Dr Oliver Mathematics
Further Mathematics
Poisson Distribution
Past Examination Questions

This booklet consists of 35 questions across a variety of examination topics.
The total number of marks available is 321.

	Symbol	Expectation	Variance	Continuity Correction?
Poisson	$\text{Po}(\lambda)$	$\lambda = np$	$\lambda = np$	No
Normal	$\text{N}(\mu, \sigma^2)$	$\mu = np$	$\sigma^2 = np$	Yes

1. A botanist suggests that the number of a particular variety of weed growing in a meadow can be modelled by a Poisson distribution.

   (a) Write down two conditions that must apply for this model to be applicable. (2)

   Assuming this model and a mean of 0.7 weeds per m^2, find

   (b) the probability that in a randomly chosen plot of size 4 m^2 there will be fewer than 3 of these weeds. (4)

   (c) Using a suitable approximation, find the probability that in a plot of 100 m^2 there will be more than 66 of these weeds. (6)

2. The number of breakdowns per day in a large fleet of hire cars has a Poisson distribution with mean $\frac{1}{7}$.

   (a) Find the probability that on a particular day there are fewer than 2 breakdowns. (3)

   (b) Find the probability that during a 14-day period there are at most 4 breakdowns. (3)

3. Minor defects occur in a particular make of carpet at a mean rate of 0.05 per m^2.

   (a) Suggest a suitable model for the distribution of the number of defects in this make of carpet. Give a reason for your answer. (3)

   A carpet fitter has a contract to fit this carpet in a small hotel. The hotel foyer requires 30 m^2 of this carpet. Find the probability that the foyer carpet contains

   (b) exactly 2 defects, (3)

   (c) more than 5 defects. (3)

   The carpet fitter orders a total of 355 m^2 of the carpet for the whole hotel.

(d) Using a suitable approximation, find the probability that this total area of carpet contains 22 or more defects. (6)

4. The random variables S are distributed as $S \sim \text{Po}(7.5)$. (1)
   Find $\text{P}(S = 5)$.

5. Over a long period of time, accidents happened on a stretch of road at random at a rate of 3 per month.

   Find the probability that

   (a) in a randomly chosen month, more than 4 accidents occurred, (3)

   (b) in a three-month period, more than 4 accidents occurred. (2)

6. The random variable X is the number of misprints per page in the first draft of a novel.

   (a) State two conditions under which a Poisson distribution is a suitable model for X. (2)

   The number of misprints per page has a Poisson distribution with mean 2.5. Find the probability that

   (b) a randomly chosen page has no misprints, (2)

   (c) the total number of misprints on 2 randomly chosen pages is more than 7. (3)

   The first chapter contains 20 pages.

   (d) Using a suitable approximation find, to 2 decimal places, the probability that the chapter will contain less than 40 misprints. (7)

7. Accidents on a particular stretch of motorway occur at an average rate of 1.5 per week.

   (a) Write down a suitable model to represent the number of accidents per week on this stretch of motorway. (1)

   Find the probability that

   (b) there will be 2 accidents in the same week, (2)

   (c) there is at least one accident per week for 3 consecutive weeks, (3)

   (d) there are more than 4 accidents in a two-week period. (3)

8. An estate agent sells properties at a mean rate of 7 per week.

   (a) Suggest a suitable model to represent the number of properties sold in a randomly chosen week. Give two reasons to support your model. (3)

   (b) Find the probability that in any randomly chosen week the estate agent sells exactly 5 properties. (2)

   (c) Using a suitable approximation find the probability that during a 24 week period the estate agent sells more than 181 properties. (6)

9. Breakdowns occur on a particular machine at random at a mean rate of 1.25 per week. (4)

Find the probability that fewer than 3 breakdowns occurred in a randomly chosen week.

10. The random variable J has a Poisson distribution with mean 4. (2)

Find $\mathrm{P}(J \geqslant 10)$.

11. (a) State the condition under which the normal distribution may be used as an approximation to the Poisson distribution. (1)

(b) Explain why a continuity correction must be incorporated when using the normal distribution as an approximation to the Poisson distribution. (1)

A company has yachts that can only be hired for a week at a time. All hiring starts on a Saturday. During the winter the mean number of yachts hired per week is 5.

(c) Calculate the probability that fewer than 3 yachts are hired on a particular Saturday in winter. (2)

During the summer the mean number of yachts hired per week increases to 25. The company has only 30 yachts for hire.

(d) Using a suitable approximation find the probability that the demand for yachts cannot be met on a particular Saturday in the summer. (6)

In the summer there are 16 Saturdays on which a yacht can be hired.

(e) Estimate the number of Saturdays in the summer that the company will not be able to meet the demand for yachts. (2)

12. An engineering company manufactures an electronic component. At the end of the manufacturing process, each component is checked to see if it is faulty. Faulty components are detected at a rate of 1.5 per hour.

(a) Suggest a suitable model for the number of faulty components detected per hour. (1)

(b) Describe, in the context of this question, two assumptions you have made in part (a) for this model to be suitable. (2)

(c) Find the probability of 2 faulty components being detected in a 1 hour period. (2)

(d) Find the probability of at least one faulty component being detected in a 3 hour period. (3)

13. (a) State two conditions under which a Poisson distribution is a suitable model to use in statistical work. (2)

The number of cars passing an observation point in a 10 minute interval is modelled by a Poisson distribution with mean 1.

(b) Find the probability that in a randomly chosen 60 minute period there will be

(i) exactly 4 cars passing the observation point, (3)

(ii) at least 5 cars passing the observation point. (2)

The number of other vehicles, other than cars, passing the observation point in a 60 minute interval is modelled by a Poisson distribution with mean 12.

(c) Find the probability that exactly 1 vehicle, of any type, passes the observation point in a 10 minute period. (4)

14. A call centre agent handles telephone calls at a rate of 18 per hour.

(a) Give two reasons to support the use of a Poisson distribution as a suitable model for the number of calls per hour handled by the agent. (2)

(b) Find the probability that in any randomly selected 15 minute interval the agent handles

(i) exactly 5 calls, (3)

(ii) more than 8 calls. (2)

15. A botanist is studying the distribution of daisies in a field. The field is divided into a number of equal sized squares. The mean number of daisies per square is assumed to be 3. The daisies are distributed randomly throughout the field.

Find the probability that, in a randomly chosen square there will be

(a) more than 2 daisies, (3)

(b) either 5 or 6 daisies. (2)

The botanist decides to count the number of daisies, x, in each of 80 randomly selected squares within the field. The results are summarised below:

$$\Sigma x = 295 \text{ and } \Sigma x^2 = 1386.$$

(c) Calculate the mean and the variance of the number of daisies per square for the 80 squares. Give your answers to 2 decimal places. (3)

(d) Explain how the answers from part (c) support the choice of a Poisson distribution as a model. (1)

(e) Using your mean from part (c), estimate the probability that exactly 4 daisies will be found in a randomly selected square. (2)

16. An administrator makes errors in her typing randomly at a rate of 3 errors every 1000 words.

(a) In a document of 2000 words find the probability that the administrator makes 4 or more errors. (3)

The administrator is given an 8000 word report to type and she is told that the report will only be accepted if there are 20 or fewer errors.

(b) Use a suitable approximation to calculate the probability that the report is accepted. (7)

17. A cloth manufacturer knows that faults occur randomly in the production process at a rate of 2 every 15 metres.

    (a) Find the probability of exactly 4 faults in a 15 metre length of cloth. (2)

    (b) Find the probability of more than 10 faults in 60 metres of cloth. (3)

    A retailer buys a large amount of this cloth and sells it in pieces of length x metres. He chooses x so that the probability of no faults in a piece is 0.80.

    (c) Write down an equation for x and show that $x = 1.7$ to 2 significant figures. (4)

    The retailer sells 1200 of these pieces of cloth. He makes a profit of 60p on each piece of cloth that does not contain a fault but a loss of £1.50 on any pieces that do contain faults.

    (d) Find the retailer's expected profit. (4)

18. A robot is programmed to build cars on a production line. The robot breaks down at random at a rate of once every 20 hours.

    (a) Find the probability that it will work continuously for 5 hours without a breakdown. (2)

    Find the probability that, in an 8 hour period,

    (b) the robot will break down at least once, (3)

    (c) there are exactly 2 breakdowns. (2)

    In a particular 8 hour period, the robot broke down twice.

    (d) Write down the probability that the robot will break down in the following 8 hour period. Give a reason for your answer. (2)

19. A cafe serves breakfast every morning. Customers arrive for breakfast at random at a rate of 1 every 6 minutes.

    Find the probability that

    (a) fewer than 9 customers arrive for breakfast on a Monday morning between 10 am and 11 am. (3)

    The cafe serves breakfast every day between 8 am and 12 noon.

    (b) Using a suitable approximation, estimate the probability that more than 50 customers arrive for breakfast next Tuesday. (6)

20. A company has a large number of regular users logging onto its website. On average 4 users every hour fail to connect to the company's website at their first attempt.

(a) Explain why the Poisson distribution may be a suitable model in this case. (1)

Find the probability that, in a randomly chosen 2 hour period,

(b) (i) all users connect at their first attempt, (3)

(ii) at least 4 users fail to connect at their first attempt. (2)

21. Cars arrive at a motorway toll booth at an average rate of 150 per hour.

(a) Suggest a suitable distribution to model the number of cars arriving at the toll booth, X, per minute. (2)

(b) State clearly any assumptions you have made by suggesting this model. (2)

Using your model,

(c) find the probability that in any given minute

(i) no cars arrive, (1)

(ii) more than 3 cars arrive. (2)

(d) In any given 4 minute period, find m such that $\mathrm{P}(X > m) = 0.0487$. (3)

(e) Using a suitable approximation find the probability that fewer than 15 cars arrive in any given 10 minute period. (6)

22. Defects occur at random in planks of wood with a constant rate of 0.5 per 10 cm length. Jim buys a plank of length 100 cm.

(a) Find the probability that Jim?s plank contains at most 3 defects. (2)

Shivani buys 6 planks each of length 100 cm.

(b) Find the probability that fewer than 2 of Shivani's planks contain at most 3 defects. (5)

(c) Using a suitable approximation, estimate the probability that the total number of defects on Shivani?s 6 planks is less than 18. (6)

23. The probability of a telesales representative making a sale on a customer call is 0.15.

Find the probability that

(a) no sales are made in 10 calls, (2)

(b) more than 3 sales are made in 20 calls. (2)

Representatives are required to achieve a mean of at least 5 sales each day.

(c) Find the least number of calls each day a representative should make to achieve this requirement. (2)

(d) Calculate the least number of calls that need to be made by a representative for the probability of at least 1 sale to exceed 0.95. (3)

24. A website receives hits at a rate of 300 per hour.

(a) State a distribution that is suitable to model the number of hits obtained during a 1 minute interval. (1)

(b) State two reasons for your answer to part (a). (2)

Find the probability of

(c) 10 hits in a given minute, (3)

(d) at least 15 hits in 2 minutes. (3)

The website will go down if there are more than 70 hits in 10 minutes.

(e) Using a suitable approximation, find the probability that the website will go down in a particular 10 minute interval. (7)

25. The number of houses sold by an estate agent follows a Poisson distribution, with a mean of 2 per week.

(a) Find the probability that in the next 4 weeks the estate agent sells

(i) exactly 3 houses, (3)

(ii) more than 5 houses. (2)

The estate agent monitors sales in periods of 4 weeks.

(b) Find the probability that in the next twelve of the 4 week periods there are exactly nine periods in which more than 5 houses are sold. (3)

The estate agent will receive a bonus if he sells more than 25 houses in the next 10 weeks.

(c) Use a suitable approximation to estimate the probability that the estate agent receives a bonus. (6)

26. In a village, power cuts occur randomly at a rate of 3 per year.

(a) Find the probability that in any given year there will be

(i) exactly 7 power cuts, (3)

(ii) at least 4 power cuts. (2)

(b) Use a suitable approximation to find the probability that in the next 10 years the number of power cuts will be less than 20. (6)

27. The number of defects per metre in a roll of cloth has a Poisson distribution with mean 0.25.

Find the probability that

(a) a randomly chosen metre of cloth has 1 defect, (2)

(b) the total number of defects in a randomly chosen 6 metre length of cloth is more than 2. (3)

A tailor buys 300 metres of cloth.

(c) Using a suitable approximation find the probability that the tailor's cloth will contain less than 90 defects. (5)

28. An online shop sells a computer game at an average rate of 1 per day.

(a) Find the probability that the shop sells more than 10 games in a 7 day period. (3)

Once every 7 days the shop has games delivered before it opens.

(b) Find the least number of games the shop should have in stock immediately after a delivery so that the probability of running out of the game before the next delivery is less than 0.05. (3)

29. In a village shop the customers must join a queue to pay. The number of customers joining the queue in a 10 minute interval is modelled by a Poisson distribution with mean 3.

Find the probability that

(a) exactly 4 customers join the queue in the next 10 minutes, (2)

(b) more than 10 customers join the queue in the next 20 minutes. (3)

30. Patients arrive at a hospital accident and emergency department at random at a rate of 6 per hour.

(a) Find the probability that, during any 90 minute period, the number of patients arriving at the hospital accident and emergency department is

(i) exactly 7, (3)

(ii) at least 10. (2)

A patient arrives at 11.30a.m.

(b) Find the probability that the next patient arrives before 11.45a.m. (3)

31. A company claims that it receives emails at a mean rate of 2 every 5 minutes. (2)

Give two reasons why a Poisson distribution could be a suitable model for the number of emails received.

32. Accidents occur randomly at a road junction at a rate of 18 every year. The random variable X represents the number of accidents at this road junction in the next 6 months.

(a) Write down the distribution of X. (2)

(b) Find $\mathrm{P}(X > 7)$. (2)

(c) Show that the probability of at least one accident in a randomly selected month is 0.777 (correct to 3 decimal places). (3)

(d) Find the probability that there is at least one accident in exactly 4 of the next 6 months. (3)

33. In a survey it is found that barn owls occur randomly at a rate of 9 per 1000 km^2.

(a) Find the probability that in a randomly selected area of 1000 km^2 there are at least 10 barn owls. (2)

(b) Find the probability that in a randomly selected area of 200 km^2 there are exactly 2 barn owls. (3)

(c) Using a suitable approximation, find the probability that in a randomly selected area of 50 000 km^2 there are at least 470 barn owls. (6)

34. The number of cherries in a *Rays* fruit cake follows a Poisson distribution with mean 1.5.

A *Rays* fruit cake is to be selected at random.

Find the probability that it contains

(a) (i) exactly 2 cherries, (2)

(ii) at least 1 cherry. (2)

Rays fruit cakes are sold in packets of 5.

(b) Show that the probability that there are more than 10 cherries, in total, in a randomly selected packet of *Rays* fruit cakes, is 0.1378 correct to 4 decimal places. (3)

35. A company receives telephone calls at random at a mean rate of 2.5 per hour.

(a) Find the probability that the company receives (5)

(i) at least 4 telephone calls in the next hour,

(ii) exactly 3 telephone calls in the next 15 minutes.

(b) Find, to the nearest minute, the maximum length of time the telephone can be left unattended so that the probability of missing a telephone call is less than 0.2. (3)