow y = 4^{2x+3} \\ &\Rightarrow y = 4^3 \times 4^{2x} \\ &\Rightarrow y = 64 \times (4^2)^x \\ &\Rightarrow y = 64 \times 16^x; \end{aligned}$$

hence, $\underline{\underline{a = 64}}$ and $\underline{\underline{b = 16}}$.

12. A function f is defined on the set of real numbers by

$$\mathrm{f}(x) = x^3 + 2x^2 - 4x + 1.$$

(a) Find the x-coordinates of the stationary points on the curve with equation $y = \mathrm{f}(x)$. (3)

Solution

Well,

$$\mathrm{f}(x) = x^3 + 2x^2 - 4x + 1 \Rightarrow \mathrm{f}'(x) = 3x^2 + 4x - 4$$

and

$$\mathrm{f}'(x) = 0 \Rightarrow 3x^2 + 4x - 4 = 0$$

$$\left.\begin{array}{ll} \text{add to:} & +4 \\ \text{multiply to:} & (+3) \times (-4) = -12 \end{array}\right\} +6, -2$$

e.g.,

$$\begin{aligned} &\Rightarrow 3x^2 + 6x - 2x - 4 = 0 \\ &\Rightarrow 3x(x+2) - 2(x+2) = 0 \\ &\Rightarrow (3x-2)(x+2) = 0 \\ &\Rightarrow 3x - 2 = 0 \text{ or } x + 2 = 0 \\ &\Rightarrow \underline{\underline{x = \tfrac{2}{3} \text{ or } x = -2}}. \end{aligned}$$

(b) Hence, determine the maximum and minimum values of $\mathrm{f}(x)$ in the interval (3)

$$-1 \leqslant x \leqslant 1.$$

Solution

Now,

$$\begin{aligned} \mathrm{f}(-1) &= -1 + 2 + 4 + 1 = 6, \\ \mathrm{f}(\tfrac{2}{3}) &= \tfrac{8}{27} + \tfrac{8}{9} - \tfrac{8}{3} + 1 = -\tfrac{13}{27}, \\ \mathrm{f}(1) &= 1 + 2 - 4 + 1 = 0. \end{aligned}$$

Hence, the maximum is $\underline{\underline{6}}$ and the minimum value is $\underline{\underline{-\tfrac{13}{27}}}$.

13. (a) Express (4)

$$2\sin x^\circ - 7\cos x^\circ$$

in the form

$$k\sin(x-a)^\circ,$$

where $k > 0$ and $0 < a < 360$.

Solution

Well,

$$\begin{aligned} k\sin(x-a)^\circ &\equiv k(\sin x^\circ \cos a^\circ - \cos x^\circ \sin a^\circ) \\ &\equiv (k\cos a^\circ)\sin x^\circ - (k\sin a^\circ)\cos x^\circ, \end{aligned}$$

and

$$k\cos a^\circ \equiv 2 \text{ and } k\sin a^\circ \equiv 7.$$

Now,

$$\begin{aligned} k &= \sqrt{k^2} \\ &= \sqrt{k^2(\cos^2 a^\circ + \sin^2 a^\circ)} \\ &= \sqrt{(k\cos a^\circ)^2 + (k\sin a^\circ)^2} \\ &= \sqrt{2^2 + 7^2} \\ &= \underline{\underline{\sqrt{53}}} \end{aligned}$$

and

$$\begin{aligned} \tan a^\circ = \frac{k\sin a^\circ}{k\cos a^\circ} &\Rightarrow \tan a^\circ = \tfrac{7}{2} \\ &\Rightarrow a = 74.054\,604\,1 \text{ (FCD)}. \end{aligned}$$

Hence,

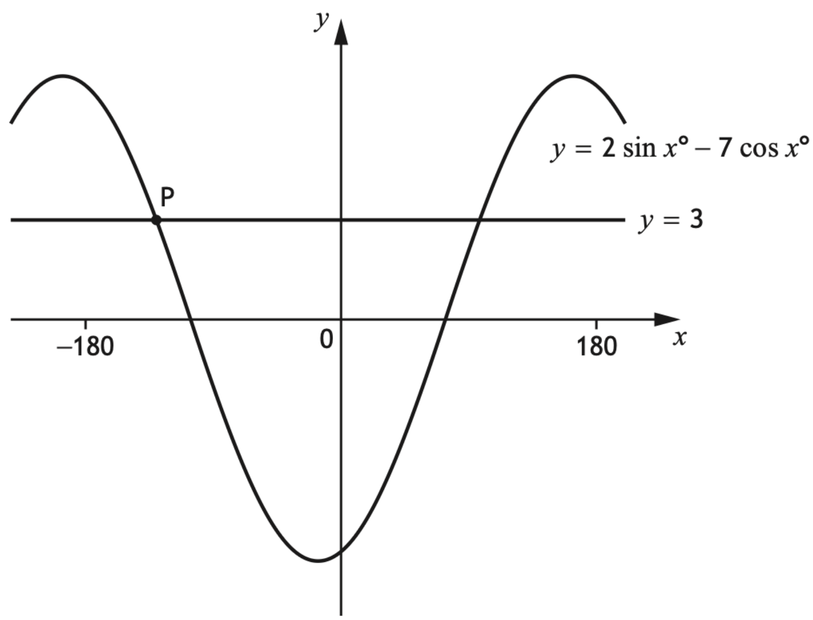
$$2\sin x^\circ - 7\cos x^\circ = \underline{\underline{\sqrt{53}\sin(x - 74.054\ldots)^\circ}}.$$

The diagram shows part of the curve with equation

$$y = 2\sin x^\circ - 7\cos x^\circ$$

and the line with equation $y = 3$.

The curve and the line intersect at the point P.



(b) Find the x-coordinate of P. (3)

Solution

Well,

$$\begin{aligned} 2\sin x^\circ - 7\cos x^\circ = 3 &\Rightarrow \sqrt{53}\sin(x - 174.054\ldots)^\circ = 3 \\ &\Rightarrow \sin(x - 74.054\ldots)^\circ = \tfrac{3}{\sqrt{53}} \\ &\Rightarrow x - 74.054\ldots = -204.335\,670\,68 \text{ (FCD)} \\ &\Rightarrow x = -130.281\,066\,6 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{x = -130 \text{ (3 sf)}}}. \end{aligned}$$

14. During a metal-working process a heated steel rod is cooled by dropping it into a container of oil.

The temperature of the rod is given by

$$T = 800\,\mathrm{e}^{-0.124t} + 25,$$

where T is the temperature of the rod, in degrees Celsius, t seconds after it is dropped into the oil.

(a) Determine the temperature of the rod as it is dropped into the oil. (1)

Solution
Well,

$$\begin{aligned} t = 0 &\Rightarrow T = 800 + 25 \\ &\Rightarrow \underline{\underline{T = 825^\circ\mathrm{C}}}. \end{aligned}$$

(b) Calculate the time taken for the temperature of the rod to cool to 150°C. (4)

Solution
Now,

$$\begin{aligned} T = 150 &\Rightarrow 150 = 800\,\mathrm{e}^{-0.124t} + 25 \\ &\Rightarrow 125 = 800\,\mathrm{e}^{-0.124t} \\ &\Rightarrow \mathrm{e}^{-0.124t} = \tfrac{5}{32} \\ &\Rightarrow -0.124t = \ln\tfrac{5}{32} \\ &\Rightarrow t = -\tfrac{1}{0.124}\ln\tfrac{5}{32} \\ &\Rightarrow t = 14.970\,145\,08 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{t = 15.0 \text{ s (3 sf)}}}. \end{aligned}$$

15. Express (3)

$$(3\sin\theta + \cos\theta)(\sin\theta + 3\cos\theta)$$

in the form

$$a + b\sin c\theta,$$

where a, b, and c are integers.

Solution
Well,

$\times$	$3\sin\theta$	$+\cos\theta$
$\sin\theta$	$3\sin^2\theta$	$+\sin\theta\cos\theta$
$+3\cos\theta$	$+9\sin\theta\cos\theta$	$+3\cos^2\theta$

and so

$$\begin{aligned}(3\sin\theta+\cos\theta)(\sin\theta+3\cos\theta) &\equiv 3\sin^2\theta+10\sin\theta\cos\theta+3\cos^2\theta\\ &\equiv 3(\sin^2\theta+\cos^2\theta)+5(2\sin\theta\cos\theta)\\ &\equiv \underline{\underline{3+5\sin 2\theta}};\end{aligned}$$

hence,

$$\underline{\underline{a=3,\ b=5,\ \text{and}\ c=2}}.$$