Dr Oliver Mathematics
AQA GCSE Mathematics
2015 November Paper 1: Non-Calculator
1 hour 30 minutes

The total number of marks available is 70.
You must write down all the stages in your working.

1. Divide 270 in the ratio (3)

$$3 : 2 : 1.$$

Solution

Well,

$$3 + 2 + 1 = 6$$

and so the 'shares' are

$$\begin{aligned} \tfrac{3}{6} \times 270 &= 3 \times 45 = 135, \\ \tfrac{2}{6} \times 270 &= 2 \times 45 = 90, \\ \tfrac{1}{6} \times 270 &= 45; \end{aligned}$$

hence,

$$\underline{\underline{135 : 90 : 45}}.$$

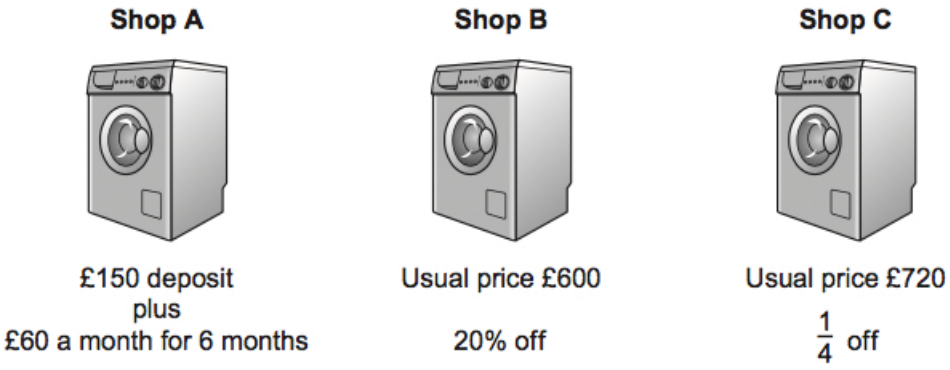
2. Solve (3)

$$\frac{4x - 1}{7} = 2x.$$

Solution

$$\begin{aligned} \frac{4x - 1}{7} = 2x &\Rightarrow 4x - 1 = 14x \\ &\Rightarrow -1 = 10x \\ &\Rightarrow \underline{\underline{x = -\tfrac{1}{10}}}. \end{aligned}$$

3. Three shops sell the same washing machine. (5)



In which shop is the washing machine cheapest?
You **must** show your working.

Solution

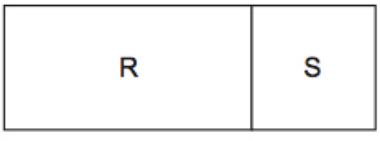
$$\underline{\text{Shop A}}: \quad 150 + (60 \times 6)$$
$$= 150 + 360$$
$$= 510.$$

$$\underline{\text{Shop B}}: \quad \tfrac{80}{100} \times 600$$
$$= 80 \times 6$$
$$= 480.$$

$$\underline{\text{Shop C}}: \quad \tfrac{3}{4} \times 720$$
$$= 3 \times 180$$
$$= 540.$$

Hence, the washing machine cheapest in Shop B.

4. A shape is made from a rectangle R and a square S. (4)



Not drawn accurately

The shape has a perimeter of 44 cm.
The area of the square is 36 cm^2.

Work out the area of the shape.

Solution

For R, let the height be r cm and the let the length be s cm.
For S, let the height (and length) be r cm.

Now,

$$r^2 = 36 \Rightarrow r = 6$$

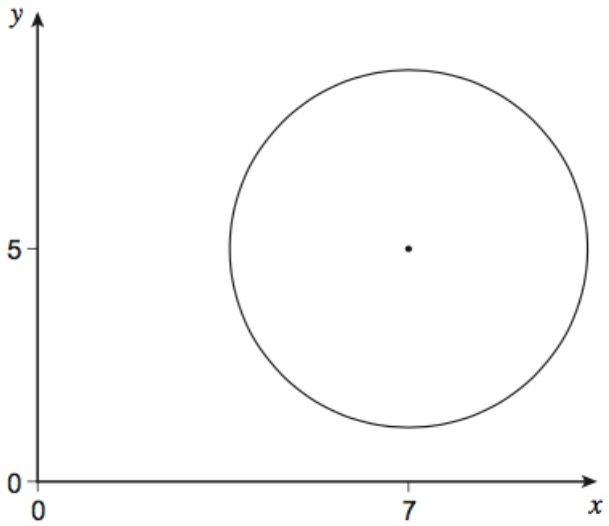
and

$$\begin{aligned} 6 + s + 6 + 6 + 6 + s = 44 &\Rightarrow 24 + 2s = 44 \\ &\Rightarrow 2s = 20 \\ &\Rightarrow s = 10. \end{aligned}$$

Finally,

$$\begin{aligned} \text{area of the shape} &= 6(10 + 6) \\ &= 6 \times 16 \\ &= \underline{\underline{96 \text{ cm}^2}}. \end{aligned}$$

5. A circle radius 3 units, centre $(7, 5)$ is shown. (2)



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Work out the coordinates of **any** point that lies on the circumference of the circle.
You **must** show your working, which may be on the diagram.

Solution

Well, any of $\underline{\underline{(7,2)}}$, $\underline{\underline{(7,8)}}$, $\underline{\underline{(4,5)}}$, or $\underline{\underline{(10,5)}}$ work. As do others of the form

$$(x-7)^2+(y-5)^2=9.$$

6. Fay is testing an ordinary six-sided dice to see if it is biased.

She throws the dice 120 times.

(a) Work out the number of times the dice is expected to land on 1. (1)

Solution

$$120\times\tfrac{1}{6}=\underline{\underline{20}}.$$

Here are the actual results.

Number on dice	1	2	3	4	5	6	Total
Frequency	5	19	17	20	21	38	120

(b) Is the dice biased? (2)
Tick a box.

Yes ☐ No ☐ Cannot tell ☐

Give a reason for your answer.

Solution
Yes: if it was a fair dice then we would have expected to land 20 times on 1, rather than just 5. Also, the number of times the dice landed on a 6 (38) rather than a 1 (5) make us believe that the dice is biased.

7. These expressions represent four numbers. (5)

$2x + 2$	$3x - 1$	$4x - 6$	$5x + 2$

The sum of the first two expressions is 36.

Work out the value of the median of the four numbers.

Solution
Well,

$$\begin{aligned}(2x + 2) + (3x - 1) = 36 &\Rightarrow 5x + 1 = 36\\ &\Rightarrow 5x = 35\\ &\Rightarrow x = 7,\end{aligned}$$

as and four numbers are 16, 20, 22, and 37. Hence, the value of the median is

$$\frac{20 + 22}{2} = 21.$$

8. (a) Expand and simplify fully (3)

$$4(x - 2) - 2(3 - 5x).$$

Solution

$$4(x-2)-2(3-5x) = 4x-8-6+10x$$
$$= \underline{\underline{14x-14}}.$$

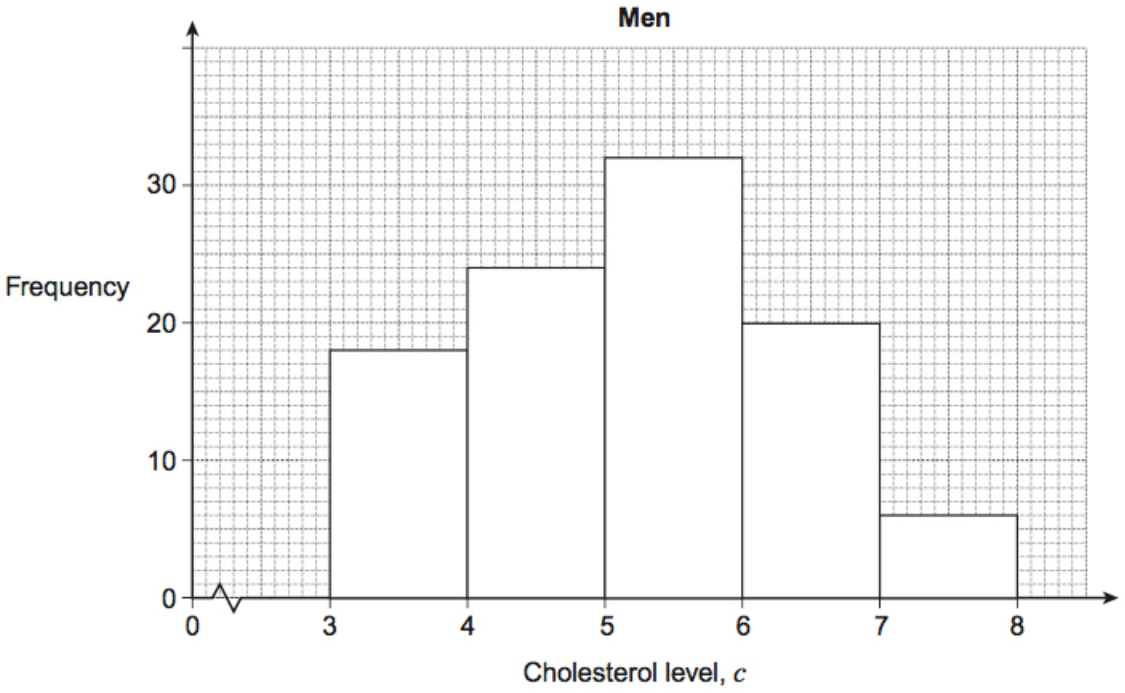
(b) Simplify fully (3)

$$\frac{8a^2+10ab}{12a+15b}.$$

Solution

$$\frac{8a^2+10ab}{12a+15b} = \frac{2a(4a+5b)}{3(4a+5b)}$$
$$= \underline{\underline{\tfrac{2a}{3}}}.$$

9. The frequency diagram shows information about the cholesterol level of 100 men.



(a) Work out an estimate of the median cholesterol level of the men. (3)

Solution

c	Cumulative Frequency
$3-4$	18
$3-5$	$18+24=42$
$3-6$	$42+32=74$
$3-7$	$64+20=94$
$3-8$	$94+6=90$

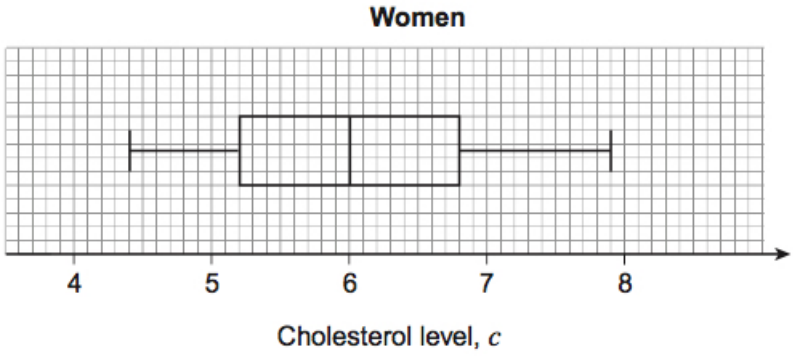
We want the $50\frac{1}{2}$th number, i.e., the $8\frac{1}{2}$th number in the $5-6$ category. Hmm: that makes it

$$5+\frac{8\frac{1}{2}}{32}=5\underline{\underline{\frac{17}{64}}}$$

or

$$\approx 5+\frac{8}{32}=\underline{\underline{5.25}}.$$

The box plot shows information about the cholesterol level of 100 women.



None of these 100 women have a cholesterol level of 6.8.

(b) Estimate how many of the **200 people** have a cholesterol level above 6.8. (3)
You **must** show your working.

Solution

Men: An estimate is

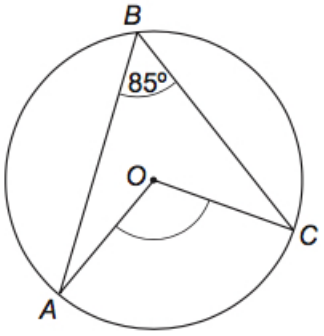
$$(\tfrac{2}{10}\times 20)+6=4+6=10.$$

Women: one-quarter of the women so 25.

So, an estimate of 200 people have a cholesterol level above 6.8 is

$$10 + 25 = \underline{\underline{35}}.$$

10. The diagram shows a circle, centre O.



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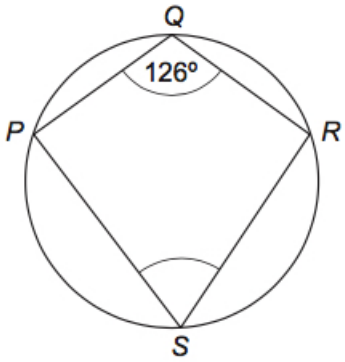
(a) Work out the size of angle AOC. (2)
Give a reason for your answer.

Solution

$$2 \times 85 = \underline{\underline{170^\circ}}$$

as the angle at the centre is twice the angle at the circumference.

P, Q, R, and S are points on the circumference of a circle.



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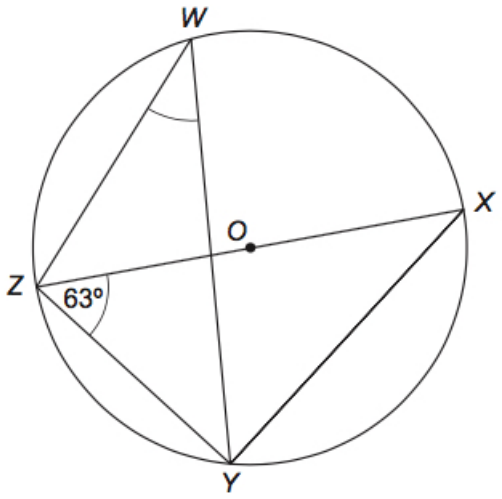
(b) Work out the size of angle PSR. (2)
Give a reason for your answer.

Solution

$$180 - 126 = \underline{\underline{54^\circ}}$$

as the opposite angles in a cyclic quadrilateral are supplementary.

(c) W, X, Y, and Z are points on the circumference of a circle centre O. (2)
ZX is a diameter.
Angle $YZX = 63^\circ$.



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Work out the size of angle ZWY.
You **must** show your working, which may be on the diagram.

Solution

Well, $\angle XYZ = 90^\circ$ (since ZOX is a diameter)
$\angle YXZ = 90 - 63 = 27^\circ$ (completing the triangle)
$\angle ZWY = \underline{\underline{27^\circ}}$ (angles in the same arc).

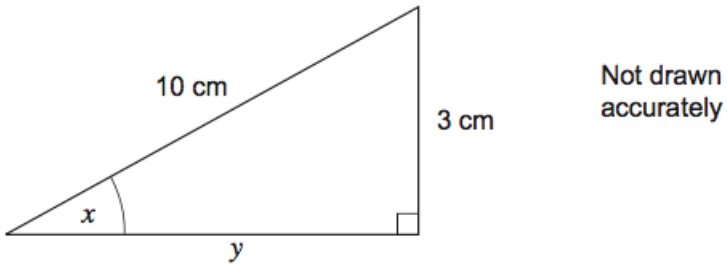
11. Solve (3)

$$2(7x + 3) < 4x - 1.$$

Solution

$$2(7x+3) < 4x - 1 \Rightarrow 14x + 6 < 4x - 1$$
$$\Rightarrow 10x < -7$$
$$\Rightarrow \underline{\underline{x < -\tfrac{7}{10}}}.$$

12. Here is a triangle.



(a) Work out the length y in the form $\sqrt{a}$ where a is an integer. (2)

Solution

Pythagoras' Theorem:

$$y^2 + 3^2 = 10^2 \Rightarrow y^2 + 9 = 100$$
$$\Rightarrow y^2 = 91$$
$$\Rightarrow y = \sqrt{91}.$$

(b) Write down the value of $\tan x$. (1)

Solution

$$\tan x = \underline{\underline{\frac{3}{\sqrt{91}}}}.$$

13. The square number sequence is (4)

$$1 \quad 4 \quad 9 \quad 16 \quad 25 \quad \ldots$$

Prove algebraically that the difference of two consecutive square numbers is an odd number.

Solution

Let the nth square number be n^2 where $n \in \mathbb{N}$.

$\times$	n	$+1$
n	n^2	$+n$
$+1$	$+n$	$+1$

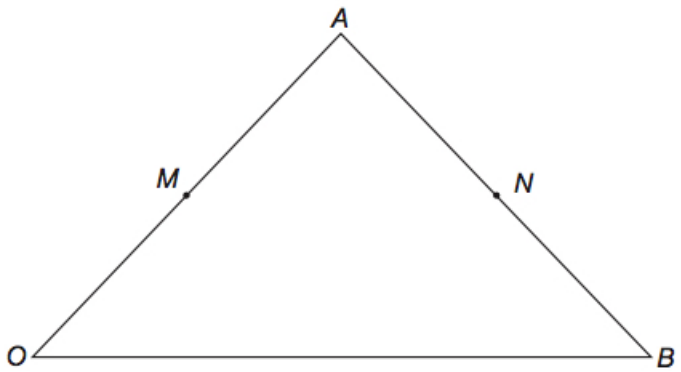
Then

$$\begin{aligned}(n+1)^2 - n^2 &= (n^2 + 2n + 1) - n^2 \\ &= 2n + 1;\end{aligned}$$

hence, the difference of two consecutive square numbers is an odd number.

14. In triangle OAB,

- M is the midpoint of OA,
- N is the midpoint of AB,
- $\overrightarrow{OA} = 2\mathbf{a}$, and
- $\overrightarrow{OB} = 2\mathbf{b}$.



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(a) Write down $\overrightarrow{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. (1)

Solution

$$\begin{aligned}\overrightarrow{AB} &= \overrightarrow{AO} + \overrightarrow{OB} \\ &= -\overrightarrow{OA} + \overrightarrow{OB} \\ &= \underline{\underline{-2\mathbf{a} + 2\mathbf{b}}}.\end{aligned}$$

(b) Show that $\overrightarrow{MN} = \mathbf{b}$. (2)

Solution

$$\begin{aligned}\overrightarrow{MN} &= \overrightarrow{MO} + \overrightarrow{OB} + \overrightarrow{BN} \\ &= \tfrac{1}{2}\overrightarrow{AO} + \overrightarrow{OB} + \tfrac{1}{2}\overrightarrow{BA} \\ &= -\mathbf{a} + 2\mathbf{b} + (\mathbf{a} - \mathbf{b}) \\ &= \underline{\underline{\mathbf{b}}},\end{aligned}$$

as required.

(c) Explain why triangles AMN and AOB are similar. (2)

Solution

$OA = 2AM$, $OB = 2AN$, and $OB = 2MN$.
Since they have A in common, triangles AMN and AOB are <u>similar</u>.

15. (a) Circle the value of (1)

$$(5\sqrt{3})^2$$

$$15 \qquad 25\sqrt{3} \qquad 75 \qquad 225$$

Solution

$$(5\sqrt{3})^2 = 25 \times 3 = 75.$$

$$15 \qquad 25\sqrt{3} \qquad \underline{\underline{75}} \qquad 225$$

(b) Simplify fully (3)

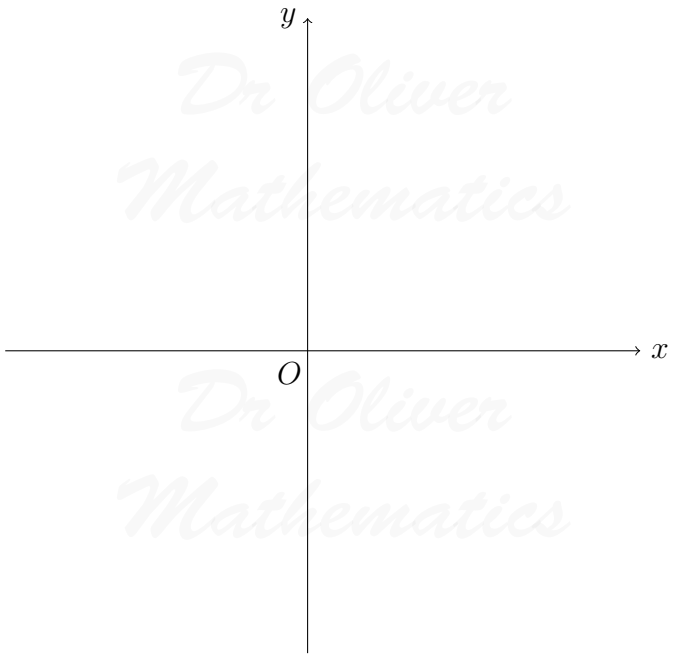
$$(16x^4y^{12})^{\frac{3}{4}}.$$

Solution

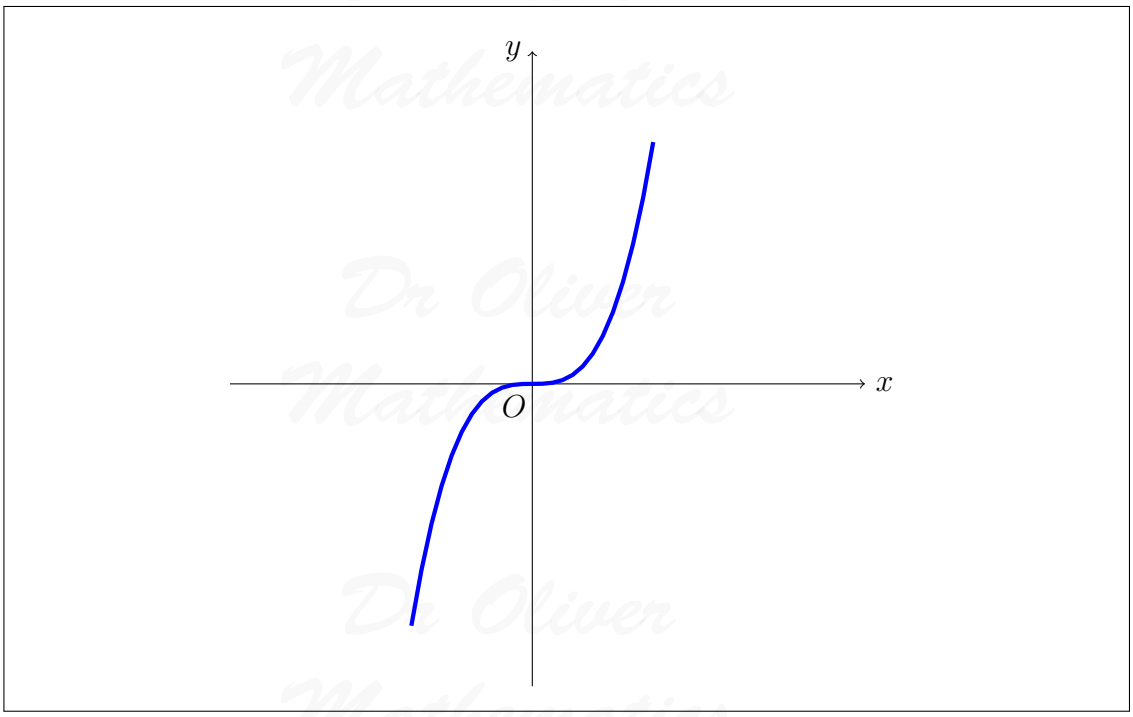
$$\begin{aligned}(16x^4y^{12})^{\frac{3}{4}} &= \left[(16x^4y^{12})^{\frac{1}{4}}\right]^3 \\ &= \left[2xy^3\right]^3 \\ &= \underline{\underline{8x^3y^9}}.\end{aligned}$$

16. (a) Sketch the graph of (1)

$$y = x^3.$$

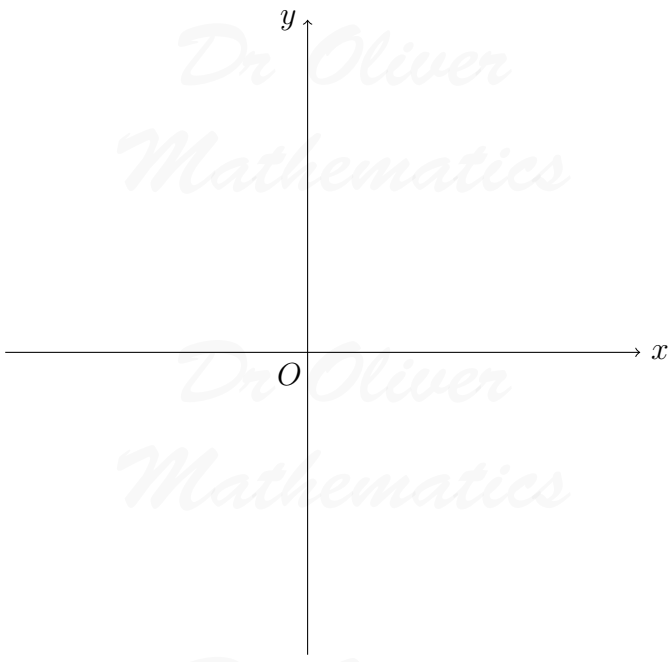


Solution

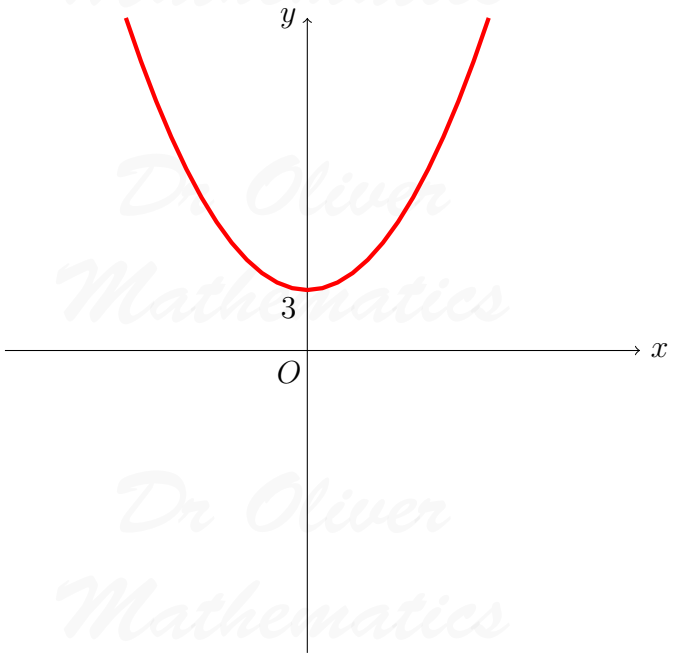


(b) Sketch the graph of (1)

$$y = x^2 + 3.$$

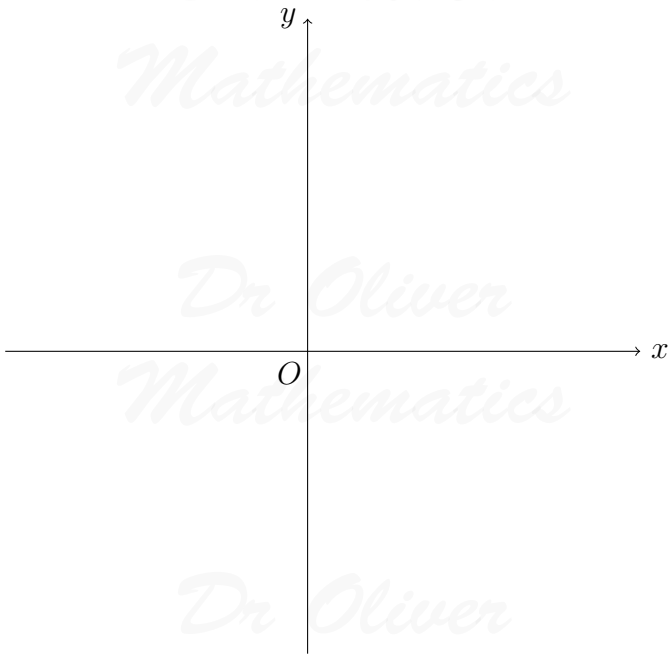


Solution

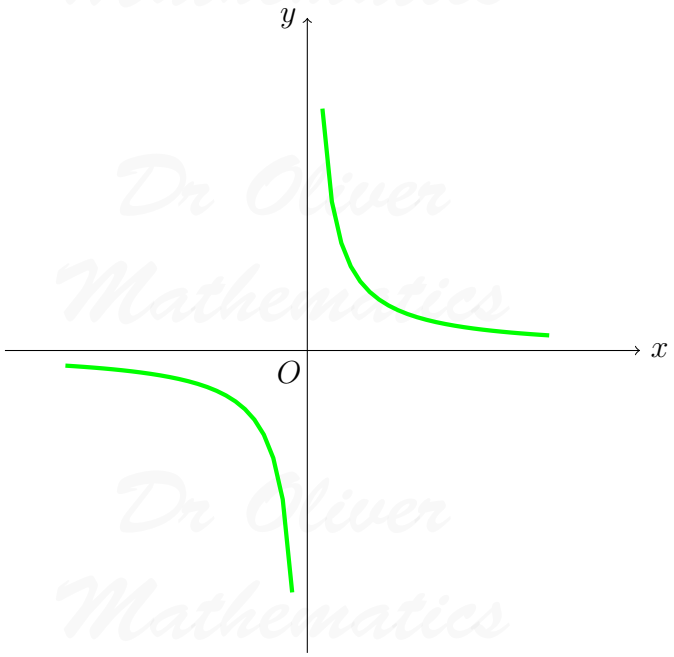


(c) Sketch the graph of (1)

$$y = \frac{1}{x}.$$



Solution



17. y is inversely proportional to x. (3)
When $y = 2$, $x = 5$.

Work out an equation connecting y and x.

Solution

$$y \propto \frac{1}{x} \Rightarrow y = \frac{k}{x},$$

for some constant k. Now,

$$x = 5, y = 2 \Rightarrow 2 = \frac{k}{5}$$
$$\Rightarrow k = 10$$

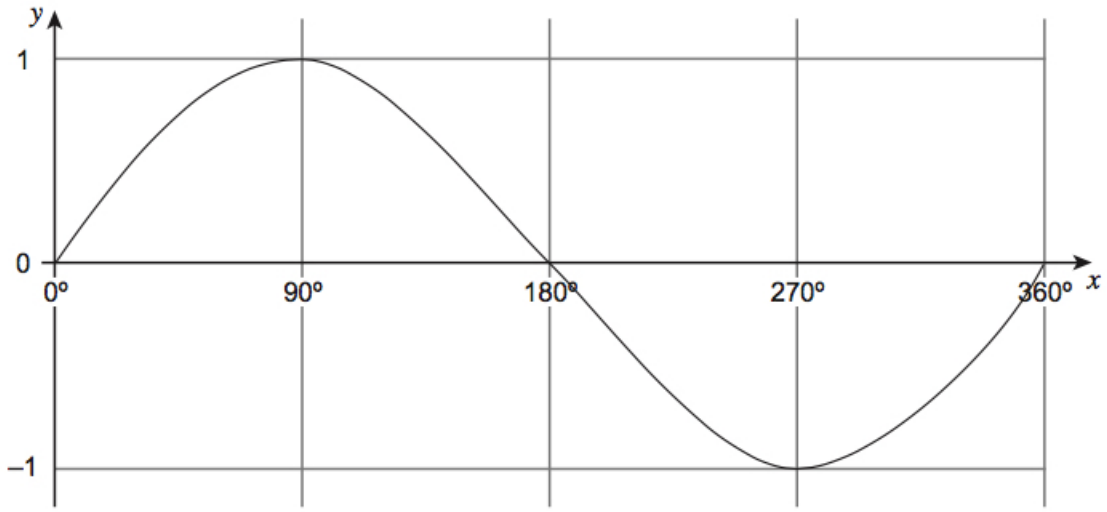
and

$$\underline{\underline{y = \frac{10}{x}}}.$$

18. This is a sketch graph of

$$y = \sin x,$$

for $0° \leqslant x \leqslant 360°$.



(a) Write down the number of solutions for (1)

$$\sin x = 0.5,$$

for $0° \leqslant x \leqslant 360°$.

Solution

2.

$$\sin x = \sin 10.$$

(b) Write down the value of x for $90° \leqslant x \leqslant 180°$. (1)

Solution

$x = 170$.