Dr Oliver Mathematics
Cambridge O Level Additional Mathematics
2007 June Paper 2: Calculator
2 hours

The total number of marks available is 80.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You must write down all the stages in your working.

1. A triangle has a base of length $(13 - 2x)$ m and a perpendicular height of x m. (3)

   Calculate the range of values of x for which the area of the triangle is greater than 3 m^2.

Solution

Well,

$$\begin{aligned}\tfrac{1}{2} \times (13 - 2x) \times x > 3 &\Rightarrow x(13 - 2x) > 6 \\ &\Rightarrow 13x - 2x^2 > 6 \\ &\Rightarrow -13x + 2x^2 < -6 \\ &\Rightarrow 2x^2 - 13x + 6 < 0\end{aligned}$$

$$\left.\begin{array}{ll}\text{add to:} & -13 \\ \text{multiply to:} & (+2) \times (+6) = +12\end{array}\right\} -12, -1$$

e.g.,

$$\begin{aligned}&\Rightarrow 2x^2 - 12x - x - 6 < 0 \\ &\Rightarrow 2x(x - 6) - 1(x + 6) < 0 \\ &\Rightarrow (2x - 1)(x - 6) < 0\end{aligned}$$

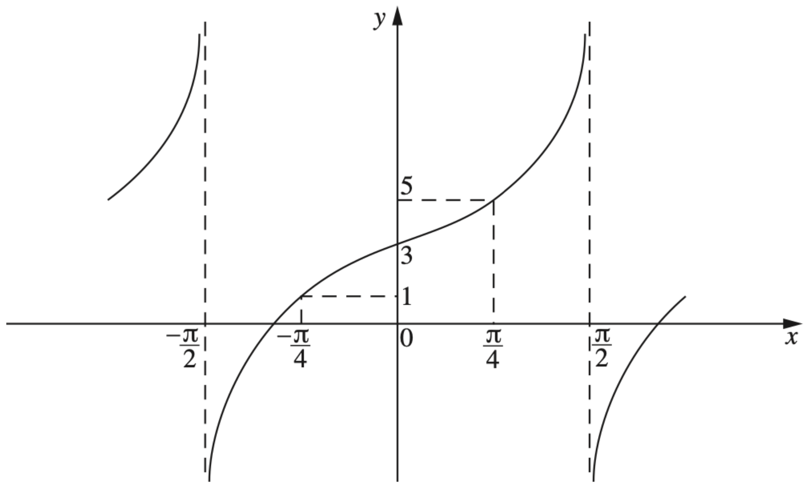
we need a 'table of signs':

	$x < \frac{1}{2}$	$x = \frac{1}{2}$	$\frac{1}{2} < x < 6$	$x = 6$	$x > 6$
$2x - 1$	$-$	0	$+$	$+$	$+$
$x - 6$	$-$	$-$	$-$	0	$+$
$(2x - 1)(x - 6)$	$+$	0	$-$	0	$+$

$$\Rightarrow \underline{\underline{\tfrac{1}{2} < x < 6}}.$$

2. The diagram shows part of the graph of

$$y = a\tan(bx) + c.$$



Find the value of

(a) c, (1)

Solution

$\underline{\underline{c = 3}}$.

(b) b, (1)

Solution

$\underline{\underline{b = 1}}$.

(c) a. (1)

Solution

$\underline{\underline{a = 4}}$.

3. The roots of the equation (5)

$$x^2 - \sqrt{28}x + 2 = 0$$

are p and q, where $p > q$.

Without using a calculator, express $\dfrac{p}{q}$ in the form $m + \sqrt{n}$, where m and n are integers.

Solution

$$\begin{aligned} x^2 - \sqrt{28}x + 2 = 0 &\Rightarrow x^2 - 2\sqrt{7}x + 2 = 0 \\ &\Rightarrow x^2 - 2\sqrt{7}x = -2 \\ &\Rightarrow x^2 - 2\sqrt{7}x + 7 = -2 + 7 \\ &\Rightarrow (x - \sqrt{7})^2 = 5 \\ &\Rightarrow x - \sqrt{7} = \pm\sqrt{5} \\ &\Rightarrow x = \sqrt{7} \pm \sqrt{5}; \end{aligned}$$

so $p = \sqrt{7} + \sqrt{5}$ and $q = \sqrt{7} - \sqrt{5}$. Now,

$$\begin{aligned} \frac{p}{q} &= \frac{\sqrt{7} + \sqrt{5}}{\sqrt{7} - \sqrt{5}} \\ &= \frac{\sqrt{7} + \sqrt{5}}{\sqrt{7} - \sqrt{5}} \times \frac{\sqrt{7} + \sqrt{5}}{\sqrt{7} + \sqrt{5}} \end{aligned}$$

$\times$	$\sqrt{7}$	$+\sqrt{5}$
$\sqrt{7}$	7	$\sqrt{35}$
$\pm\sqrt{5}$	$\pm\sqrt{35}$	± 5

$$\begin{aligned} &= \frac{12 + 2\sqrt{35}}{2} \\ &= \underline{\underline{6 + \sqrt{35}}}; \end{aligned}$$

hence, $\underline{\underline{m = 6}}$ and $\underline{\underline{n = 35}}$.

4. An artist has 6 watercolour paintings and 4 oil paintings.

She wishes to select 4 of these 10 paintings for an exhibition.

(a) Find the number of different selections she can make. (2)

Solution

$$\binom{10}{4} = \underline{\underline{210}}.$$

(b) In how many of these selections will there be more watercolour paintings than oil paintings? (3)

Solution

$$\begin{aligned}\text{P(more watercolour paintings)} &= \text{P(3 watercolour)} + \text{P(4 watercolour)} \\ &= \left[\binom{6}{3} \times 4\right] + \binom{6}{4} \\ &= 80 + 15 \\ &= \underline{\underline{95}}.\end{aligned}$$

5. (a) Express (1)

$$\frac{1}{\sqrt{32}}$$

as a power of 2.

Solution

$$\begin{aligned}\frac{1}{\sqrt{32}} &= \frac{1}{\sqrt{2^5}} \\ &= \frac{1}{2^{\frac{5}{2}}} \\ &= \underline{\underline{2^{-\frac{5}{2}}}}.\end{aligned}$$

(b) Express (1)

$$(64)^{\frac{1}{x}}$$

as a power of 2.

Solution

$$\begin{aligned}(64)^{\frac{1}{x}} &= (2^6)^{\frac{1}{x}} \\ &= \underline{\underline{2^{\frac{6}{x}}}}.\end{aligned}$$

(c) Hence solve the equation (3)

$$\frac{(64)^{\frac{1}{x}}}{2^x} = \frac{1}{\sqrt{32}}.$$

Solution

$$\begin{aligned}\frac{(64)^{\frac{1}{x}}}{2^x} = \frac{1}{\sqrt{32}} &\Rightarrow \frac{2^{\frac{6}{x}}}{2^x} = 2^{-\frac{5}{2}} \\ &\Rightarrow 2^{\frac{6}{x}-x} = 2^{-\frac{5}{2}} \\ &\Rightarrow \frac{6}{x} - x = -\tfrac{5}{2}\end{aligned}$$

multiply by $2x$:

$$\begin{aligned}&\Rightarrow 12 - 2x^2 = -5x \\ &\Rightarrow 2x^2 - 5x - 12 = 0\end{aligned}$$

$$\left.\begin{array}{ll}\text{add to:} & -5 \\ \text{multiply to:} & (+2) \times (-12) = -24\end{array}\right\} -8, +3$$

e.g.,

$$\begin{aligned}&\Rightarrow 2x^2 - 8x + 3x - 12 = 0 \\ &\Rightarrow 2x(x-4) + 3(x-4) = 0 \\ &\Rightarrow (2x+3)(x-4) = 0 \\ &\Rightarrow 2x+3 = 0 \text{ or } x-4 = 0 \\ &\Rightarrow \underline{\underline{x = -1\tfrac{1}{2} \text{ or } x = 4}}.\end{aligned}$$

6. (a) Differentiate (2)

$$x^2 \ln x$$

with respect to x.

Solution

Product rule:

$$\begin{aligned}u = x^2 &\Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 2x \\ v = \ln x &\Rightarrow \frac{\mathrm{d}v}{\mathrm{d}x} = \frac{1}{x}\end{aligned}$$

so

$$\frac{\mathrm{d}y}{\mathrm{d}x} = (x^2)\left(\frac{1}{x}\right) + (2x)(\ln x)$$
$$= \underline{\underline{x + 2x\ln x}}.$$

(b) Use your result to show that (4)

$$\int_1^{\mathrm{e}} 4x\ln x\,\mathrm{d}x = \mathrm{e}^2 + 1.$$

Solution

$$\begin{aligned}\int_1^{\mathrm{e}} 4x\ln x\,\mathrm{d}x &= \int_1^{\mathrm{e}} 2\left[(x + 2x\ln x) - x\right]\,\mathrm{d}x \\ &= 2\int_1^{\mathrm{e}} \left[(x + 2x\ln x) - x\right]\,\mathrm{d}x \\ &= 2\left[x^2\ln x - \tfrac{1}{2}x^2\right]_{x=1}^{\mathrm{e}} \\ &= 2\left\{\left(\mathrm{e}^2 - \tfrac{1}{2}e^2\right) - \left(0 - \tfrac{1}{2}\right)\right\} \\ &= 2\left(\tfrac{1}{2}e^2 + \tfrac{1}{2}\right) \\ &= \underline{\underline{\mathrm{e}^2 + 1}},\end{aligned}$$

as required.

7. Given that

$$\mathbf{A} = \begin{pmatrix} 2 & 3 \\ -2 & -1 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 8 & 10 \\ -4 & 2 \end{pmatrix},$$

find the matrices $\mathbf{X}$ and $\mathbf{Y}$ such that

(a) $\mathbf{X} = \mathbf{A}^2 + 2\mathbf{B}$, (3)

Solution

$$\begin{aligned}\mathbf{X} &= \mathbf{A}^2 + 2\mathbf{B} \\ &= \begin{pmatrix} 2 & 3 \\ -2 & -1 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ -2 & -1 \end{pmatrix} + 2\begin{pmatrix} 8 & 10 \\ -4 & 2 \end{pmatrix} \\ &= \begin{pmatrix} -2 & 3 \\ -2 & -5 \end{pmatrix} + \begin{pmatrix} 16 & 20 \\ -8 & 4 \end{pmatrix} \\ &= \underline{\underline{\begin{pmatrix} 14 & 23 \\ -10 & -1 \end{pmatrix}}}.\end{aligned}$$

(b) $\mathbf{YA} = \mathbf{B}$. (4)

Solution

Well,

$$\mathbf{A}^{-1} = \tfrac{1}{4}\begin{pmatrix} -1 & -3 \\ 2 & 2 \end{pmatrix}$$

and

$$\begin{aligned}\mathbf{YA} = \mathbf{B} &\Rightarrow \mathbf{Y} = \mathbf{BA}^{-1} \\ &\Rightarrow \mathbf{Y} = \begin{pmatrix} 8 & 10 \\ -4 & 2 \end{pmatrix} \cdot \tfrac{1}{4}\begin{pmatrix} -1 & -3 \\ 2 & 2 \end{pmatrix} \\ &\Rightarrow \mathbf{Y} = \tfrac{1}{4}\begin{pmatrix} 12 & -4 \\ 8 & 16 \end{pmatrix} \\ &\Rightarrow \mathbf{Y} = \underline{\underline{\begin{pmatrix} 3 & -1 \\ 2 & 4 \end{pmatrix}}}.\end{aligned}$$

8. The equation of the curve C is

$$2y = x^2 + 4.$$

The equation of the line L is

$$y = 3x - k,$$

where k is an integer.

(a) Find the largest value of the integer k for which L intersects C. (4)

Solution

Well,

$$y = 3x - k \Rightarrow 2y = 6x - 2k$$

and we insert L into C:

$$x^2 + 4 = 6x - 2k \Rightarrow x^2 - 6x + (4 + 2k) = 0.$$

Now, they meet if

$$\begin{aligned} b^2 - 4ac \geqslant 0 &\Rightarrow (-6)^2 - 4(1)(4 + 2k) \geqslant 0 \\ &\Rightarrow 36 \geqslant 4(4 + 2k) \\ &\Rightarrow 9 \geqslant 4 + 2k \\ &\Rightarrow 5 \geqslant 2k \\ &\Rightarrow k \leqslant 2\tfrac{1}{2}. \end{aligned}$$

Since k is an integer, $\underline{\underline{k = 2}}$.

(b) In the case where $k = -2$, show that the line joining the points of intersection of L and C is bisected by the line $y = 2x + 5$. (4)

Solution

In the case $k = -2$, we have

$$\begin{aligned} x^2 - 6x = 0 &\Rightarrow x(x - 6) = 0 \\ &\Rightarrow x = 0 \text{ or } x = 6 \\ &\Rightarrow y = 2 \text{ or } y = 20; \end{aligned}$$

so, $(0, 2)$ and $(6, 20)$ are on the line. Now, the midpoint is $(3, 11)$ and

$$x = 3 \Rightarrow 3(3) + 2 = 11 \quad \checkmark;$$

the line joining the points of intersection of L and C is $\underline{\underline{\text{bisected}}}$ by the line $y = 2x + 5$.

9. The position vectors, relative to an origin O, of three points P, Q, and R are $\mathbf{i} + 3\mathbf{j}$, $5\mathbf{i} + 11\mathbf{j}$, and $9\mathbf{i} + 9\mathbf{j}$ respectively.

(a) By finding the magnitude of the vectors $\overrightarrow{PR}$, $\overrightarrow{RQ}$, and $\overrightarrow{QP}$, show that angle PQR is 90°. (4)

Solution

Well,

$$\begin{aligned}\overrightarrow{PR} &= \overrightarrow{PO} + \overrightarrow{OR}\\ &= -\overrightarrow{OP} + \overrightarrow{OR}\\ &= -(\mathbf{i} + 3\mathbf{j}) + (9\mathbf{i} + 9\mathbf{j})\\ &= 8\mathbf{i} + 6\mathbf{j}\end{aligned}$$

and

$$\begin{aligned}PR^2 &= 8^2 + 6^2\\ &= 100;\end{aligned}$$

$$\begin{aligned}\overrightarrow{PQ} &= \overrightarrow{PO} + \overrightarrow{OQ}\\ &= -\overrightarrow{OP} + \overrightarrow{OQ}\\ &= -(\mathbf{i} + 3\mathbf{j}) + (5\mathbf{i} + 11\mathbf{j})\\ &= 4\mathbf{i} + 8\mathbf{j}\end{aligned}$$

and

$$\begin{aligned}PQ^2 &= 4^2 + 8^2\\ &= 80;\end{aligned}$$

$$\begin{aligned}\overrightarrow{QR} &= \overrightarrow{QO} + \overrightarrow{OR}\\ &= -\overrightarrow{OQ} + \overrightarrow{OR}\\ &= -(5\mathbf{i} + 11\mathbf{j}) + (9\mathbf{i} + 9\mathbf{j})\\ &= 4\mathbf{i} - 2\mathbf{j}\end{aligned}$$

and

$$\begin{aligned}QR^2 &= 4^2 + (-2)^2\\ &= 20;\end{aligned}$$

Now,

$$\begin{aligned}PQ^2 + QR^2 &= 80 + 20\\ &= 100\\ &= PR^2,\end{aligned}$$

and so the angle PQR is $\underline{\underline{90^\circ}}$.

(b) Find the unit vector parallel to $\overrightarrow{PR}$. (2)

Solution
A unit vector parallel to $\overrightarrow{PR}$ is

$$\tfrac{1}{10}(8\mathbf{i} + 6\mathbf{j}) = \underline{\underline{\tfrac{1}{5}(4\mathbf{i} + 3\mathbf{j})}}.$$

(c) Given that (3)

$$\overrightarrow{OQ} = m\overrightarrow{OP} + n\overrightarrow{OR},$$

where m and n are constants, find the value of m and of n.

Solution

$$\begin{aligned}\overrightarrow{OQ} = m\overrightarrow{OP} + n\overrightarrow{OR} &\Rightarrow \begin{pmatrix} 5 \\ 11 \end{pmatrix} = m\begin{pmatrix} 1 \\ 3 \end{pmatrix} + n\begin{pmatrix} 9 \\ 9 \end{pmatrix} \\ &\Rightarrow \begin{pmatrix} 5 \\ 11 \end{pmatrix} = \begin{pmatrix} m + 9n \\ 3m + 9n \end{pmatrix}\end{aligned}$$

and so

$$\begin{aligned} m + 9n &= 5 \quad (1) \\ 3m + 9n &= 11 \quad (2). \end{aligned}$$

Do (2) − (1):

$$\begin{aligned} 2m = 6 &\Rightarrow \underline{\underline{m = 3}} \\ &\Rightarrow 3 + 9n = 5 \\ &\Rightarrow 9n = 2 \\ &\Rightarrow \underline{\underline{n = \tfrac{2}{9}}}. \end{aligned}$$

10. The functions f and g are defined, for $x \in \mathbb{R}$, by

$$\begin{aligned} \mathrm{f}: &x \mapsto 3x - 2, \\ \mathrm{g}: &x \mapsto \frac{7x - a}{x + 1}, \text{ where } x \neq -1 \text{ and } a \text{ is a positive constant.} \end{aligned}$$

(a) Obtain expressions for f^{-1} and g^{-1}. (3)

Solution

$$\begin{aligned} y = 3x - 2 &\Rightarrow y + 2 = 3x \\ &\Rightarrow \tfrac{1}{3}(y+2) = x \end{aligned}$$

and

$$\mathrm{f}^{-1}(x) = \underline{\underline{\tfrac{1}{3}(x+2)}}.$$

$$\begin{aligned} y = \frac{7x - a}{x+1} &\Rightarrow y(x+1) = 7x - a \\ &\Rightarrow xy + y = 7x - a \\ &\Rightarrow xy - 7x = -y - a \\ &\Rightarrow x(y-7) = -y - a \\ &\Rightarrow x = \frac{-y-a}{y-7} \end{aligned}$$

and

$$\mathrm{g}^{-1}(x) = \underline{\underline{\frac{-x-a}{x-7},\ x \neq 7}}.$$

(b) Determine the value of a for which (3)

$$\mathrm{f}^{-1}\,\mathrm{g}(4) = 2.$$

Solution

$$\begin{aligned} \mathrm{f}^{-1}\,\mathrm{g}(4) = 2 &\Rightarrow \mathrm{g}(4) = \mathrm{f}(2) \\ &\Rightarrow \frac{7(4) - a}{4+1} = 3(2) - 2 \\ &\Rightarrow \frac{28 - a}{5} = 4 \\ &\Rightarrow 28 - a = 20 \\ &\Rightarrow \underline{\underline{a = 8}}. \end{aligned}$$

(c) If $a = 9$, show that there is only one value of x for which (3)

$$\mathrm{g}(x) = \mathrm{g}^{-1}(x).$$

Solution

If $a = 9$,

$$\frac{7x-9}{x+1} = \frac{-x-9}{x-7} \Rightarrow (7x-9)(x-7) = (x+1)(-x-9)$$

$\times$	$7x$	-9
x	$7x^2$	$-9x$
-7	$-49x$	$+63$

$\times$	x	$+1$
$-x$	$-x^2$	$-x$
-9	$-9x$	-9

$$\Rightarrow 7x^2 - 58x + 63 = -x^2 - 10x - 9$$
$$\Rightarrow 8x^2 - 48x + 72 = 0$$
$$\Rightarrow 8(x^2 - 6x + 9) = 0$$

$$\left.\begin{array}{ll}\text{add to:} & -6\\ \text{multiply to:} & +9\end{array}\right\} -3 \text{ (repeated)}$$

$$\Rightarrow 8(x-3)^2 = 0$$
$$\Rightarrow \underline{\underline{x = 3}}.$$

11. A particle, moving in a straight line, passes through a fixed point O with velocity 14 ms^{-1}.

The acceleration, a ms^{-2} of the particle, t seconds after passing through O, is given by particle by

$$a = 2t - 9.$$

The particle subsequently comes to instantaneous rest, firstly at A and later at B.

Find

(a) the acceleration of the particle at A and at B, (4)

Solution

Well,

$$a = 2t - 9 \Rightarrow v = t^2 - 9t + c,$$

for some constant c. Now,

$$\begin{aligned} t = 0, v = 14 &\Rightarrow 14 = 0 + 0 + c \\ &\Rightarrow c = 14 \end{aligned}$$

and

$$v = t^2 - 9t + 14.$$

Next,

$$v = 0 \Rightarrow t^2 - 9t + 14 = 0$$

$$\left.\begin{array}{ll} \text{add to:} & -9 \\ \text{multiply to:} & +14 \end{array}\right\} -2, -7$$

$$\begin{aligned} &\Rightarrow (t-2)(t-7) = 0 \\ &\Rightarrow t = 2 \text{ or } t = 7. \end{aligned}$$

Finally,

$$t = 2 \Rightarrow a_{t=2} = 2(2) - 9 = \underline{\underline{-5 \text{ ms}^{-2}}}$$

and

$$t = 7 \Rightarrow a_{t=2} = 2(7) - 9 = \underline{\underline{5 \text{ ms}^{-2}}}.$$

(b) the greatest speed of the particle as it travels from A to B, (2)

Solution

Well, the greatest speed of the particle occurs when

$$t = \frac{2+7}{2} = 4\tfrac{1}{2} \text{ s}$$

(quadratic!) and

$$v = (4\tfrac{1}{2})^2 - 9(4\tfrac{1}{2}) + 14 = -6\tfrac{1}{4};$$

So, the greatest speed is

$$\underline{\underline{6\tfrac{1}{4}\text{ ms}^{-1}}}.$$

(c) the distance AB. (4)

Solution

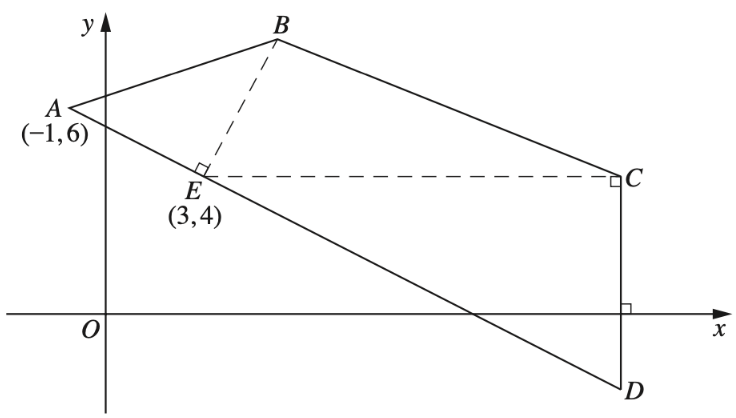
$$v = t^2 - 9t + c \Rightarrow s = \tfrac{1}{3}t^3 - \tfrac{9}{2}t^2 + 14t + d,$$

for some constant d. Now,

$$\begin{aligned} s_{AB} &= \left|\left[\tfrac{1}{3}t^3 - \tfrac{9}{2}t^2 + 14t\right]_{t=2}^{7}\right| \\ &= \left|\left(\tfrac{343}{3} - \tfrac{441}{2} + 98\right) - \left(\tfrac{8}{3} - 18 + 28\right)\right| \\ &= \left|-8\tfrac{1}{6} - 12\tfrac{2}{3}\right| \\ &= \underline{\underline{20\tfrac{5}{6}\text{ m}}}. \end{aligned}$$

EITHER

12. The diagram shows a quadrilateral $ABCD$.



- The point E lies on AD such that angle $AEB = 90°$.
- The line EC is parallel to the x-axis and the line CD is parallel to the y-axis.
- The points A and E are $(-1, 6)$ and $(3, 4)$ respectively.

Given that the gradient of AB is $\frac{1}{3}$,

(a) find the coordinates of B. (5)

Solution

Well, the equation of AB is

$$y - 6 = \tfrac{1}{3}(x+1) \Rightarrow y - 6 = \tfrac{1}{3}x + \tfrac{1}{3}$$
$$\Rightarrow \boxed{y = \tfrac{1}{3}x + \tfrac{19}{3} \quad (1).}$$

Now,

$$\begin{aligned} m_{AE} &= \frac{4-6}{3-(-1)} \\ &= \frac{-2}{4} \\ &= -\tfrac{1}{2} \end{aligned}$$

and so

$$m_{BE} = -\frac{1}{-\frac{1}{2}} = 2.$$

Next, the equation of BE is

$$y - 4 = 2(x-3) \Rightarrow y - 4 = 2x - 6$$
$$\Rightarrow \boxed{y = 2x - 2 \quad (2).}$$

Do (1) = (2):

$$\begin{aligned} \tfrac{1}{3}x + \tfrac{19}{3} = 2x - 2 &\Rightarrow \tfrac{25}{3} = \tfrac{5}{3}x \\ &\Rightarrow x = 5 \\ &\Rightarrow y = 8; \end{aligned}$$

hence, $\underline{\underline{B(5,8)}}$.

Given also that the area of triangle EBC is 24 units^2,

(b) find the coordinates of C, (3)

Solution

Let $F(5,4)$ and $BF = 4$. Now,

$$\begin{aligned} \text{area} = 24 &\Rightarrow \tfrac{1}{2} \times EC \times BF = 24 \\ &\Rightarrow 4EC = 48 \\ &\Rightarrow EC = 12; \end{aligned}$$

since $E(3,4)$, $\underline{\underline{C(15,4)}}$.

(c) find the coordinates of D. (2)

Solution
Now, the equation of AE is

$$\begin{aligned} y - 4 = -\tfrac{1}{2}(x-3) &\Rightarrow y - 4 = -\tfrac{1}{2}x + \tfrac{3}{2} \\ &\Rightarrow y = -\tfrac{1}{2}x + \tfrac{11}{2} \end{aligned}$$

and

$$\begin{aligned} x = 15 &\Rightarrow y = -\tfrac{1}{2}(15) + \tfrac{11}{2} \\ &\Rightarrow y = -2; \end{aligned}$$

so, $\underline{\underline{D(15,-2)}}$.

OR

13. (a) The expression

$$f(x) = x^3 + ax^2 + bx + c$$

leaves the same remainder, R, when it is divided by $(x+2)$ and when it is divided by $(x-2)$.

(i) Evaluate b. (2)

Solution
We use synthetic division twice:

2	1	a	b	c
	$\downarrow$	2	$2a+4$	$4a+2b+8$
	1	$a+2$	$2a+b+4$	$4a+2b+c+8$

and

-2	1	a	b	c
	$\downarrow$	-2	$-2a+4$	$4a+2b-8$
	1	$a-2$	$-2a+b+4$	$4a-2b+c-8$

As we have R twice,

$$4a+2b+c+8 = 4a-2b+c-8 \Rightarrow 4b = -16$$
$$\Rightarrow \underline{\underline{b = -4}}.$$

f(x) also leaves the same remainder, R, when divided by $(x-1)$.

(ii) Evaluate a. (2)

Solution

1	1	a	-4	c
	$\downarrow$	1	$a+1$	$a-3$
	1	$a+1$	$a-3$	$a+c-3$

So we have two values of R, $4a+c$ and $a+c-3$:

$$4a+c = a+c-3 \Rightarrow 3a = -3$$
$$\Rightarrow \underline{\underline{a = -1}}.$$

f(x) leaves a remainder of 4 when divided by $(x-3)$.

(iii) Evaluate c. (1)

Solution

3	1	-1	-4	c
	$\downarrow$	3	6	6
	1	2	2	$c+6$

Finally,

$$c+6 = 4 \Rightarrow \underline{\underline{c = -2}}.$$

(b) Solve the equation (5)

$$x^3 + 3x^2 = 2,$$

giving your answers to 2 decimal places where necessary.

Solution

$$x^3 + 3x^2 = 2 \Rightarrow x^3 + 3x^2 - 2 = 0$$

and let

$$\text{f}(x) = x^3 + 3x^2 - 2.$$

Now,

$$\text{f}(1) = 1 + 3 - 2 = 2$$
$$\text{f}(-1) = -1 + 3 - 2 = 0$$

and so $(x + 1)$ is a factor of $\text{f}(x)$.

-1	1	3	0	-2
	$\downarrow$	-1	-2	2
	1	2	-2	0

and so

$$\text{f}(x) = (x + 2)(x^2 + 2x - 2).$$

Next,

$$\begin{aligned} x^2 + 2x - 2 = 0 &\Rightarrow x^2 + 2x = 2 \\ &\Rightarrow x^2 + 2x + 1 = 2 + 1 \\ &\Rightarrow (x + 1)^2 = 3 \\ &\Rightarrow x + 1 = \pm\sqrt{3} \\ &\Rightarrow x = -1 \pm \sqrt{3} \end{aligned}$$

and so the solutions are

$$-1,\ -1 \pm \sqrt{3} = \underline{\underline{-2.73,\ -1,\ \text{or } 0.73 \text{ (2 dp)}}}.$$