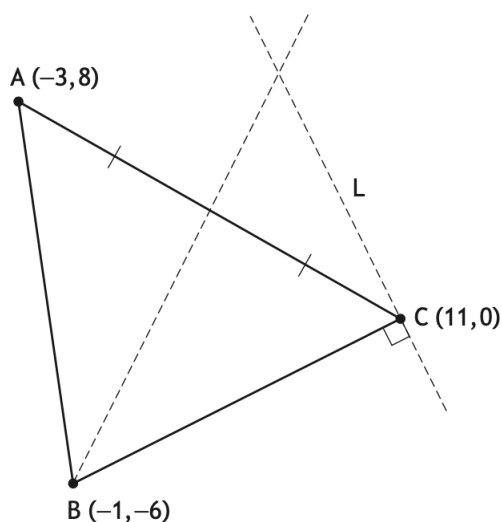


**Dr Oliver Mathematics**  
**Mathematics: Higher**  
**2024 Paper 2: Calculator**  
**1 hour 30 minutes**

The total number of marks available is 65.

You must write down all the stages in your working.

1. Triangle  $ABC$  has vertices  $A(3, 8)$ ,  $B(1, 6)$ , and  $C(11, 0)$ .



- (a) Find the equation of the median through  $B$ . (3)
- (b) Find the equation of  $L$ , the line perpendicular to  $BC$  passing through  $C$ . (3)
- (c) Determine the coordinates of the point of intersection of the median through  $B$  and the line  $L$ . (2)
2. A curve has equation (5)

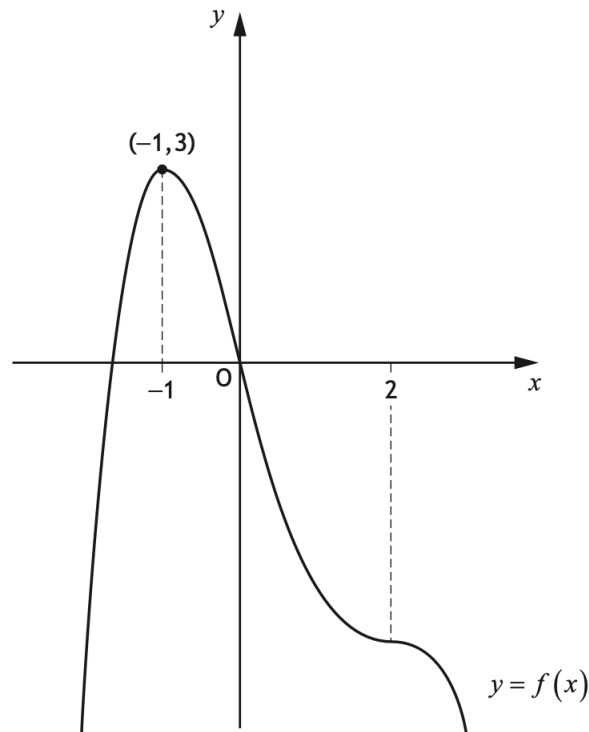
$$y = \frac{8}{x^3}, x > 0.$$

Find the equation of the tangent to this curve at the point where  $x = 2$ .

3. The coordinates of points  $D$ ,  $E$ , and  $F$  are given by  $D(2, -3, 4)$ ,  $E(1, 1, -2)$ , and  $F(3, 2, 1)$ .
- (a) Express  $\overrightarrow{ED}$  and  $\overrightarrow{EF}$  in component form. (2)
- (b) (i) Calculate (1)
- $$\overrightarrow{ED} \cdot \overrightarrow{EF}.$$

(ii) Hence, or otherwise, calculate the size of angle  $DEF$ . (4)

4. The diagram shows the graph of a quartic function  $y = f(x)$ .



- A maximum turning point occurs at  $(-1, 3)$ .
- The graph of  $y = f(x)$  also has a point of inflection at  $x = 2$ .

(a) Determine the coordinates of the maximum turning point on the graph of (2)

$$y = f(x - 4) + 2.$$

(b) Sketch the graph of (3)

$$y = f'(x).$$

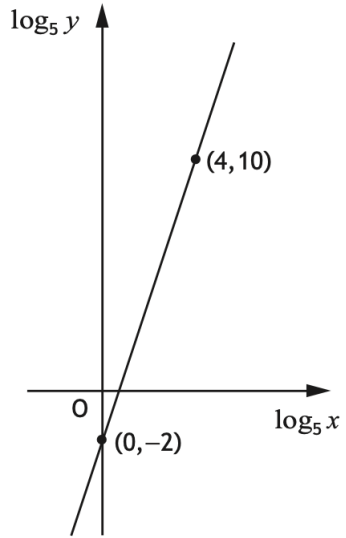
5. Evaluate (3)

$$\int_0^{\frac{1}{7}\pi} \sin 5x \, dx.$$

6. Two variables,  $x$  and  $y$ , are connected by the equation (5)

$$y = ax^b.$$

The graph of  $\log_5 y$  against  $\log_5 x$  is a straight line as shown.



Find the values of  $a$  and  $b$ .

7. The diagram shows the curve with equation

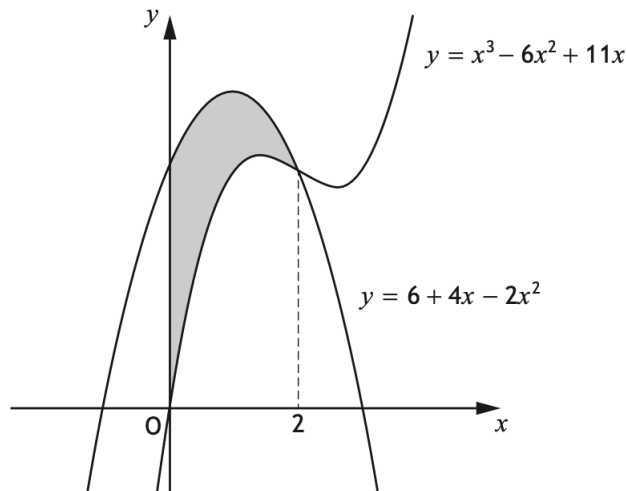
$$y = x^3 - 6x^2 + 11x$$

(5)

intersecting the curve with equation

$$y = 6 + 4x - 2x^2$$

at  $x = 2$ .



Calculate the shaded area.

8. Functions  $f$  and  $g$  are defined on  $\mathbb{R}$ , the set of real numbers, by:

- $f(x) = 2x^2 - 18$ .

- $g(x) = x + 1$ .

(a) Find an expression for  $f(g(x))$ . (2)

(b) Find the values of  $x$  for which (2)

$$\frac{1}{f(g(x))}$$

is undefined.

9. (a) Determine the coordinates of the stationary points on the curve with equation (4)

$$y = \frac{1}{3}x^3 - x^2 - 3x + 1.$$

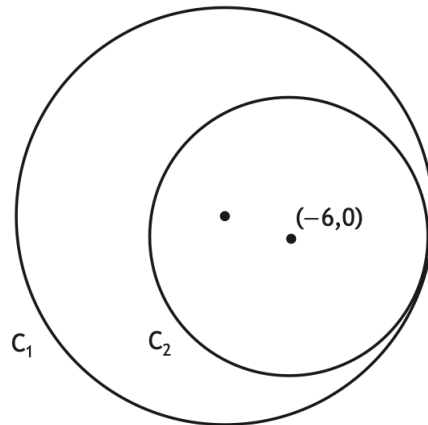
(b) Hence, determine the greatest and least values of  $y$  in the interval  $-1 \leq x \leq 6$ . (2)

10. The circle  $C_1$  has equation

$$x^2 + y^2 + 18x - 2y - 8 = 0.$$

(a) Find the centre and radius of  $C_1$ . (2)

A second circle,  $C_2$ , touches  $C_1$  internally.



The centre of  $C_2$  is  $(6, 0)$ .

(b) Determine the equation of  $C_2$ . (2)

11. The number of electric vehicles worldwide can be modelled by

$$N = 6.8e^{kt},$$

where:

- $N$  is the estimated number of vehicles in millions,
- $t$  is the number of years since the end of 2020, and
- $k$  is a constant.

(a) Use the model to estimate the number of electric vehicles worldwide at the end of 2020. (1)

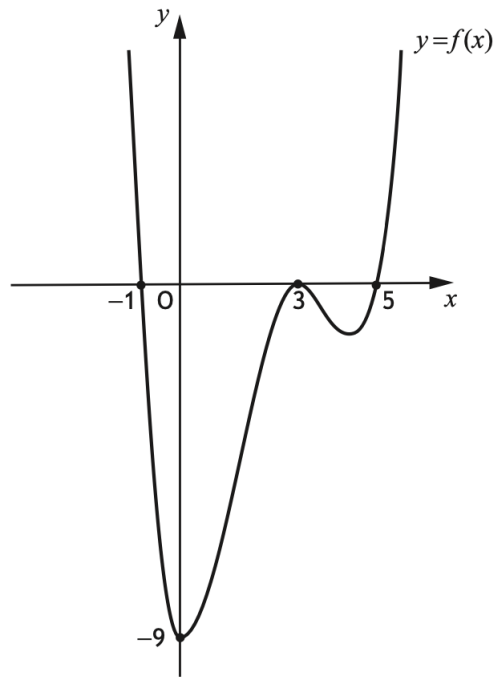
At the end of 2030, it is estimated there will be 125 million electric vehicles worldwide.

(b) Determine the value of  $k$ . (4)

12. Solve the equation (5)

$$2 \sin 2x^\circ - \sin^2 x^\circ = 0, 0 \leq x < 360.$$

13. The diagram shows the graph of  $y = f(x)$ , where  $f(x)$  is a quartic function. (3)



Express  $f(x)$  in the form

$$f(x) = k(x + a)^2(x + b)(x + c).$$