Dr Oliver Mathematics
Mathematics
Statics and Dynamics
Past Examination Questions

This booklet consists of 68 questions across a variety of examination topics.
The total number of marks available is 735.

1. In Figure 1, $\angle AOC = 90°$ and $\angle BOC = \theta°$.

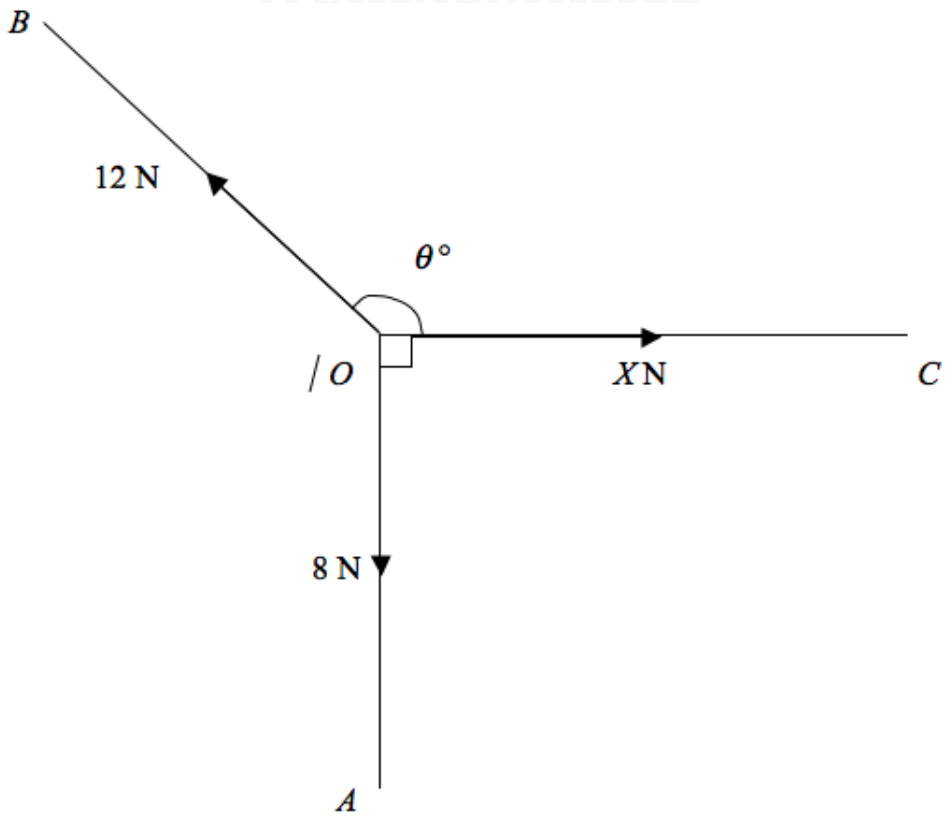


Figure 1: three planar forces

A particle at O is in equilibrium under the action of three coplanar forces. The three forces have magnitude 8 N, 12 N, and X N and act along OA, OB, and OC respectively. Calculate

(a) the value, to one decimal place, of θ, (3)

Solution

$$\begin{aligned} 8 = 12\cos(\theta - 90)^\circ &\Rightarrow \cos(\theta - 90) = \tfrac{2}{3} \\ &\Rightarrow \theta - 90 = 48.189\,685\,1 \text{ (FCD)} \\ &\Rightarrow \theta = 138.189\,685\,1 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{\theta = 138.2 \text{ (1 dp)}}}. \end{aligned}$$

(b) the value, to 2 decimal places, of X. (3)

Solution

$$\begin{aligned} X = 12\cos(180 - \theta)^\circ &\Rightarrow X = 8.944\,271\,91 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{X = 8.94 \text{ (2 dp)}}}. \end{aligned}$$

2. A box of mass 1.5 kg is placed on a plane which is inclined at an angle of 30° to the horizontal. The coefficient of friction between the box and plane is $\frac{1}{3}$. The box is kept in equilibrium by a light string which lies in a vertical plane containing a line of greatest slope of the plane. The string makes an angle of 20° with the plane, as shown in Figure 2. (10)

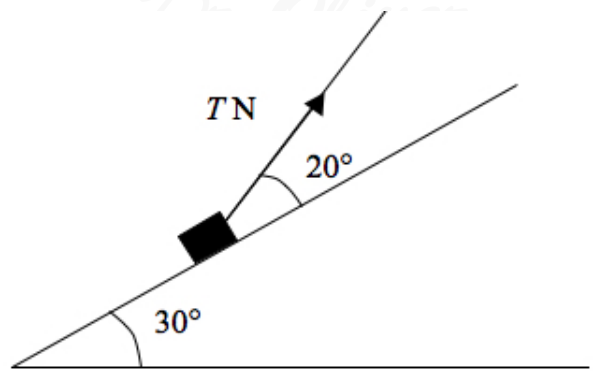


Figure 2: a box of mass 1.5 kg

The box is in limiting equilibrium and is about to move up the plane. The tension in the string is T newtons. The box is modelled as a particle.

Find the value of T.

Solution

Let F N be the friction and let R N be the normal reaction.

$$\begin{aligned} \text{Parallel:} \quad & T\cos 20^\circ = F + 1.5g\sin 30^\circ \\ \text{Perpendicular:} \quad & R + T\sin 20^\circ = 1.5g\cos 30^\circ \\ F = \mu R: \quad & F = \tfrac{1}{3}R. \end{aligned}$$

Now,

$$\begin{aligned} & T\cos 20^\circ = \tfrac{1}{3}R + 1.5g\sin 30^\circ \\ \Rightarrow \quad & T\cos 20^\circ = \tfrac{1}{3}(1.5g\cos 30^\circ - T\sin 20^\circ) + 1.5g\sin 30^\circ \\ \Rightarrow \quad & T\cos 20^\circ = 0.5g\cos 30^\circ - \tfrac{1}{3}T\sin 20^\circ + 1.5g\sin 30^\circ \\ \Rightarrow \quad & T\cos 20^\circ + \tfrac{1}{3}T\sin 20^\circ = 0.5g\cos 30^\circ + 1.5g\sin 30^\circ \\ \Rightarrow \quad & T(\cos 20^\circ + \tfrac{1}{3}\sin 20^\circ) = \tfrac{1}{2}g(\cos 30^\circ + 3\sin 30^\circ) \\ \Rightarrow \quad & T = \frac{\tfrac{1}{2}g(\cos 30^\circ + 3\sin 30^\circ)}{\cos 20^\circ + \tfrac{1}{3}\sin 20^\circ} \\ \Rightarrow \quad & T = 11.002\,687\,48 \text{ (FCD)} \\ \Rightarrow \quad & \underline{\underline{T = 11 \text{ (2 sf)}}}. \end{aligned}$$

3. A particle A of mass 0.8 kg rests on a horizontal table and is attached to one end of a light inextensible string. The string passes over a small smooth pulley P fixed at the edge of the table. The other end of the string is attached to a particle B of mass 1.2 kg which hangs freely below the pulley, as shown in Figure 3.

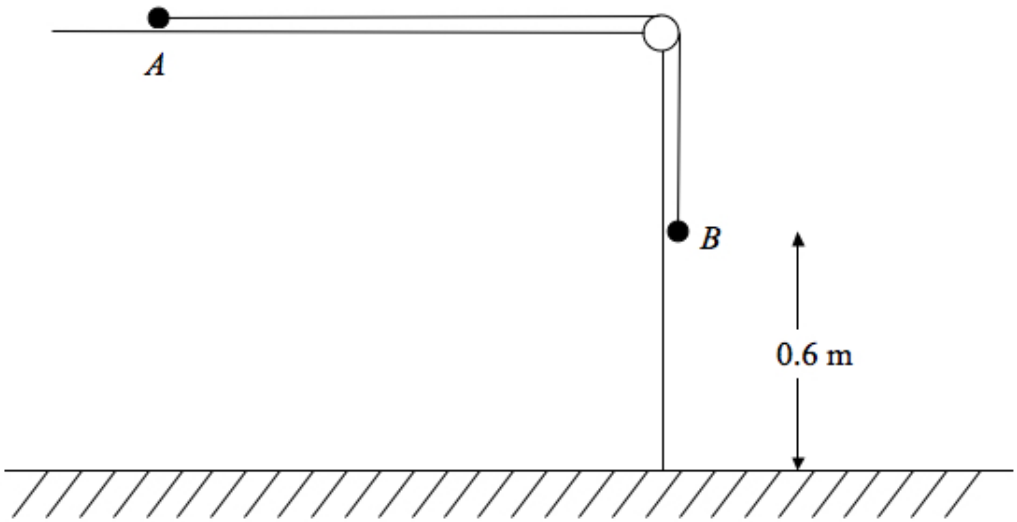


Figure 3: two particles, A and B

The system is released from rest with the string taut and with B at a height of 0.6 m above the ground. In the subsequent motion A does not reach P before B reaches the ground. In an initial model of the situation, the table is assumed to be smooth. Using this model, find

(a) the tension in the string before B reaches the ground, (5)

Solution

Let T N be the tension.

$$\begin{aligned} A: &\quad T = 0.8a \\ B: &\quad 1.2g - T = 1.2a. \end{aligned}$$

Now,

$$\begin{aligned} T = 0.8 \times \tfrac{5}{6} \times (1.2g - T) &\Rightarrow T = \tfrac{2}{3}(1.2g - T) \\ &\Rightarrow T = \tfrac{4}{5}g - \tfrac{2}{3}T \\ &\Rightarrow \tfrac{5}{3}T = \tfrac{4}{5}g \\ &\Rightarrow T = \tfrac{12}{25}g \\ &\Rightarrow T = 4.704 \\ &\Rightarrow \underline{\underline{T = 4.7 \text{ (2 sf)}}}. \end{aligned}$$

(b) the time taken by B to reach the ground. (3)

Solution

We need a:

$$T = 0.8a \Rightarrow a = 5.88.$$

For B, $s = 0.6, u = 0, v = ?, a = 5.88, t = ?$:

$$\begin{aligned} s = ut + \tfrac{1}{2}at^2 &\Rightarrow 0.6 = 2.94t^2 \\ &\Rightarrow t^2 = \tfrac{10}{49} \\ &\Rightarrow t = \pm 0.451\,753\,951\,5 \text{ (FCD)}; \end{aligned}$$

clearly, $\underline{\underline{t = 0.45 \text{ (2 sf)}}}$.

In a refinement of the model, it is assumed that the table is rough and that the coefficient of friction between A and the table is $\frac{1}{5}$. Using this refined model,

(c) find the time taken by B to reach the ground. (8)

Solution

Let F N be the friction and let R N be the normal reaction:

$$R = 0.8g.$$

$$\begin{aligned} A: &\quad T - F = 0.8a \\ B: &\quad 1.2g - T = 1.2a \\ F = \mu R: &\quad F = \tfrac{1}{5}R. \end{aligned}$$

Now,

$$\begin{aligned} T - \tfrac{1}{5}R = \tfrac{2}{3}(1.2g - T) &\Rightarrow T - \tfrac{4}{25}g = \tfrac{2}{3}(1.2g - T) \\ &\Rightarrow T - \tfrac{4}{25}g = \tfrac{4}{5}g - \tfrac{2}{3}T \\ &\Rightarrow \tfrac{5}{3}T = \tfrac{24}{25}g \\ &\Rightarrow T = \tfrac{72}{125}g \\ &\Rightarrow a = 5.096. \end{aligned}$$

Finally, $s = 0.6, u = 0, v = ?, a = 5.096, t = ?$:

$$\begin{aligned} s = ut + \tfrac{1}{2}at^2 &\Rightarrow 0.6 = 2.548^2 \\ &\Rightarrow t^2 = \tfrac{150}{637} \\ &\Rightarrow t = \pm 0.485\,261\,586\,1 \text{ (FCD)}; \end{aligned}$$

clearly, $\underline{\underline{t = 0.49 \text{ (2 sf)}}}$.

4. A particle P of mass 2.5 kg rests in equilibrium on a rough plane under the action of a force of magnitude X newtons acting up a line of greatest slope of the plane, as shown in Figure 4.

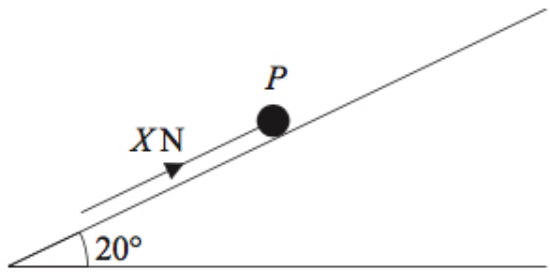


Figure 4: a particle P of mass 2.5 kg

The plane is inclined at 20° to the horizontal. The coefficient of friction between P and the plane is 0.4. The particle is in limiting equilibrium and is on the point of moving up the plane. Calculate

(a) the normal reaction of the plane on P, (2)

Solution

Let R the normal reaction.

$$R = 2.5g\cos 20^\circ \Rightarrow R = 23.022\,469\,21 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{R = 23 \text{ (2 sf)}}}.$$

(b) the value of X. (4)

Solution

$$X = 0.4R + 2.5g\sin 20^\circ \Rightarrow X = 17.588\,481\,2 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{X = 18 \text{ (2 sf)}}}.$$

The force of magnitude X newtons is now removed.

(c) Show that P remains in equilibrium on the plane. (4)

Solution

Now,

$$F = 2.5g\sin 20^\circ = 8.379\,495\,511 \text{ (FCD)}$$

while

$$\mu R = 0.4 \times 2.5g\cos 20^\circ = 9.208\,987\,684 \text{ (FCD)};$$

P remains in equilibrium as $F < \mu R$.

5. A block of wood A of mass 0.5 kg rests on a rough horizontal table and is attached to one end of a light inextensible string. The string passes over a small smooth pulley P fixed at the edge of the table. The other end of the string is attached to a ball B of mass 0.8 kg which hangs freely below the pulley, as shown in Figure 5.

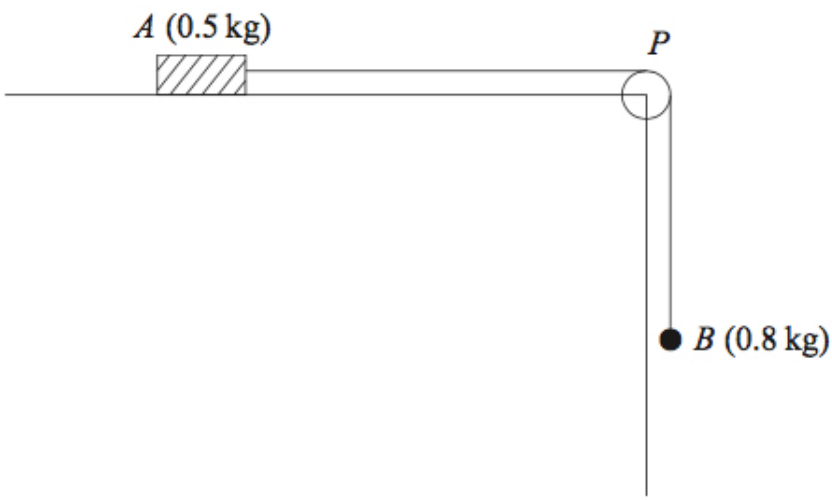


Figure 5: two masses, A and B

The coefficient of friction between A and the table is μ. The system is released from rest with the string taut. After release, B descends a distance of 0.4 m in 0.5 s. Modelling A and B as particles, calculate

(a) the acceleration of B, (3)

Solution

For B, $s = 0.4, u = 0, v = ?, a = ?, t = 0.5$:

$$s = ut + \tfrac{1}{2}at^2 \Rightarrow 0.4 = 0 + \tfrac{1}{2} \times a \times 0.5^2$$
$$\Rightarrow a = 3.2 \text{ ms}^{-2}.$$

(b) the tension in the string, (4)

Solution
Let T be the tension. Then

$$\begin{aligned} F = ma &\Rightarrow 0.8g - T = 0.8 \times 3.2 \\ &\Rightarrow T = 5.28 \\ &\Rightarrow \underline{\underline{T = 5.3 \text{ N (2 sf)}}}. \end{aligned}$$

(c) the value of μ. (5)

Solution
Let R N be the normal reaction. Then $R = 0.5g$ and

$$\begin{aligned} T - \mu R = 0.5 \times 3.2 &\Rightarrow \mu R = 5.28 - 1.6 \\ &\Rightarrow 4.9\mu = 3.68 \\ &\Rightarrow \mu = 0.751\,020\,040\,82 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{\mu = 0.75 \text{ (2 sf)}}}. \end{aligned}$$

(d) State how in your calculations you have used the information that the string is inextensible. (1)

Solution
We have the <u>same acceleration</u> for A and B.

6. A smooth bead B is threaded on a light inextensible string. The ends of the string are attached to two fixed points A and C on the same horizontal level. The bead is held in equilibrium by a horizontal force of magnitude 6 N acting parallel to AC. The bead B is vertically below C and $\angle BAC = \alpha$, as shown in Figure 6.

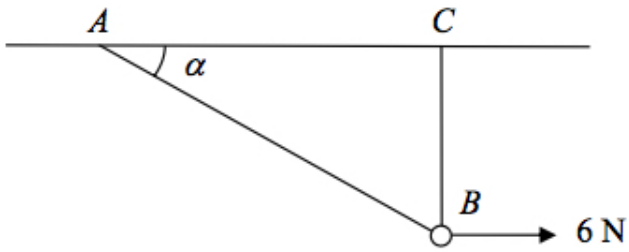


Figure 6: a smooth bead

Given that $\tan\alpha = \frac{3}{4}$, find

(a) the tension in the string, (3)

Solution

Let T N be the tension in the string. Then

$$\sin\alpha = \tfrac{3}{5} \text{ and } \cos\alpha = \tfrac{4}{5}.$$

Then

$$\begin{aligned} T\cos\alpha = 6 &\Rightarrow \tfrac{4}{5}T = 6 \\ &\Rightarrow \underline{\underline{T = 7.5 \text{ N}}}. \end{aligned}$$

(b) the weight of the bead. (4)

Solution

Let W be the weight of the bead. Then

$$\begin{aligned} T\sin\alpha + T = W &\Rightarrow W = \tfrac{3}{5} \times 7.5 + 7.5 \\ &\Rightarrow W = 4.5 + 7.5 \\ &\Rightarrow \underline{\underline{W = 12 \text{ N}}}. \end{aligned}$$

7. A box of mass 2 kg is pulled up a rough plane face by means of a light rope. The plane is inclined at an angle of 20° to the horizontal, as shown in Figure 7.

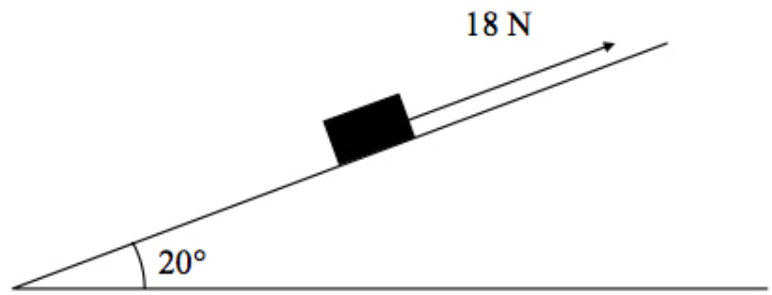


Figure 7: a box of mass 2 kg

The rope is parallel to a line of greatest slope of the plane. The tension in the rope is 18 N. The coefficient of friction between the box and the plane is 0.6. By modelling the box as a particle, find

(a) the normal reaction of the plane on the box, (3)

Solution

Let R N be the normal reaction, let F N be the friction, and let a ms^{-2} be the acceleration.

$$\begin{aligned} \text{Parallel:} \quad & 18 - F - 2g\sin 20^\circ = 2a \\ \text{Perpendicular:} \quad & R = 2g\cos 20^\circ \\ F = \mu R: \quad & F = 0.6R. \end{aligned}$$

Now,

$$\begin{aligned} R &= 2g\cos 20^\circ \\ &= 18.417\,975\,37 \text{ (FCD)} \\ &= \underline{\underline{18 \text{ (2 sf)}}}. \end{aligned}$$

(b) the acceleration of the box. (5)

Solution

$$\begin{aligned} 18 - F - 2g\sin 20^\circ = 2a &\Rightarrow 18 - 0.6R - 2g\sin 20^\circ = 2a \\ &\Rightarrow a = 9 - 0.3R - g\sin 20^\circ \\ &\Rightarrow a = 0.122\,809\,985\,2 \text{ (FCD)} \\ &= \underline{\underline{0.12 \text{ (2 sf)}}}. \end{aligned}$$

8. Figure 8 shows a lorry of mass 1600 kg towing a car of mass 900 kg along a straight horizontal road.

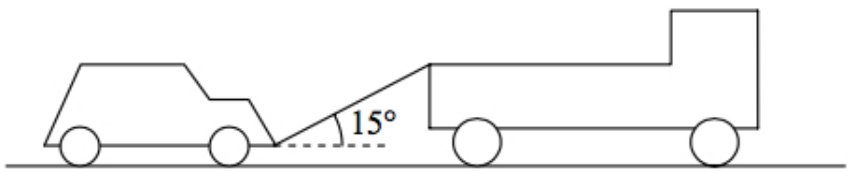


Figure 8: a lorry and a car

The two vehicles are joined by a light towbar which is at an angle of 15° to the road. The lorry and the car experience constant resistances to motion of magnitude 600 N and 300 N respectively. The lorry's engine produces a constant horizontal force on the lorry of magnitude 1500 N. Find

(a) the acceleration of the lorry and the car, (3)

Solution

Let T be the tension and let a be the acceleration.

$$\begin{aligned} \text{Lorry}: &\quad 1500 - 600 - T\cos 15^\circ = 1600a \\ \text{Car}: &\quad T\cos 15^\circ - 300 = 900a. \end{aligned}$$

Add:

$$\begin{aligned} & 1600a + 900a = (1500 - 600 - T\cos 15^\circ) + (T\cos 15^\circ - 300) \\ \Rightarrow\ & 2500a = 300 \\ \Rightarrow\ & \underline{\underline{a = 0.24}}. \end{aligned}$$

(b) the tension in the towbar. (4)

Solution

You could use either the car or lorry: we will pick the car.

$$\begin{aligned} T\cos 15^\circ - 300 = 900a &\Rightarrow T\cos 15^\circ = 516 \\ &\Rightarrow T = 534.202\,509\,1 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{T = 530 \text{ (2 sf)}}}. \end{aligned}$$

When the speed of the vehicles is 6 ms^{-1}, the towbar breaks. Assuming that the resistance to the motion of the car remains of constant magnitude 300 N,

(c) find the distance moved by the car from the moment the towbar breaks to the moment when the car comes to rest. (4)

Solution

$$F = ma \Rightarrow -300 = 900a$$
$$\Rightarrow a = -\tfrac{1}{3}.$$

$s = ?, u = 6, v = 0, a = -\frac{1}{3}, t = ?$:

$$v^2 = u^2 + 2as \Rightarrow 0 = 6^2 - \tfrac{2}{3}s$$
$$\Rightarrow \tfrac{2}{3}s = 36$$
$$\Rightarrow \underline{\underline{s = 54 \text{ m}}}.$$

(d) State whether, when the towbar breaks, the normal reaction of the road on the car is increased, decreased or remains constant. Give a reason for your answer. (2)

Solution

The vertical component of T is no longer there. Hence, the normal reaction is now <u>increased</u>.

9. Two forces $\mathbf{P}$ and $\mathbf{Q}$ act on a particle. The force $\mathbf{P}$ has magnitude 7 N and acts due north. The resultant of $\mathbf{P}$ and $\mathbf{Q}$ is a force of magnitude 10 N acting in a direction with bearing 120°. Find

(a) the magnitude of $\mathbf{Q}$, (6)

Solution

Let $\mathbf{R}$ N be the resultant. Then

$$\mathbf{R} = 10\cos 30^\circ \mathbf{i} - 10\sin 30^\circ \mathbf{j}$$
$$= 5\sqrt{3}\mathbf{i} - 5\mathbf{j}.$$

Now,

$$\mathbf{Q} = \mathbf{R} - \mathbf{P}$$
$$= (5\sqrt{3}\mathbf{i} - 5\mathbf{j}) - 7\mathbf{j}$$
$$= 5\sqrt{3}\mathbf{i} - 12\mathbf{j}.$$

Finally,

$$\begin{aligned}|\mathbf{Q}| &= \sqrt{\left(5\sqrt{3}\right)^2 + (-12)^2} \\ &= \sqrt{75 + 144} \\ &= \underline{\underline{\sqrt{219}}}.\end{aligned}$$

(b) the direction of $\mathbf{Q}$, giving your answer as a bearing. (3)

Solution

We want the angle with $\mathbf{i}$:

$$\arctan\left(\frac{-12}{5\sqrt{3}}\right) = -54.182\,474\,436 \text{ (FCD)}$$

and we add to 90:

$$\text{angle} = 90 + 54.182\ldots = \underline{\underline{144^\circ \text{ (nearest angle)}}}.$$

10. A parcel of weight 10 N lies on a rough plane inclined at an angle of 30° to the horizontal. A horizontal force of magnitude P newtons acts on the parcel, as shown in Figure 9.

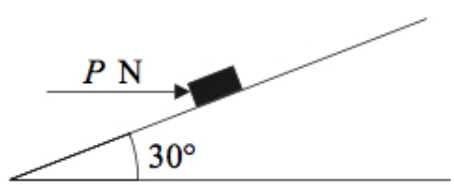


Figure 9: a parcel

The parcel is in equilibrium and on the point of slipping up the plane. The normal reaction of the plane on the parcel is 18 N. The coefficient of friction between the parcel and the plane is μ. Find

(a) the value of P, (4)

Solution

Let F N be the friction.

$$\begin{aligned} \text{Parallel:} \quad & F + 10\sin 30^\circ - P\cos 30^\circ = 0 \\ \text{Perpendicular:} \quad & P\sin 30^\circ + 10\cos 30^\circ = 18 \\ F = \mu R: \quad & F = 18\mu \end{aligned}$$

Now,

$$\begin{aligned} P\sin 30^\circ + 10\cos 30^\circ = 18 &\Rightarrow P\sin 30^\circ = 18 - 5\sqrt{3} \\ &\Rightarrow P = 18.679\,491\,92 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{P = 19 \text{ (2 sf)}}}. \end{aligned}$$

(b) the value of μ. (5)

Solution

$$\begin{aligned} F + 10\sin 30^\circ - P\cos 30^\circ = 0 &\Rightarrow 18\mu = P\cos 30^\circ - 5 \\ &\Rightarrow \mu = 0.620\,939\,696\,5 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{\mu = 0.62 \text{ (2 sf)}}}. \end{aligned}$$

The horizontal force is removed.

(c) Determine whether or not the parcel moves. (5)

Solution

Normal reaction is now $10\cos 30^\circ = 5\sqrt{3}$.
Component of weight down the plane $10\sin 30^\circ = 5$.
Now,

$$F = \mu R = 5.377\,495\,514 \text{ (FCD)};$$

no, <u>the parcel does not slide</u> as $5.377\ldots > 5$.

11. A fixed wedge has two plane faces, each inclined at 30° to the horizontal. Two particles A and B, of mass $3m$ and m respectively, are attached to the ends of a light inextensible string. Each particle moves on one of the plane faces of the wedge. The string passes over a small smooth light pulley fixed at the top of the wedge. The face on which A moves is smooth. The face on which B moves is rough. The coefficient of friction between B and this face is μ. Particle A is held at rest with the string taut. The string lies in the

same vertical plane as lines of greatest slope on each plane face of the wedge, as shown in Figure 10.

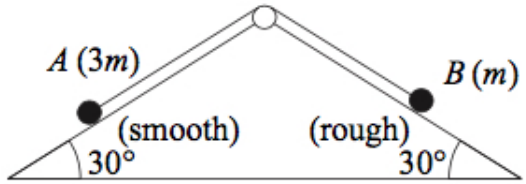


Figure 10: two particles, A and B

The particles are released from rest and start to move. Particle A moves downwards and B moves upwards. The accelerations of A and B each have magnitude $\frac{1}{10}g$.

(a) By considering the motion of A, find, in terms of m and g, the tension in the string. (3)

Solution

Let T N be the tension and let R_1 N be the normal reaction.

$$\begin{aligned} \text{Parallel:} \quad & 3m\sin 30^\circ - T = \tfrac{3}{10}mg \\ \text{Perpendicular:} \quad & R_1 = 3mg\cos 30^\circ. \end{aligned}$$

Now,

$$\begin{aligned} 3mg\sin 30^\circ - T = \tfrac{3}{10}mg &\Rightarrow T = 3mg\sin 30^\circ - \tfrac{3}{10}mg \\ &\Rightarrow \underline{\underline{T = \tfrac{6}{5}mg}}. \end{aligned}$$

(b) By considering the motion of B, find the value of μ. (8)

Solution

Let R_2 N be the normal reaction.

$$\begin{aligned} \text{Parallel:} \quad & T - F - mg\sin 30^\circ = \tfrac{1}{10}mg \\ \text{Perpendicular:} \quad & R_2 = mg\cos 30^\circ \\ F = \mu R: \quad & F = \mu R_2. \end{aligned}$$

Then,

$$\begin{aligned} T - F - mg\sin 30^\circ = \tfrac{1}{10}mg &\Rightarrow \tfrac{6}{5}mg - \mu R_2 - mg\sin 30^\circ = \tfrac{1}{10}mg \\ &\Rightarrow \tfrac{6}{5}mg - \mu mg\cos 30^\circ - mg\sin 30^\circ = \tfrac{1}{10}mg \\ &\Rightarrow \tfrac{6}{5} - \mu\cos 30^\circ - \sin 30^\circ = \tfrac{1}{10} \\ &\Rightarrow \mu\cos 30^\circ = \tfrac{6}{5} - \tfrac{1}{10} - \tfrac{1}{2} \\ &\Rightarrow \tfrac{\sqrt{3}}{2}\mu = \tfrac{3}{5} \\ &\Rightarrow \underline{\underline{\mu = \tfrac{2\sqrt{3}}{5}}}. \end{aligned}$$

(c) Find the resultant force exerted by the string on the pulley, giving its magnitude and direction. (3)

Solution

$2T\cos 60^\circ = \underline{\underline{\tfrac{6}{5}mg}}$ and it is <u>vertically downwards</u>.

12. A particle P of mass 0.5 kg is on a rough plane inclined at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$. The particle is held at rest on the plane by the action of a force of magnitude 4 N acting up the plane in a direction parallel to a line of greatest slope of the plane, as shown in Figure 11.

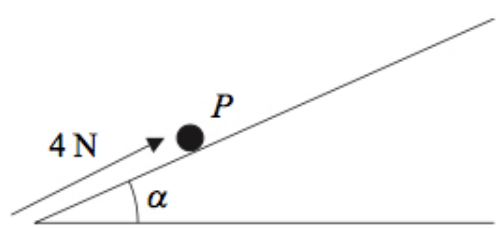


Figure 11: a particle P of mass 0.5 kg

The particle is on the point of slipping up the plane.

(a) Find the coefficient of friction between P and the plane. (7)

Solution

Let R N be the normal reaction and let F N be the friction. Then

$$\sin\alpha = \tfrac{3}{5} \text{ and } \cos\alpha = \tfrac{4}{5}.$$

$$\begin{aligned} \text{Parallel:} \quad & 4 - F - \tfrac{3}{10}g = 0 \\ \text{Perpendicular:} \quad & R = \tfrac{2}{5}g \\ F = \mu R: \quad & F = \mu R. \end{aligned}$$

Then,

$$\begin{aligned} 4 - F - \tfrac{3}{10}g = 0 &\Rightarrow 4 - \mu R - \tfrac{3}{10}g = 0 \\ &\Rightarrow \tfrac{2}{5}\mu g = 4 - \tfrac{3}{10}g \\ &\Rightarrow \mu = 0.270\,408\,163\,3 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{\mu = 0.27 \text{ (2 sf)}}}. \end{aligned}$$

The force of magnitude 4 N is removed.

(b) Find the acceleration of P down the plane. (4)

Solution

$$\begin{aligned} \text{Parallel:} \quad & \tfrac{3}{10}g - F = \tfrac{1}{2}a \\ \text{Perpendicular:} \quad & R = \tfrac{2}{5}g. \end{aligned}$$

Now,

$$\begin{aligned} \tfrac{3}{10}g - F = \tfrac{1}{2}a &\Rightarrow \tfrac{3}{10}g - \tfrac{2}{5}\mu g = \tfrac{1}{2}a \\ &\Rightarrow \underline{\underline{a = 3.76 \text{ ms}^{-2}}}. \end{aligned}$$

13. A car is towing a trailer along a straight horizontal road by means of a horizontal tow-rope. The mass of the car is 1400 kg. The mass of the trailer is 700 kg. The car and the trailer are modelled as particles and the tow-rope as a light inextensible string. The resistances to motion of the car and the trailer are assumed to be constant and of magnitude 630 N and 280 N respectively. The driving force on the car, due to its engine, is 2380 N. Find

(a) the acceleration of the car, (3)

Solution

Let T N be the tension and let a ms^{-2} be the acceleration.

$$\begin{aligned} \text{Car}: &\quad 2380 - 630 - T = 1400a \\ \text{Trailer}: &\quad T - 280 = 700a. \end{aligned}$$

Add:

$$\begin{aligned} 1400a + 700a = 2380 - 630 - 280 &\Rightarrow 2100a = 1470 \\ &\Rightarrow \underline{\underline{a = 0.7}}. \end{aligned}$$

(b) the tension in the tow-rope. (3)

Solution

You could use either the car or trailer: we will pick the trailer.

$$\begin{aligned} T - 280 = 700a &\Rightarrow T = 280 + 490 \\ &\Rightarrow \underline{\underline{T = 770}}. \end{aligned}$$

When the car and trailer are moving at 12 ms^{-1}, the tow-rope breaks. Assuming that the driving force on the car and the resistances to motion are unchanged,

(c) find the distance moved by the car in the first 4 s after the tow-rope breaks. (6)

Solution

$$\begin{aligned} F = ma &\Rightarrow 2380 - 630 = 1400a \\ &\Rightarrow 1400a = 1750 \\ &\Rightarrow a = 1.25 \end{aligned}$$

$s = ?, u = 12, v = ?, a = 1.25, t = 4$:

$$\begin{aligned} s &= ut + \tfrac{1}{2}at^2 \\ &= 12 \times 4 + \tfrac{1}{2}1.25 \times 4^2 \\ &= \underline{\underline{58 \text{ m}}}. \end{aligned}$$

(d) State how you have used the modelling assumption that the tow-rope is inextensible. (1)

Solution

We have the <u>same acceleration</u> for the car and trailer.

14. A particle of weight 24 N is held in equilibrium by two light inextensible strings. One string is horizontal. The other string is inclined at an angle of 30° to the horizontal, as shown in Figure 12.

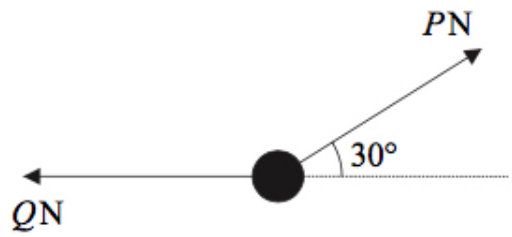


Figure 12: a particle of weight 24 N

The tension in the horizontal string is Q newtons and the tension in the other string is P newtons. Find

(a) the value of P, (3)

Solution

$$P \sin 30^\circ = 24 \Rightarrow \tfrac{1}{2}P = 24$$
$$\Rightarrow \underline{\underline{P = 48}}.$$

(b) the value of Q. (3)

Solution

$$Q = P \cos 30^\circ = \underline{\underline{24\sqrt{3}}}.$$

15. A box of mass 30 kg is being pulled along rough horizontal ground at a constant speed using a rope. The rope makes an angle of 20° with the ground, as shown in Figure 13.

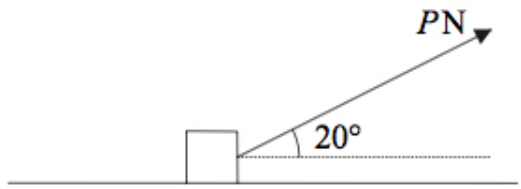


Figure 13: a box of mass 30 kg

The coefficient of friction between the box and the ground is 0.4. The box is modelled as a particle and the rope as a light, inextensible string. The tension in the rope is P newtons.

(a) Find the value of P. (8)

Solution

$$\begin{aligned} \text{Parallel:} \quad & P\cos 20^\circ - F = 0 \\ \text{Perpendicular:} \quad & P\sin 20^\circ + R = 30g \\ F = \mu R: \quad & F = 0.4R. \end{aligned}$$

Now,

$$\begin{aligned} P\cos 20^\circ = F + R &\Rightarrow P\cos 20^\circ = 0.4R \\ &\Rightarrow R = \tfrac{5}{2}P\cos 20^\circ \\ &\Rightarrow 30g - P\sin 20^\circ = \tfrac{5}{2}P\cos 20^\circ \\ &\Rightarrow 30g = P\sin 20^\circ + \tfrac{5}{2}P\cos 20^\circ \\ &\Rightarrow 30g = P(\sin 20^\circ + \tfrac{5}{2}\cos 20^\circ) \\ &\Rightarrow P = \frac{30g}{\sin 20^\circ + \frac{5}{2}\cos 20^\circ} \\ &\Rightarrow P = 109.242\,848 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{P = 110 \text{ (2 sf)}}}. \end{aligned}$$

The tension in the rope is now increased to 150 N.

(b) Find the acceleration of the box. (6)

Solution

$$\begin{aligned} \text{Parallel:} \quad & 150\cos 20^\circ - F = 30a \\ \text{Perpendicular:} \quad & 150\sin 20^\circ + R = 30g \\ F = \mu R: \quad & F = 0.4R. \end{aligned}$$

Now,

$$R = 30g - 150\sin 20^\circ$$

and

$$\begin{aligned} F = ma: \quad & 150\cos 20^\circ - 0.4R = 30a \\ \Rightarrow \quad & a = \frac{150\cos 20^\circ - 0.4R}{30} \\ \Rightarrow \quad & a = 1.462\,503\,391 \text{ (FCD)} \\ \Rightarrow \quad & \underline{\underline{a = 1.5 \text{ ms}^{-2} \text{ (2 sf)}}}. \end{aligned}$$

16. Figure 14 shows two particles P and Q, of mass 3 kg and 2 kg respectively, connected by a light inextensible string.

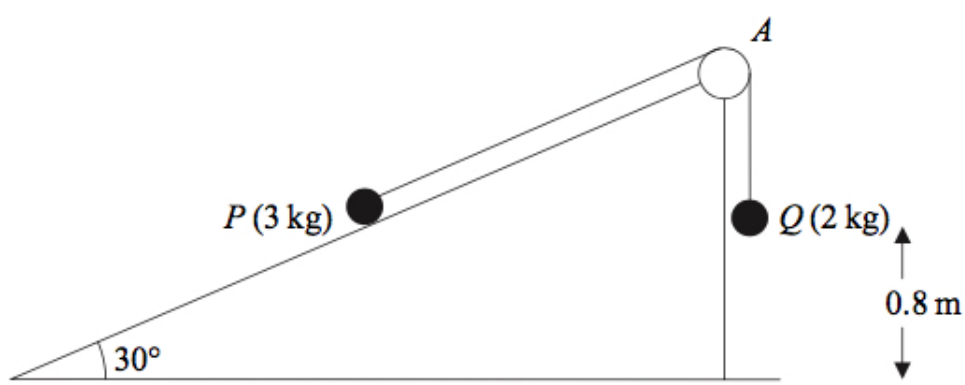


Figure 14: two particles, P and Q

Initially P is held at rest on a fixed smooth plane inclined at 30° to the horizontal. The string passes over a small smooth light pulley A fixed at the top of the plane. The part of the string from P to A is parallel to a line of greatest slope of the plane. The particle Q hangs freely below A. The system is released from rest with the string taut.

(a) Write down an equation of motion for P and an equation of motion for Q. (4)

Solution

Let T N be the tension and let a ms^{-2} be the acceleration.
Newton's Second Law:
for P,

$$\underline{\underline{T - 3g\sin 30^\circ = 3a}}$$

and, for Q,

$$\underline{\underline{2g - T = 2a}}.$$

(b) Hence show that the acceleration of Q is 0.98 ms^{-2} (2)

Solution
Add:

$$5a = 2g - 3g\sin 30^\circ \Rightarrow 5a = \tfrac{1}{2}g \Rightarrow \underline{\underline{a = 0.98}}.$$

(c) Find the tension in the string. (2)

Solution
$T = 2g - 2a = 17.64 = \underline{\underline{18 \text{ (2 sf)}}}$.

(d) State where in your calculations you have used the information that the string is inextensible. (1)

Solution
We have the <u>same acceleration</u> for P and Q.

On release, Q is at a height of 0.8 m above the ground. When Q reaches the ground, it is brought to rest immediately by the impact with the ground and does not rebound. The initial distance of P from A is such that in the subsequent motion P does not reach A. Find

(e) the speed of Q as it reaches the ground, (2)

Solution

$s = 0.8, u = 0, v = ?, a = 0.98, t = ?$:

$$\begin{aligned} v^2 = u^2 + 2as &\Rightarrow v^2 = 0 + 2 \times 0.98 \times 0.8 \\ &\Rightarrow v^2 = 1.568 \\ &\Rightarrow v = 1.252\,198\,067 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{v = 1.3 \text{ (2 sf)}}}. \end{aligned}$$

(f) the time between the instant when Q reaches the ground and the instant when the string becomes taut again. (5)

Solution

Newton's Second Law for P:

$$-3g \sin 30^\circ = 3a \Rightarrow a = -\tfrac{1}{2}g.$$

$s = 0, u = \frac{14\sqrt{5}}{25}, v = ?, a = -\frac{1}{2}g, t = ?$:

$$\begin{aligned} s = ut + \tfrac{1}{2}at^2 &\Rightarrow 0 = \tfrac{14\sqrt{5}}{25}t - \tfrac{1}{2} \times \tfrac{1}{2}g \times t^2 \\ &\Rightarrow 0 = \tfrac{14\sqrt{5}}{25}t - \tfrac{1}{4}gt^2 \\ &\Rightarrow 0 = t\left(\tfrac{14\sqrt{5}}{25} - \tfrac{1}{4}gt\right) \\ &\Rightarrow (t = 0 \text{ or })\ t = \tfrac{56\sqrt{5}}{25g} \\ &\Rightarrow t = 0.511\,101\,252 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{t = 0.51 \text{ (2 sf)}}}. \end{aligned}$$

17. A particle P is attached to one end of a light inextensible string. The other end of the string is attached to a fixed point O. A horizontal force of magnitude 12 N is applied to P. The particle P is in equilibrium with the string taut and OP making an angle of 20° with the downward vertical, as shown in Figure 15.

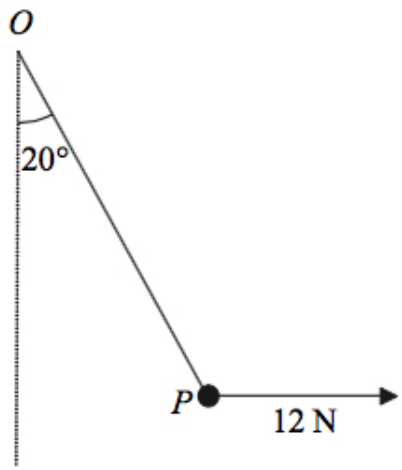


Figure 15: a particle P is attached to one end of a light inextensible string

Find

(a) the tension in the string, (3)

Solution

Let W N be the weight.

$$\begin{aligned} \text{horizontal}: \quad & T\cos 70^\circ = 12 \\ \text{vertical}: \quad & T\sin 70^\circ = W. \end{aligned}$$

Now,

$$\begin{aligned} T\cos 70^\circ = 12 &\Rightarrow T = 35.085\,652\,8 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{T = 35 \text{ (2 sf)}}}. \end{aligned}$$

(b) the weight of P. (4)

Solution

$$\begin{aligned} W &= T\sin 70^\circ \\ &= 32.969\,729\,03 \text{ (FCD)} \\ &= \underline{\underline{33 \text{ (2 sf)}}}. \end{aligned}$$

18. A small ring of mass 0.25 kg is threaded on a fixed rough horizontal rod. The ring is

pulled upwards by a light string which makes an angle 40° with the horizontal, as shown in Figure 16.

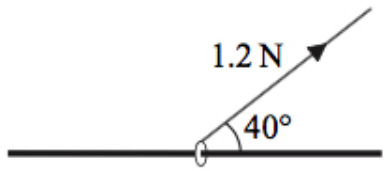


Figure 16: a small ring of mass 0.25 kg

The string and the rod are in the same vertical plane. The tension in the string is 1.2 N and the coefficient of friction between the ring and the rod is μ. Given that the ring is in limiting equilibrium, find

(a) the normal reaction between the ring and the rod, (4)

Solution

$$\begin{aligned} \text{Parallel:} \quad & 1.2\cos 40^\circ - F = 0 \\ \text{Perpendicular:} \quad & 1.2\sin 40^\circ + R = 0.25g \\ F = \mu R: \quad & \end{aligned}$$

Now,

$$\begin{aligned} 1.2\sin 40^\circ + R = 0.25g &\Rightarrow R = 0.25g - 1.2\sin 40^\circ \\ &\Rightarrow R = 1.678\,654\,868 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{R = 1.7 \text{ (2 sf)}}}. \end{aligned}$$

(b) the value of μ. (6)

Solution

$$\begin{aligned} 1.2\cos 40^\circ - F = 0 &\Rightarrow \mu R = 1.2\cos 40^\circ \\ &\Rightarrow \mu = \frac{1.2\cos 40^\circ}{R} \\ &\Rightarrow \mu = 0.547\,613\,061\,5 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{\mu = 0.55 \text{ (2 sf)}}}. \end{aligned}$$

19. Two particles P and Q have mass 0.5 kg and m kg respectively, where $m < 0.5$. The particles are connected by a light inextensible string which passes over a smooth, fixed pulley. Initially, P is 3.15 m above horizontal ground. The particles are released from rest with the string taut and the hanging parts of the string vertical, as shown in Figure 17.

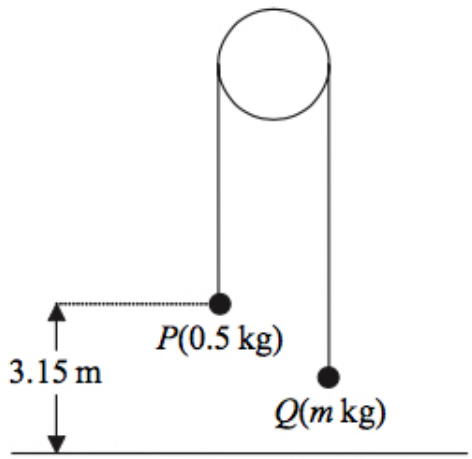


Figure 17: two particles, P and Q

After P has been descending for 1.5 s, it strikes the ground. Particle P reaches the ground before Q has reached the pulley.

(a) Show that the acceleration of P as it descends is 2.8 ms^{-2}. (3)

Solution

Let a ms^{-2} be the acceleration.
$s = 3.15, u = 0, v = ?, a = ?, t = 1.5$:

$$s = ut + \tfrac{1}{2}at^2 \Rightarrow 3.15 = \tfrac{9}{8}a$$
$$\Rightarrow \underline{\underline{a = 2.8}}.$$

(b) Find the tension in the string as P descends. (3)

Solution

Let T N be the tension.

$$P: \quad 0.5g - T = 0.5a$$
$$Q: \quad T - mg = ma.$$

Now,

$$0.5g - T = 0.5a \Rightarrow T = 0.5g - 0.5a$$
$$\Rightarrow \underline{\underline{T = 3.5}}.$$

(c) Show that $m = \frac{5}{18}$. (4)

Solution

$$T - mg = ma \Rightarrow 3.5 - mg = 2.8m$$
$$\Rightarrow 12.6m = 3.5$$
$$\Rightarrow \underline{\underline{m = \frac{5}{18}}}.$$

(d) State how you have used the information that the string is inextensible. (1)

Solution
We have the <u>same acceleration</u> for P and Q.

When P strikes the ground, P does not rebound and the string becomes slack. Particle Q then moves freely under gravity, without reaching the pulley, until the string becomes taut again.

(e) Find the time between the instant when P strikes the ground and the instant when the string becomes taut again. (6)

Solution
Let t s be the time.
$s = ?, u = 0, v = ?, a = 2.8, t = 1.5$:

$$v = u + at = 0 + 2.8 \times 1.5 = 4.2.$$

$s = ?, u = -4.2, v = 4.2, a = 9.8, t = ?$:

$$v = u + at \Rightarrow 4.2 = -4.2 + 9.8t$$
$$\Rightarrow 9.8t = 8.4$$
$$\Rightarrow \underline{\underline{t = \frac{6}{7}}}.$$

20. A particle P of mass 6 kg lies on the surface of a smooth plane. The plane is inclined at an angle of $30°$ to the horizontal. The particle is held in equilibrium by a force of magnitude 49 N, acting at an angle θ to the plane, as shown in Figure 18.

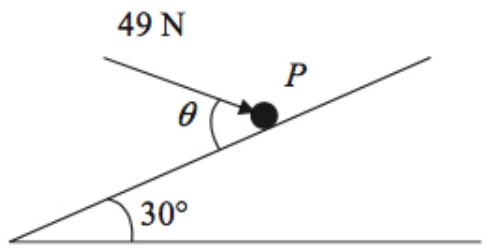


Figure 18: a particle P of mass 6 kg

The force acts in a vertical plane through a line of greatest slope of the plane.

(a) Show that $\cos\theta = \frac{3}{5}$. (3)

Solution

$$\begin{aligned} \text{Parallel:} \quad & 49\cos\theta^\circ = 6g\sin 30^\circ \\ \text{Perpendicular:} \quad & R = 6g\cos 30^\circ + 49\sin\theta^\circ. \end{aligned}$$

Now,

$$\begin{aligned} 49\cos\theta^\circ = 6g\sin 30^\circ &\Rightarrow \cos\theta^\circ = \frac{3g}{49} \\ &\Rightarrow \underline{\underline{\cos\theta^\circ = \tfrac{3}{5}}}. \end{aligned}$$

(b) Find the normal reaction between P and the plane. (4)

Solution

$$\begin{aligned} R &= 6g\cos 30^\circ + 49\sin\theta^\circ \\ &= 90.122\,293\,374 \text{ (FCD)} \\ &= \underline{\underline{90 \text{ (2 sf)}}}. \end{aligned}$$

The direction of the force of magnitude 49 N is now changed. It is now applied horizontally to P so that P moves up the plane. The force again acts in a vertical plane through a line of greatest slope of the plane.

(c) Find the initial acceleration of P. (4)

Solution

Let a ms^{-2} be the acceleration. Then

$$\begin{aligned} \text{Parallel:} \quad & 49\cos 30^\circ - 6g\sin 30^\circ = 6a \\ \text{Perpendicular:} \quad & R = 6g\cos 30^\circ + 49\sin 30^\circ. \end{aligned}$$

Now,

$$\begin{aligned} 49\cos 30^\circ - 6g\sin 30^\circ = 6a \Rightarrow a &= \frac{49\cos 30^\circ - 6g\sin 30^\circ}{6} \\ &\Rightarrow a = 2.172\,540\,798 \text{ (FCD)} \\ &= \underline{\underline{a = 2.2 \text{ (2 sf)}}}. \end{aligned}$$

21. Two particles A and B, of mass m and $2m$ respectively, are attached to the ends of a light inextensible string. The particle A lies on a rough horizontal table. The string passes over a small smooth pulley P fixed on the edge of the table. The particle B hangs freely below the pulley, as shown in Figure 19.

Figure 19: two particles, A and B

The coefficient of friction between A and the table is μ. The particles are released from rest with the string taut. Immediately after release, the magnitude of the acceleration of A and B is $\frac{4}{9}g$. By writing down separate equations of motion for A and B,

(a) find the tension in the string immediately after the particles begin to move, (3)

Solution

$$\begin{aligned} A: \quad & T - F = \tfrac{4}{9}mg \\ B: \quad & 2mg - T = \tfrac{8}{9}mg \\ F = \mu R: \quad & \end{aligned}$$

Now,

$$2mg - T = \tfrac{8}{9}mg \Rightarrow \underline{\underline{T = \tfrac{10}{9}mg}}.$$

(b) show that $\mu = \frac{2}{3}$. (5)

Solution

$$\begin{aligned} T - F = \tfrac{4}{9}mg &\Rightarrow \tfrac{10}{9}mg - \mu R = \tfrac{4}{9}mg \\ &\Rightarrow \tfrac{10}{9}mg - \mu mg = \tfrac{4}{9}mg \\ &\Rightarrow \tfrac{10}{9} - \mu = \tfrac{4}{9} \\ &\Rightarrow \mu = \tfrac{6}{9} \\ &\Rightarrow \underline{\underline{\mu = \tfrac{2}{3}}}. \end{aligned}$$

When B has fallen a distance h, it hits the ground and does not rebound. Particle A is then a distance $\frac{1}{3}h$ from P.

(c) Find the speed of A as it reaches P. (6)

Solution

$s = h, u = 0, v = ?, a = \frac{4}{9}g, t = ?$:

$$\begin{aligned} v^2 = u^2 + 2as &\Rightarrow v^2 = \tfrac{8}{9}gh \\ &\Rightarrow v = \sqrt{\tfrac{8}{9}gh}. \end{aligned}$$

Acceleration:

$$ma = -\tfrac{2}{3}mg \Rightarrow a = -\tfrac{2}{3}g.$$

Speed of B:

$$\begin{aligned} v^2 = u^2 + 2as &\Rightarrow v^2 = \tfrac{8}{9}gh - 2 \times \tfrac{2}{3}g \times \tfrac{1}{3}h \\ &\Rightarrow v^2 = \tfrac{4}{9}gh \\ &\Rightarrow \underline{\underline{v = \sqrt{\tfrac{4}{9}gh}}}. \end{aligned}$$

(d) State how you have used the information that the string is light. (1)

Solution

We have the <u>same tension</u> for A and B.

22. Two forces **P** and **Q** act on a particle at a point O. The force **P** has magnitude 15 N and the force **Q** has magnitude X newtons. The angle between **P** and **Q** is 150°, as shown in Figure 20.

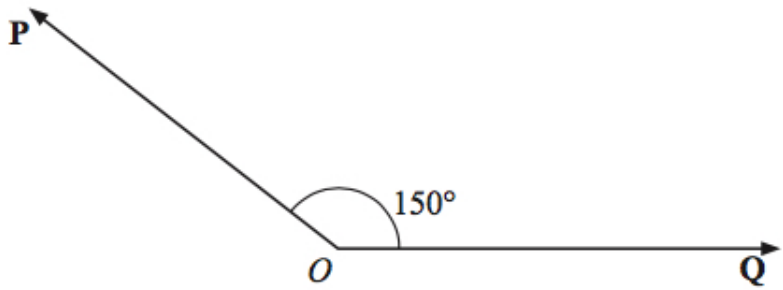


Figure 20: two forces **P** and **Q** act on a particle at a point O

The resultant of **P** and **Q** is **R**.

Given that the angle between **R** and **Q** is 50°, find

(a) the magnitude of the **R**, (4)

Solution

$$\begin{aligned} \text{Horizontal:} \quad & X = 15\cos 30^\circ + R\cos 50^\circ \\ \text{Vertical:} \quad & 15\sin 30^\circ = R\sin 50^\circ. \end{aligned}$$

Solve the vertical:

$$\begin{aligned} 15\sin 30^\circ = R\sin 50^\circ &\Rightarrow R = 9.790\,554\,67 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{R = 9.8 \text{ (2 sf)}}}. \end{aligned}$$

(b) the value of X. (5)

Solution

Solve the horizontal:

$$X = 15\cos 30^\circ + R\cos 50^\circ \Rightarrow X = 19.283\,628\,29 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{X = 19 \text{ (2 sf)}}}.$$

23. A package of mass 4 kg lies on a rough plane inclined at 30° to the horizontal. The package is held in equilibrium by a force of magnitude 45 N acting at an angle of 50° to the plane, as shown in Figure 21.

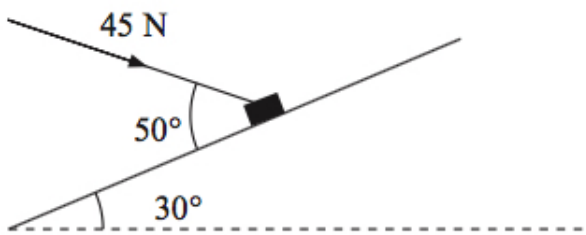


Figure 21: a package of mass 4 kg

The force is acting in a vertical plane through a line of greatest slope of the plane. The package is in equilibrium on the point of moving up the plane. The package is modelled as a particle. Find

(a) the magnitude of the normal reaction of the plane on the package, (5)

Solution

$$\begin{aligned} \text{Parallel:} \quad & 45\cos 50^\circ - F - 4g\sin 30^\circ = 0 \\ \text{Perpendicular:} \quad & R = 45\sin 50^\circ + 4g\cos 30^\circ \\ F = \mu R: \quad & \end{aligned}$$

Now,

$$\begin{aligned} R &= 45\sin 50^\circ + 4g\cos 30^\circ \\ &= 68.420\,195\,77 \text{ (FCD)} \\ &= \underline{\underline{68 \text{ (2 sf)}}}. \end{aligned}$$

(b) the coefficient of friction between the plane and the package. (6)

Solution

$$\begin{aligned} 45\cos 50^\circ - F - 4g\sin 30^\circ = 0 &\Rightarrow 45\cos 50^\circ - \mu R - 4g\sin 30^\circ = 0 \\ &\Rightarrow \mu R = 45\cos 50^\circ - 4g\sin 30^\circ \\ &\Rightarrow \mu = \frac{45\cos 50^\circ - 4g\sin 30^\circ}{R} \\ &\Rightarrow \mu = 0.136\,296\,634\,8 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{\mu = 0.14 \text{ (2 sf)}}}. \end{aligned}$$

24. Two particles P and Q, of mass 2 kg and 3 kg respectively, are joined by a light inextensible string. Initially the particles are at rest on a rough horizontal plane with the string taut. A constant force $\mathbf{F}$ of magnitude 30 N is applied to Q in the direction PQ, as shown in Figure 22.

Figure 22: two particles P and Q

The force is applied for 3 s and during this time Q travels a distance of 6 m. The coefficient of friction between each particle and the plane is mu. Find

(a) the acceleration of Q, (2)

Solution

Let a ms^{-2} be the acceleration.
$s = 6, u = 0, v = ?, a = ?, t = 3$:

$$\begin{aligned} s = ut + \tfrac{1}{2}at^2 &\Rightarrow 6 = \tfrac{9}{2}a \\ &\Rightarrow \underline{\underline{a = \tfrac{4}{3}}}. \end{aligned}$$

(b) the value of μ, (4)

Solution

Let T N be the tension. Then

$$\begin{aligned} P: &\quad T - 2\mu g = \tfrac{4}{3} \times 2 \\ Q: &\quad 30 - T - 3\mu g = \tfrac{4}{3} \times 3. \end{aligned}$$

Add:

$$\begin{aligned} 30 - 5\mu g = \tfrac{20}{3} &\Rightarrow 5\mu g = 30 - \tfrac{20}{3} \\ &\Rightarrow \mu = \frac{30 - \frac{20}{3}}{5g} \\ &\Rightarrow \underline{\underline{\mu = \tfrac{10}{21}}}. \end{aligned}$$

(c) the tension in the string. (4)

Solution

$$T = \tfrac{8}{3} + 2\mu g = \underline{\underline{12}}.$$

(d) State how in your calculation you have used the information that the string is inextensible. (1)

Solution

We have the <u>same tension</u> for P and Q.

When the particles have moved for 3 s, the force **F** is removed.

(e) Find the time between the instant that the force is removed and the instant that Q comes to rest. (4)

Solution

$s = 6, u = 0, v = ?, a = \frac{4}{3}, t = 3$:

$$v = u + at = 0 + \tfrac{4}{3} \times 3 = 4.$$

$s = ?, u = 4, v = 0, a = -\mu g, t = ?$:

$$v = u + at \Rightarrow 0 = 4 - \mu g t \Rightarrow \underline{\underline{t = \tfrac{6}{7}}}.$$

25. A small package of mass 1.1 kg is held in equilibrium on a rough plane by a horizontal force. The plane is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$. The force acts in a vertical plane containing a line of greatest slope of the plane and has magnitude P newtons, as shown in Figure 23.

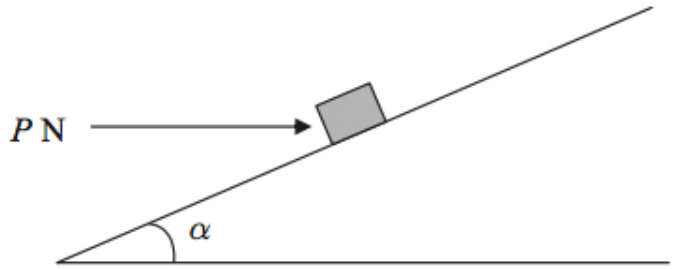
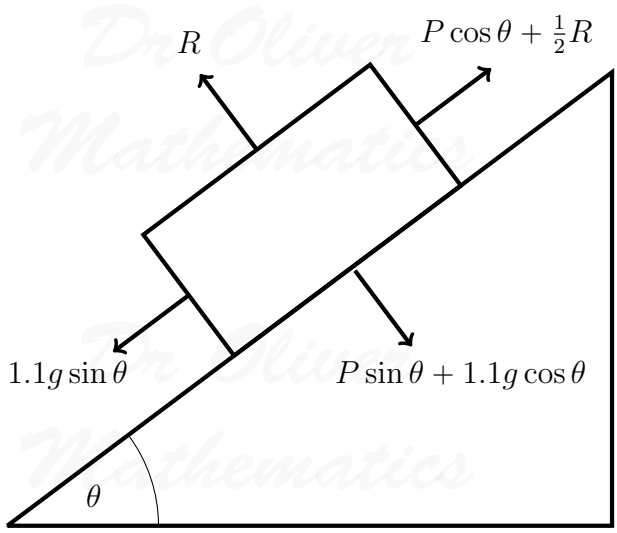


Figure 23: a small package of mass 1.1 kg

The coefficient of friction between the package and the plane is 0.5 and the package is modelled as a particle. The package is in equilibrium and on the point of slipping down the plane.

(a) Draw all the forces acting on the package, showing their directions clearly. (2)

Solution



(b) (i) Find the magnitude of the normal reaction between the package and the plane. (6)

Solution

Let R N be the normal reaction and let F N be the friction. Now,

$$\sin\theta = \tfrac{3}{5} \text{ and } \cos\theta = \tfrac{4}{5}$$

and we have

$$\begin{aligned} \text{Parallel:} \quad & \tfrac{4}{5}P + F = \tfrac{33}{50}g \\ \text{Perpendicular:} \quad & R = \tfrac{3}{5}P + \tfrac{22}{25}g \\ F = \mu R: \quad & F = \tfrac{1}{2}R. \end{aligned}$$

Then,

$$\begin{aligned} \tfrac{4}{5}P + F = \tfrac{33}{50}g &\Rightarrow \tfrac{4}{5}P = \tfrac{33}{50}g - \tfrac{1}{2}R \\ &\Rightarrow P = \tfrac{33}{40}g - \tfrac{5}{8}R \\ &\Rightarrow R = \tfrac{3}{5}\left(\tfrac{33}{40}g - \tfrac{5}{8}R\right) + \tfrac{22}{25}g \\ &\Rightarrow R = \tfrac{11}{8}g - \tfrac{3}{8}R \\ &\Rightarrow \tfrac{11}{8}R = \tfrac{11}{8}g \\ &\Rightarrow R = g \\ &\Rightarrow \underline{\underline{R = 9.8}}. \end{aligned}$$

(ii) Find the value of P. (5)

Solution

$$P = \tfrac{33}{40}g - \tfrac{5}{8}R = \underline{\underline{1.96}}.$$

26. One end of a light inextensible string is attached to a block P of mass 5 kg. The block P is held at rest on a smooth fixed plane which is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{3}{5}$. The string lies along a line of greatest slope of the plane and passes over a smooth light pulley which is fixed at the top of the plane. The other end of the string is attached to a light scale pan which carries two blocks Q and R, with block Q on top of block R, as shown in Figure 24.

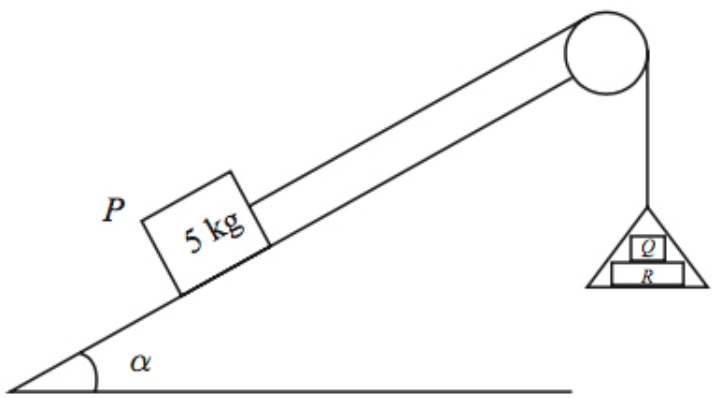


Figure 24: to a block P of mass 5 kg

The mass of block Q is 5 kg and the mass of block R is 10 kg. The scale pan hangs at rest and the system is released from rest. By modelling the blocks as particles, ignoring air resistance and assuming the motion is uninterrupted, find

(a) (i) the acceleration of the scale pan, (6)

Solution
Let T N be the tension, let R N be the normal reaction, and let a ms^{-2} be the acceleration.

$$\begin{aligned} \text{Parallel:} \quad & T - 5g\sin\alpha = 5a \\ \text{Perpendicular:} \quad & R = 5g\cos\alpha \\ Q + R: \quad & 15g - T = 15a \end{aligned}$$

Add:

$$\begin{aligned} (15g - T) + (T - 5g\sin\alpha) = 5a + 15a &\Rightarrow 20a = 15g - 3g \\ &\Rightarrow 20a = 12g \\ &\Rightarrow a = \tfrac{3}{5}g \\ &\Rightarrow \underline{\underline{a = 5.88}}. \end{aligned}$$

(ii) the tension in the string, (2)

Solution

$$\begin{aligned} T - 5g\sin\alpha = 5a &\Rightarrow T = 5a + 3g \\ &\Rightarrow T = 6g \\ &\Rightarrow \underline{\underline{T = 58.8}}. \end{aligned}$$

(b) the magnitude of the force exerted on block Q by block R, (3)

Solution

Let S N be the tension. Then

$$\begin{aligned} 5g - S = 5 \times \tfrac{3}{5}g &\Rightarrow S = 5g - 3g \\ &\Rightarrow S = 2g \\ &\Rightarrow \underline{\underline{S = 9.6}}. \end{aligned}$$

(c) the magnitude of the force exerted on the pulley by the string. (5)

Solution

Let F N be the friction. We have T and T with angle $(90^\circ - \alpha)$:

$$\begin{aligned} F &= 2T\cos\left(\frac{90^\circ - \alpha}{2}\right) \\ &= 105.184\,637\,7 \text{ (FCD)} \\ &= \underline{\underline{110 \text{ (2 sf)}}}. \end{aligned}$$

27. A small brick of mass 0.5 kg is placed on a rough plane which is inclined to the horizontal at an angle θ, where $\tan\theta = \frac{4}{3}$, and released from rest. The coefficient of friction between the brick and the plane is $\frac{1}{3}$. (9)

Find the acceleration of the brick.

Solution

Let R N be the normal reaction, let F N be the friction, and let a ms^{-2} be the acceleration. Now,

$$\sin\theta = \tfrac{4}{5} \text{ and } \cos\theta = \tfrac{3}{5}$$

and

$$\begin{aligned} \text{Parallel:} \quad & 0.5g\sin\theta - F = 0.5a \\ \text{Perpendicular:} \quad & R = 0.5g\cos\theta \\ F = \mu R: \quad & F = \tfrac{1}{3}R \end{aligned}$$

and this means

$$\begin{aligned} \tfrac{2}{5}g - \tfrac{1}{3}R = \tfrac{1}{2}a &\Rightarrow \tfrac{2}{5}g - \tfrac{1}{10}g = \tfrac{1}{2}a \\ &\Rightarrow \tfrac{1}{2}a = \tfrac{3}{10}g \\ &\Rightarrow a = \tfrac{3}{5}g \\ &\Rightarrow \underline{\underline{a = 5.88}}. \end{aligned}$$

28. A small box of mass 15 kg rests on a rough horizontal plane. The coefficient of friction between the box and the plane is 0.2. A force of magnitude P newtons is applied to the box at 50° to the horizontal, as shown in Figure 25. (9)

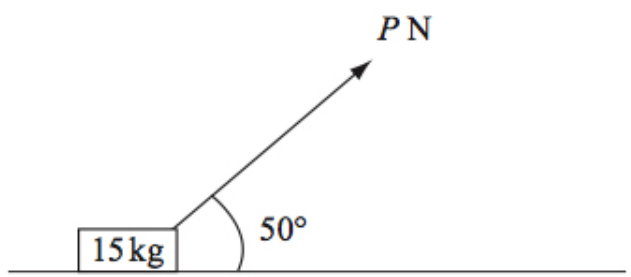


Figure 25: a block P of mass 15 kg

The box is on the point of sliding along the plane.

Find the value of P, giving your answer to 2 significant figures.

Solution

$$\begin{aligned} \text{Horizontal:} \quad & P\cos 50^\circ - F = 0 \\ \text{Vertically:} \quad & R + P\sin 50^\circ = 15g \\ F = \mu R: \quad & F = 0.2R. \end{aligned}$$

Then

$$\begin{aligned}
P\cos 50^\circ - F = 0 &\Rightarrow P\cos 50^\circ = 0.2R \\
&\Rightarrow P\cos 50^\circ = 0.2(15g - P\sin 50^\circ) \\
&\Rightarrow P\cos 50^\circ = 3g - 0.2P\sin 50^\circ \\
&\Rightarrow P\cos 50^\circ + 0.2P\sin 50^\circ = 3g \\
&\Rightarrow P(\cos 50^\circ + 0.2\sin 50^\circ) = 3g \\
&\Rightarrow P = \frac{3g}{\cos 50^\circ + 0.2\sin 50^\circ} \\
&\Rightarrow P = 36.934\,835\,85 \text{ (FCD)} \\
&\Rightarrow \underline{\underline{P = 37 \text{ (2 sf)}}}.
\end{aligned}$$

29. A car of mass 800 kg pulls a trailer of mass 200 kg along a straight horizontal road using a light towbar which is parallel to the road. The horizontal resistances to motion of the car and the trailer have magnitudes 400 N and 200 N respectively. The engine of the car produces a constant horizontal driving force on the car of magnitude 1200 N. Find

(a) the acceleration of the car and trailer, (3)

Solution

Let T N be the tension and let a ms^{-2} be the acceleration.

$$\begin{aligned}
\text{Car}: &\quad 1200 - 400 - T = 800a \\
\text{Trailer}: &\quad T - 200 = 200a.
\end{aligned}$$

Add:

$$600 = 1000a \Rightarrow \underline{\underline{a = 0.6}}.$$

(b) the magnitude of the tension in the towbar. (3)

Solution

$$T = 200 \times 0.6 + 200 = \underline{\underline{320}}.$$

The car is moving along the road when the driver sees a hazard ahead. He reduces the force produced by the engine to zero and applies the brakes. The brakes produce a force on the car of magnitude F newtons and the car and trailer decelerate. Given that the resistances to motion are unchanged and the magnitude of the thrust in the towbar is 100 N,

(c) find the value of F. (7)

Solution

Let F N be the brakes.

$$\begin{aligned} \text{Car}: \quad & 100 - 400 - F = 800a \\ \text{Trailer}: \quad & -200 - 100 = 200a. \end{aligned}$$

Now,

$$\begin{aligned} -200 - 100 = 200a &\Rightarrow 200a = -300 \\ &\Rightarrow a = -\tfrac{3}{2} \end{aligned}$$

and

$$\begin{aligned} 100 - 400 - F = 800a &\Rightarrow -F - 300 = -1200 \\ &\Rightarrow \underline{\underline{F = 900}}. \end{aligned}$$

30. A particle of mass m kg is attached at C to two light inextensible strings AC and BC. The other ends of the strings are attached to fixed points A and B on a horizontal ceiling. The particle hangs in equilibrium with AC and BC inclined to the horizontal at 30° and 60° respectively, as shown in Figure 26.

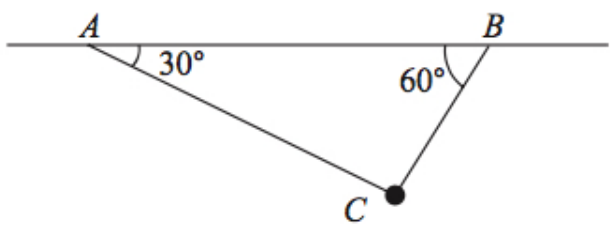


Figure 26: a particle of mass m kg

Given that the tension in AC is 20 N, find

(a) the tension in BC, (4)

Solution

Let T N be the tension.

$$\begin{aligned} \text{Horizontal:} \quad & 20\cos 30^\circ = T\cos 60^\circ \\ \text{Vertically:} \quad & 20\sin 30^\circ + T\sin 60^\circ = mg. \end{aligned}$$

$$20\cos 30^\circ = T\cos 60^\circ \Rightarrow T = \frac{20\cos 30^\circ}{\cos 60^\circ}$$
$$\Rightarrow \underline{\underline{T = 20\sqrt{3}}}.$$

(b) the value of m. (4)

Solution

$$20\sin 30^\circ + T\sin 60^\circ = mg \Rightarrow m = \frac{20\sin 30^\circ + T\sin 60^\circ}{g}$$
$$\Rightarrow \underline{\underline{m = \tfrac{200}{49}}}.$$

31. A particle of mass 0.8 kg is held at rest on a rough plane. The plane is inclined at 30° to the horizontal. The particle is released from rest and slides down a line of greatest slope of the plane. The particle moves 2.7 m during the first 3 seconds of its motion. Find

(a) the acceleration of the particle, (3)

Solution

$s = 2.7, u = 0, v = ?, a = ?, t = 3$:

$$s = ut + \tfrac{1}{2}at^2 \Rightarrow 2.7 = \tfrac{9}{2}a$$
$$\Rightarrow \underline{\underline{a = 0.6}}.$$

(b) the coefficient of friction between the particle and the plane. (5)

Solution

$$\begin{aligned} \text{Parallel:} \quad & 0.8g\sin 30^\circ - F = 0.8 \times 0.6 \\ \text{Perpendicular:} \quad & R = 0.8g\cos 30^\circ \\ F = \mu R: \quad & \end{aligned}$$

Now,

$$\begin{aligned}
0.8g\sin 30^\circ - F = 0.8 \times 0.6 &\Rightarrow 0.8g\sin 30^\circ - \mu R = 0.48 \\
&\Rightarrow \mu \times 0.8g\cos 30^\circ = 0.8g\sin 30^\circ - 0.48 \\
&\Rightarrow \mu = \frac{0.8g\sin 30^\circ - 0.48}{0.8g\cos 30^\circ} \\
&\Rightarrow \mu = 0.506\,654\,317\,9 \text{ (FCD)} \\
&\Rightarrow \underline{\underline{\mu = 0.51 \text{ (2 sf)}}}.
\end{aligned}$$

The particle is now held on the same rough plane by a horizontal force of magnitude X newtons, acting in a plane containing a line of greatest slope of the plane, as shown in Figure 27.

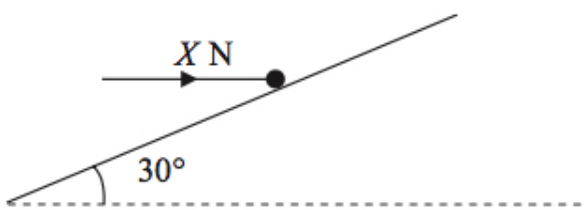


Figure 27: a particle of mass 0.8 kg

The particle is in equilibrium and on the point of moving up the plane.

(c) Find the value of X. (7)

Solution

$$\begin{aligned}
\text{Parallel:} \quad & X\cos 30^\circ - 0.8g\sin 30^\circ - F = 0 \\
\text{Perpendicular:} \quad & R = X\sin 30^\circ + 0.8g\cos 30^\circ \\
F = \mu R: \quad &
\end{aligned}$$

Now,

$$
\begin{aligned}
& X\cos 30^\circ - 0.8g\sin 30^\circ - F = 0 \\
\Rightarrow\quad & X\cos 30^\circ = 0.8g\sin 30^\circ + \mu R \\
\Rightarrow\quad & X\cos 30^\circ = 0.8g\sin 30^\circ + \mu(X\sin 30^\circ + 0.8g\cos 30^\circ) \\
\Rightarrow\quad & X\cos 30^\circ = 0.8g\sin 30^\circ + \mu X\sin 30^\circ + \mu 0.8g\cos 30^\circ \\
\Rightarrow\quad & X\cos 30^\circ - \mu X\sin 30^\circ = 0.8g\sin 30^\circ + \mu 0.8g\cos 30^\circ \\
\Rightarrow\quad & X(\cos 30^\circ - \mu\sin 30^\circ) = 0.8g\sin 30^\circ + \mu 0.8g\cos 30^\circ \\
\Rightarrow\quad & X = \frac{0.8g\sin 30^\circ + \mu 0.8g\cos 30^\circ}{\cos 30^\circ - \mu\sin 30^\circ} \\
\Rightarrow\quad & X = 12.012\,438\,52 \text{ (FCD)} \\
\Rightarrow\quad & \underline{\underline{X = 12 \text{ (2 sf)}}}.
\end{aligned}
$$

32. A small box is pushed along a floor. The floor is modelled as a rough horizontal plane and the box is modelled as a particle. The coefficient of friction between the box and the floor is $\frac{1}{2}$. The box is pushed by a force of magnitude 100 N which acts at an angle of 30° with the floor, as shown in Figure 28. (7)

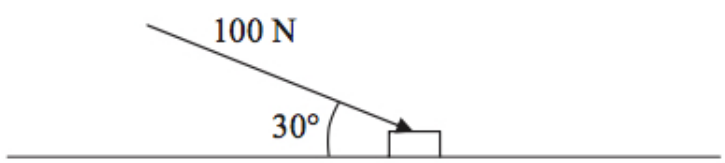


Figure 28: a small box is pushed along a floor

Given that the box moves with constant speed, find the mass of the box.

Solution

Let m kg be the mass.

$$
\begin{aligned}
\text{Parallel:}\quad & 100\cos 30^\circ - F = 0 \\
\text{Perpendicular:}\quad & R = 100\sin 30^\circ + mg \\
F = \mu R:\quad & F = \tfrac{1}{2}R
\end{aligned}
$$

Then,

$$\begin{aligned}
100\cos 30^\circ - F = 0 &\Rightarrow 100\cos 30^\circ = \tfrac{1}{2}R \\
&\Rightarrow 100\cos 30^\circ = \tfrac{1}{2}(100\sin 30^\circ + mg) \\
&\Rightarrow 100\cos 30^\circ = 50\sin 30^\circ + \tfrac{1}{2}mg \\
&\Rightarrow \tfrac{1}{2}mg = 100\cos 30^\circ - 50\sin 30^\circ \\
&\Rightarrow m = \frac{100\cos 30^\circ - 50\sin 30^\circ}{\frac{1}{2}g} \\
&\Rightarrow m = 12.571\,947\,02 \text{ (FCD)} \\
&\Rightarrow \underline{\underline{m = 13 \text{ (2sf)}}}.
\end{aligned}$$

33. A particle of mass 0.4 kg is held at rest on a fixed rough plane by a horizontal force of magnitude P newtons. The force acts in the vertical plane containing the line of greatest slope of the inclined plane which passes through the particle. The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$, as shown in Figure 29.

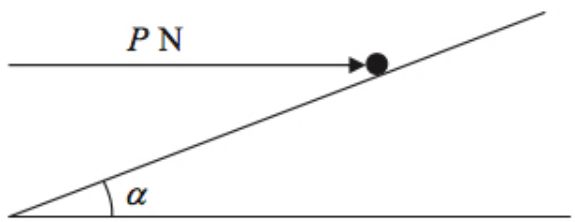


Figure 29: a particle of mass 0.4 kg

The coefficient of friction between the particle and the plane is $\frac{1}{3}$. Given that the particle is on the point of sliding up the plane, find

(a) the magnitude of the normal reaction between the particle and the plane, (5)

Solution

Well,

$$\sin\alpha = \tfrac{3}{5} \text{ and } \cos\alpha = \tfrac{4}{5}$$

and

$$\begin{aligned}
\text{Parallel:} \quad & P\cos\alpha - 0.4g\sin\alpha - F = 0 \\
\text{Perpendicular:} \quad & R = 0.4\cos\alpha + P\sin\alpha \\
F = \mu R: \quad & F = \tfrac{1}{3}R
\end{aligned}$$

Now,

$$\begin{aligned} P\cos\alpha - 0.4g\sin\alpha - F = 0 &\Rightarrow \tfrac{4}{5}P = \tfrac{6}{25}g + \tfrac{1}{3}R \\ &\Rightarrow P = \tfrac{3}{10}g + \tfrac{5}{12}R \\ &\Rightarrow R = \tfrac{8}{25}g + \tfrac{3}{5}(\tfrac{3}{10}g + \tfrac{5}{12}R) \\ &\Rightarrow R = \tfrac{1}{2}g + \tfrac{1}{4}R \\ &\Rightarrow \tfrac{3}{4}R = \tfrac{1}{2}g \\ &\Rightarrow \underline{\underline{R = \tfrac{2}{3}g}}. \end{aligned}$$

(b) the value of P. (5)

Solution

$$P = \tfrac{3}{10}g + \tfrac{5}{12} \times \tfrac{2}{3}g = \underline{\underline{\tfrac{26}{45}g}}.$$

34. Two particles A and B have mass 0.4 kg and 0.3 kg respectively. The particles are attached to the ends of a light inextensible string. The string passes over a small smooth pulley which is fixed above a horizontal floor. Both particles are held, with the string taut, at a height of 1 m above the floor, as shown in Figure 30.

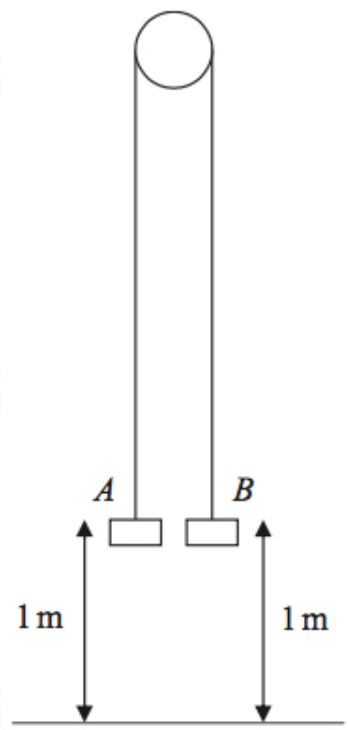


Figure 30: two particles A and B

The particles are released from rest and in the subsequent motion B does not reach the pulley.

(a) Find the tension in the string immediately after the particles are released. (6)

Solution

$$A: \quad 0.4g - T = 0.4a$$
$$B: \quad T - 0.3g = 0.3a.$$

Add:

$$0.1g = 0.7a \Rightarrow \underline{\underline{a = \tfrac{1}{7}g}}$$

and

$$T - 0.3g = 0.3a \Rightarrow T = 0.3g + 0.3 \times \tfrac{1}{7}g = \underline{\underline{\tfrac{12}{35}g}}.$$

(b) Find the acceleration of A immediately after the particles are released. (2)

Solution
See part (a).

When the particles have been moving for 0.5 s, the string breaks.

(c) Find the further time that elapses until B hits the floor. (9)

Solution
For B, $s = 1, u = 0, v = ?, a = \frac{1}{7}g, t = 0.5$:

$$\begin{aligned} v = u + at &= 0 + \tfrac{1}{7}g \times 0.5 \\ &= \tfrac{1}{14}g \end{aligned}$$

and

$$s = ut + \tfrac{1}{2}at^2 = 0 + \tfrac{1}{2} \times \tfrac{1}{7}g \times 0.5 = 0.175.$$

Now, $s = -1.175, u = \frac{1}{14}g, v = ?, a = -g, t = ?$:

$$\begin{aligned} s = ut + \tfrac{1}{2}at^2 &\Rightarrow -1.175 = \tfrac{1}{14}gt - gt^2 \\ &\Rightarrow -1.175 = 0.7t - 4.9t^2 \\ &\Rightarrow 4.9t^2 - 0.7t - 1.175 = 0 \\ &\Rightarrow t = \frac{0.7 \pm \sqrt{0.7^2 - 4 \times 4.9 \times (-1.175)}}{9.8} \\ &\Rightarrow t = \frac{0.7 \pm \sqrt{23.52}}{9.8} \\ &\Rightarrow t = -0423\ldots \text{ or } 0.566\ldots; \end{aligned}$$

hence, $\underline{\underline{t = 0.57 \text{ (2 sf)}}}$.

35. A particle of weight 120 N is placed on a fixed rough plane which is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$. The coefficient of friction between the particle and the plane is $\frac{1}{2}$. The particle is held at rest in equilibrium by a horizontal force of magnitude 30 N, which acts in the vertical plane containing the line of greatest slope of the plane through the particle, as shown in Figure 31.

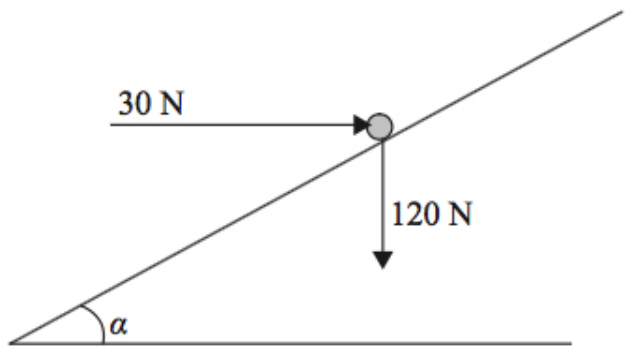


Figure 31: a particle of weight 120 N

(a) Show that the normal reaction between the particle and the plane has magnitude 114 N. (4)

Solution

Well,

$$\sin\alpha = \tfrac{3}{5} \text{ and } \cos\alpha = \tfrac{4}{5}$$

and

$$\begin{aligned} \text{Parallel:} \quad & 30\cos\alpha - 120\sin\alpha - F = 0 \\ \text{Perpendicular:} \quad & R = 120\cos\alpha + 30\sin\alpha \\ F = \mu R: \quad & F = \tfrac{1}{2}R \end{aligned}$$

Now,

$$R = 120\cos\alpha + 30\sin\alpha = 96 + 18 = \underline{\underline{114}}.$$

The horizontal force is removed and replaced by a force of magnitude P newtons acting up the slope along the line of greatest slope of the plane through the particle, as shown in Figure 32.

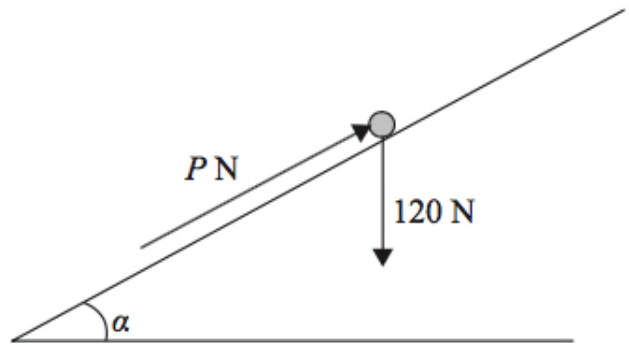


Figure 32: a force of magnitude P

The particle remains in equilibrium.

(b) Find the greatest possible value of P. (8)

Solution

$$\begin{aligned} \text{Parallel:} \quad & P - 120\sin\alpha - F = 0 \\ \text{Perpendicular:} \quad & R = 120\cos\alpha \\ F = \mu R: \quad & F = \tfrac{1}{2}R. \end{aligned}$$

Now,

$$\begin{aligned} P - 120\sin\alpha - F = 0 &\Rightarrow P - 72 - \tfrac{1}{2}R = 0 \\ &\Rightarrow P - 72 - \tfrac{1}{2}\times 96 = 0 \\ &\Rightarrow \underline{\underline{P = 120}}. \end{aligned}$$

(c) Find the magnitude and direction of the frictional force acting on the particle when $P = 30$. (3)

Solution

The magnitude is

$$30 - 72 - F = 0 \Rightarrow \underline{\underline{F = 42}}$$

and the direction <u>up the plane</u>.

36. Two particles A and B, of mass 7 kg and 3 kg respectively, are attached to the ends of a light inextensible string. Initially, B is held at rest on a rough fixed plane inclined at angle to the horizontal, where $\tan\theta = \frac{5}{12}$. The part of the string from B to P is parallel to a line of greatest slope of the plane. The string passes over a small smooth pulley, P, fixed at the top of the plane. The particle A hangs freely below P, as shown in Figure 33.

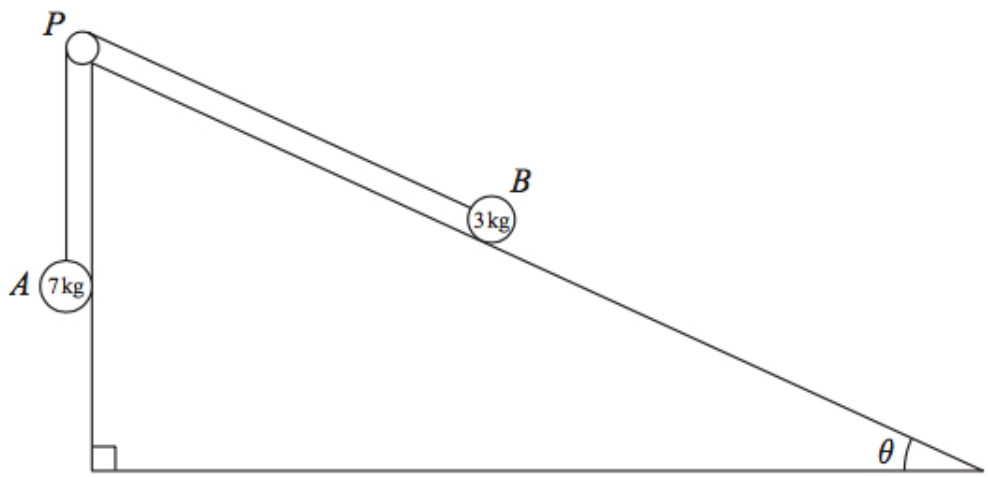


Figure 33: two particles A and B

The coefficient of friction between B and the plane is $\frac{2}{3}$. The particles are released from rest with the string taut and B moves up the plane.

(a) Find the magnitude of the acceleration of B immediately after release. (10)

Solution

Well,

$$\sin\theta = \tfrac{5}{13} \text{ and } \cos\theta = \tfrac{12}{13}$$

and

$$\begin{aligned} A: &\quad 7g - T = 7a \\ B: &\quad \text{Parallel:} \quad T - 3g\sin\theta - F = 3a \\ &\quad \text{Perpendicular:} \quad R = 3g\cos\theta \\ &\quad F = \mu R: \quad F = \tfrac{2}{3}R. \end{aligned}$$

Now,

$$\begin{aligned} T - \tfrac{15}{13}g - \tfrac{2}{3}R = 3a &\Rightarrow T - \tfrac{15}{13}g - \tfrac{24}{13}g = 3a \\ &\Rightarrow T - 3g = 3a. \end{aligned}$$

Add:

$$4g = 10a \Rightarrow \underline{\underline{a = 3.92}}.$$

(b) Find the speed of B when it has moved 1 m up the plane. (2)

Solution

$s = 1, u = 0, v = ?, a = 3.92, t = ?$:

$$\begin{aligned} v^2 = u^2 + 2as &\Rightarrow v^2 = 0 + 2 \times 3.92 \times 1 \\ &\Rightarrow v^2 = 7.84 \\ &\Rightarrow \underline{\underline{v = 2.8}}. \end{aligned}$$

When B has moved 1 m up the plane the string breaks. Given that in the subsequent motion B does not reach P,

(c) find the time between the instants when the string breaks and when B comes to instantaneous rest. (4)

Solution

$$-3g\sin\theta - F = 3a \Rightarrow -3g = 3a \Rightarrow a = -g.$$

$s = ?, u = 2.8, v = 0, a = -g, t = ?$:

$$\begin{aligned} v = u + at &\Rightarrow 0 = 2.8 - 9.8t \\ &\Rightarrow \underline{\underline{t = \tfrac{2}{7}}}. \end{aligned}$$

37. A particle of weight W newtons is held in equilibrium on a rough inclined plane by a horizontal force of magnitude 4 N. The force acts in a vertical plane containing a line of greatest slope of the inclined plane. The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$, as shown in Figure 34.

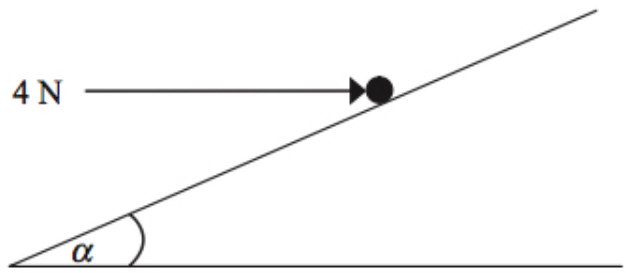


Figure 34: a particle of weight W newtons

The coefficient of friction between the particle and the plane is $\frac{1}{2}$. Given that the particle is on the point of sliding down the plane,

(a) show that the magnitude of the normal reaction between the particle and the plane is 20 N, (5)

Solution

Well,

$$\sin\alpha = \tfrac{3}{5} \text{ and } \cos\alpha = \tfrac{4}{5}$$

and

$$\begin{aligned} \text{Parallel:} \quad & 4\cos\alpha - W\sin\alpha + F = 0 \\ \text{Perpendicular:} \quad & R = W\cos\alpha + 4\sin\alpha \\ F = \mu R: \quad & F = \tfrac{1}{2}R \end{aligned}$$

Now,

$$\begin{aligned} 4\cos\alpha - W\sin\alpha + F = 0 &\Rightarrow \tfrac{16}{5} - \tfrac{3}{5}W + \tfrac{1}{2}(\tfrac{4}{5}W + \tfrac{12}{5}) = 0 \\ &\Rightarrow \tfrac{16}{5} - \tfrac{3}{5}W + \tfrac{4}{10}W + \tfrac{12}{10} = 0 \\ &\Rightarrow \tfrac{22}{5} = \tfrac{1}{5}W \\ &\Rightarrow \underline{\underline{W = 22}} \\ &\Rightarrow R = \tfrac{4}{5} \times 22 + \tfrac{12}{5} \\ &\Rightarrow \underline{\underline{R = 20}}. \end{aligned}$$

(b) find the value of W. (4)

Solution

See the above.

38. Two particles P and Q have masses 0.3 kg and m kg respectively. The particles are attached to the ends of a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a fixed rough plane. The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$. The coefficient of friction between P and the plane is $\frac{1}{2}$. The string lies in a vertical plane through a line of greatest slope of the inclined plane. The particle P is held at rest on the inclined plane and the particle Q hangs freely below the pulley with the string taut, as shown in Figure 35.

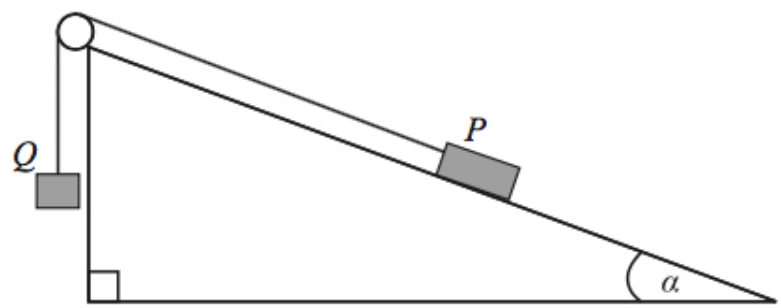


Figure 35: two particles P and Q

The system is released from rest and Q accelerates vertically downwards at 1.4 ms^{-2}. Find

(a) the magnitude of the normal reaction of the inclined plane on P, (2)

Solution

Well,

$$\sin\theta = \tfrac{3}{5} \text{ and } \cos\theta = \tfrac{4}{5}$$

and

$$\begin{aligned} P: &\quad \text{Parallel:} \quad T - 0.3g\sin\theta - F = 0.3a \\ &\quad \text{Perpendicular:} \quad R = 0.3g\cos\theta \\ &\quad F = \mu R: \quad F = \tfrac{1}{2}R \\ Q: &\quad mg - T = ma \end{aligned}$$

Now,

$$R = 0.3g\cos\theta = 0.3g \times \tfrac{4}{5} = \underline{\underline{0.24g}}.$$

(b) the value of m. (8)

Solution

$$\begin{aligned} T - 0.3g\sin\theta - F = 0.3a &\Rightarrow T - 0.18g - \tfrac{1}{2}R = 0.3 \times 1.4 \\ &\Rightarrow T - 0.18g - \tfrac{1}{2} \times 0.24g = 0.42 \\ &\Rightarrow T - 0.3g = 0.42. \end{aligned}$$

Add:

$$\begin{aligned} mg - 0.3g = 1.4m + 0.42 &\Rightarrow mg - 1.4m = 0.3g + 0.42 \\ &\Rightarrow m(g - 1.4) = 0.3g + 0.42 \\ &\Rightarrow m = \frac{0.3g + 0.42}{g - 1.4} \\ &\Rightarrow \underline{\underline{m = 0.4}}. \end{aligned}$$

When the particles have been moving for 0.5 s, the string breaks. Assuming that P does not reach the pulley,

(c) find the further time that elapses until P comes to instantaneous rest. (6)

Solution

$s = ?, u = 0, v = ?, a = 1.4, t = 0.5$:

$$v = u + at = 0 + 1.4 \times 0.5 = 0.7.$$

$$-0.3g = 0.3a \Rightarrow a = -g.$$

$s = ?, u = 0.7, v = 0, a = -g, t = ?$:

$$\begin{aligned} v = u + at &\Rightarrow 0 = 0.7 - 9.8t \\ &\Rightarrow t = \tfrac{1}{14} \\ &\Rightarrow \underline{\underline{t = 0.071 \text{ (2 sf)}}}. \end{aligned}$$

39. A car of mass 1000 kg is towing a caravan of mass 750 kg along a straight horizontal road. The caravan is connected to the car by a tow-bar which is parallel to the direction of motion of the car and the caravan. The tow-bar is modelled as a light rod. The engine of the car provides a constant driving force of 3200 N. The resistances to the motion of the car and the caravan are modelled as constant forces of magnitude 800 newtons and R newtons respectively. Given that the acceleration of the car and the caravan is 0.88 ms^{-2},

(a) show that $R = 860$, (3)

Solution

Let T N be the tension.

$$\begin{aligned} \text{Car}: &\quad 3200 - 800 - T = 880 \\ \text{Trailer}: &\quad T - R = 660. \end{aligned}$$

Add:

$$\begin{aligned} 3200 - 800 - R = 880 + 660 &\Rightarrow 2400 - R = 1540 \\ &\Rightarrow \underline{\underline{R = 860}}, \end{aligned}$$

as required.

(b) find the tension in the tow-bar. (3)

Solution

$$T - 860 = 660 \Rightarrow \underline{\underline{T = 1520}}.$$

40. A particle P of mass 4 kg is moving up a fixed rough plane at a constant speed of 16 ms^{-1} under the action of a force of magnitude 36 N. The plane is inclined at 30° to the horizontal. The force acts in the vertical plane containing the line of greatest slope of the plane through P, and acts at 30° to the inclined plane, as shown in Figure 36.

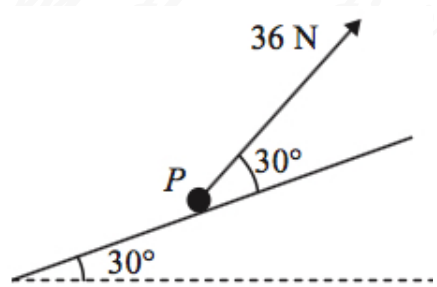


Figure 36: a particle P of mass 4 kg

The coefficient of friction between P and the plane is μ. Find

(a) the magnitude of the normal reaction between P and the plane, (4)

Solution

Let F N be the friction and let R N be the normal reaction.

$$\begin{aligned} \text{Parallel:} \quad & 36\cos 30^\circ = F + 4g\sin 30^\circ \\ \text{Perpendicular:} \quad & R + 36\sin 30^\circ = 4g\cos 30^\circ \\ F = \mu R: \quad & \end{aligned}$$

Now,

$$\begin{aligned} R + 36\sin 30^\circ = 4g\cos 30^\circ &\Rightarrow R = 4g\cos 30^\circ - 36\sin 30^\circ \\ &\Rightarrow R = 15.948\,195\,83 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{R = 16 \text{ (2 sf)}}}. \end{aligned}$$

(b) the value of μ. (5)

Solution

$$\begin{aligned} 36\cos 30^\circ = F + 4g\sin 30^\circ &\Rightarrow 36\cos 30^\circ = \mu R + 4g\sin 30^\circ \\ &\Rightarrow 36\cos 30^\circ = \mu(4g\cos 30^\circ - 36\sin 30^\circ) + 4g\sin 30^\circ \\ &\Rightarrow \mu(4g\cos 30^\circ - 36\sin 30^\circ) = 36\cos 30^\circ - 4g\sin 30^\circ \\ &\Rightarrow \mu = \frac{36\cos 30^\circ - 4g\sin 30^\circ}{4g\cos 30^\circ - 36\sin 30^\circ} \\ &\Rightarrow \mu = 0.725\,907\,473\,2 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{\mu = 0.73 \text{ (2 sf)}}}. \end{aligned}$$

The force of magnitude 36 N is removed.

(c) Find the distance that P travels between the instant when the force is removed and the instant when it comes to rest. (5)

Solution

After the force is removed,

$$R = 4g\cos 30^\circ$$

which means that

$$\begin{aligned}
-F - 4g\sin 30^\circ = 4a &\Rightarrow -\mu R - 4g\sin 30^\circ = 4a \\
&\Rightarrow -4\mu g\cos 30^\circ - 4g\sin 30^\circ = 4a \\
&\Rightarrow a = -\mu g\cos 30^\circ - g\sin 30^\circ \\
&\Rightarrow a = -11.060\,812\,26 \text{ (FCD)}.
\end{aligned}$$

$s = ?, u = 16, v = 0, a = -11.060\ldots, t = ?$:

$$\begin{aligned}
v^2 = u^2 + 2as &\Rightarrow 0 = 16^2 + 2 \times (-11.060\ldots) \times s \\
&\Rightarrow s = 11.572\,387 \text{ (FCD)} \\
&\Rightarrow \underline{\underline{s = 12 \text{ (2 sf)}}}.
\end{aligned}$$

41. A box of mass 5 kg lies on a rough plane inclined at 30° to the horizontal. The box is held in equilibrium by a horizontal force of magnitude 20 N, as shown in Figure 37.

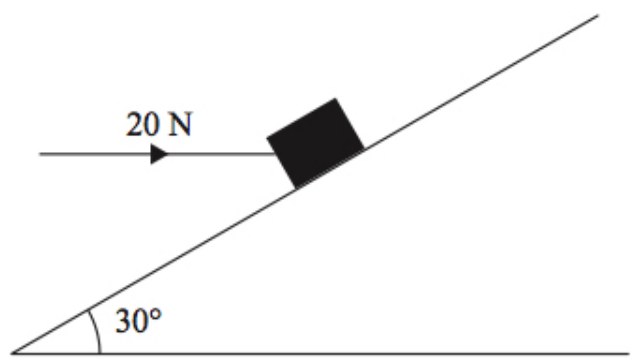


Figure 37: a box of mass 5 kg

The force acts in a vertical plane containing a line of greatest slope of the inclined plane. The box is in equilibrium and on the point of moving down the plane. The box is modelled as a particle. Find

(a) the magnitude of the normal reaction of the plane on the box, (4)

Solution

$$\begin{aligned}
\text{Parallel:} \quad & 20\cos 30^\circ - 5g\sin 30^\circ + F = 0 \\
\text{Perpendicular:} \quad & R = 5g\cos 30^\circ + 20\sin 30^\circ \\
F = \mu R: \quad &
\end{aligned}$$

Now,

$$\begin{aligned} R &= 5g\cos 30^\circ + 20\sin 30^\circ \\ &= 52.435\,244\,79 \text{ (FCD)} \\ &= \underline{\underline{52 \text{ (2 sf)}}}. \end{aligned}$$

(b) the coefficient of friction between the box and the plane. (5)

Solution

$$\begin{aligned} & 20\cos 30^\circ - 5g\sin 30^\circ + F = 0 \\ \Rightarrow\ & 20\cos 30^\circ - 5g\sin 30^\circ + \mu R = 0 \\ \Rightarrow\ & \mu(5g\cos 30^\circ + 20\sin 30^\circ) = 5g\sin 30^\circ - 20\cos 30^\circ \\ \Rightarrow\ & \mu = \frac{5g\sin 30^\circ - 20\cos 30^\circ}{5g\cos 30^\circ + 20\sin 30^\circ} \\ \Rightarrow\ & \mu = 0.136\,921\,110\,1 \text{ (FCD)} \\ \Rightarrow\ & \underline{\underline{\mu = 0.14 \text{ (2 sf)}}}. \end{aligned}$$

42. Two particles P and Q, of mass 0.3 kg and 0.5 kg respectively, are joined by a light horizontal rod. The system of the particles and the rod is at rest on a horizontal plane. At time $t = 0$, a constant force $\mathbf{F}$ of magnitude 4 N is applied to Q in the direction PQ, as shown in Figure 38.

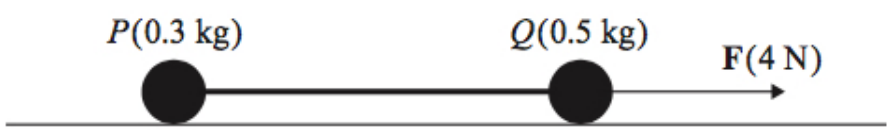


Figure 38: two particles P and Q

The system moves under the action of this force until $t = 6$ s. During the motion, the resistance to the motion of P has constant magnitude 1 N and the resistance to the motion of Q has constant magnitude 2 N. Find

(a) the acceleration of the particles as the system moves under the action of $\mathbf{F}$, (3)

Solution

$$\begin{aligned} P: &\quad T - 1 = 0.3a \\ Q: &\quad 4 - 2 - T = 0.5a. \end{aligned}$$

Add:

$$1 = 0.8a \Rightarrow a = 1.25 = \underline{\underline{1.3 \text{ (2 sf)}}}.$$

(b) the speed of the particles at $t = 6$ s, (2)

Solution

$s = ?, u = 0, v = ?, a = 1.25, t = 6$:

$$v = u + at = 0 + 1.25 \times 6 = \underline{\underline{7.5}}.$$

(c) the tension in the rod as the system moves under the action of $\mathbf{F}$. (3)

Solution

$$T - 1 = 0.3 \times 1.25 \Rightarrow T = 1.375 = \underline{\underline{1.4 \text{ (2 sf)}}}.$$

At $t = 6$ s, $\mathbf{F}$ is removed and the system decelerates to rest. The resistances to motion are unchanged. Find

(d) the distance moved by P as the system decelerates, (4)

Solution

Now, the thrust is 'reversed':

$$\begin{aligned} P: &\quad T + 1 = 0.3a \\ Q: &\quad 2 - T = 0.5a. \end{aligned}$$

Add:

$$-3 = 0.8a \Rightarrow a = -3.75.$$

$s = ?, u = 7.5, v = 0, a = -3.75, t = ?$:

$$\begin{aligned} v^2 = u^2 + 2as &\Rightarrow 0 = 7.5^2 + 2 \times (-3.75) \times s \\ &\Rightarrow \underline{\underline{s = 7.5}}. \end{aligned}$$

(e) the thrust in the rod as the system decelerates. (3)

Solution

$$T + 1 = 0.3 \times 3.75 \Rightarrow T = 0.125 = \underline{\underline{0.13 \text{ (2 sf)}}}.$$

43. A particle P of mass 2 kg is attached to one end of a light string, the other end of which is attached to a fixed point O. The particle is held in equilibrium, with OP at $30°$ to the downward vertical, by a force of magnitude F newtons. The force acts in the same vertical plane as the string and acts at an angle of $30°$ to the horizontal, as shown in Figure 39. (8)

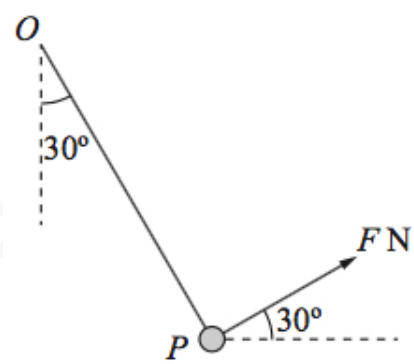


Figure 39: a particle P of mass 2 kg

Find

(a) the value of F,

Solution

$$\begin{aligned} \text{Horizontal}: \quad & T\cos 60° = F\cos 30° \\ \text{Vertical}: \quad & T\sin 60° + F\sin 30° = 2g. \end{aligned}$$

Now,

$$\begin{aligned} T\cos 60^\circ = F\cos 30^\circ &\Rightarrow T = \frac{F\cos 30^\circ}{\cos 60^\circ} \\ &\Rightarrow \frac{F\cos 30^\circ \sin 60^\circ}{\cos 60^\circ} + F\sin 30^\circ = 2g \\ &\Rightarrow \frac{F(\cos 30^\circ \sin 60^\circ + \sin 30^\circ \cos 60^\circ)}{\cos 60^\circ} = 2g \\ &\Rightarrow F = \frac{2g\cos 60^\circ}{\cos 30^\circ \sin 60^\circ + \sin 30^\circ \cos 60^\circ} \\ &\Rightarrow \underline{\underline{F = g}}, \end{aligned}$$

as

$$\cos 30^\circ \sin 60^\circ + \sin 30^\circ \cos 60^\circ = \sin 90^\circ = 1.$$

(b) the tension in the string.

Solution

$$T = \frac{F\cos 30^\circ}{\cos 60^\circ} = \underline{\underline{\sqrt{3}g}}.$$

44. A lifeboat slides down a straight ramp inclined at an angle of 15° to the horizontal. The lifeboat has mass 800 kg and the length of the ramp is 50 m. The lifeboat is released from rest at the top of the ramp and is moving with a speed of 12.6 ms^{-1} when it reaches the end of the ramp. By modelling the lifeboat as a particle and the ramp as a rough inclined plane, find the coefficient of friction between the lifeboat and the ramp. (9)

Solution

$s = 50, u = 0, v = 12.6, a = ?, t = ?$:

$$\begin{aligned} v^2 = u^2 + 2as &\Rightarrow 12.6^2 = 0 + 100a \\ &\Rightarrow a = 1.5876. \end{aligned}$$

$$\begin{aligned} \text{Parallel:} \quad & 800g\sin 15^\circ - F = 800 \times 1.5876 \\ \text{Perpendicular:} \quad & R = 800g\cos 15^\circ \\ F = \mu R: & \end{aligned}$$

Now,

$$\begin{aligned}
800g\sin 15^\circ - F = 1270.08 &\Rightarrow 800g\sin 15^\circ - \mu R = 1270.08 \\
&\Rightarrow 800g\sin 15^\circ - 800\mu g\cos 15^\circ = 1270.08 \\
&\Rightarrow 800\mu g\cos 15^\circ = 800g\sin 15^\circ - 1270.08 \\
&\Rightarrow \mu = \frac{800g\sin 15^\circ - 1270.08}{800g\cos 15^\circ} \\
&\Rightarrow \mu = 0.100\,234\,451\,2 \text{ (FCD)} \\
&\Rightarrow \underline{\underline{\mu = 0.10 \text{ (2 sf)}}}.
\end{aligned}$$

45. Figure 40 shows two particles A and B, of mass $2m$ and $4m$ respectively, connected by a light inextensible string.

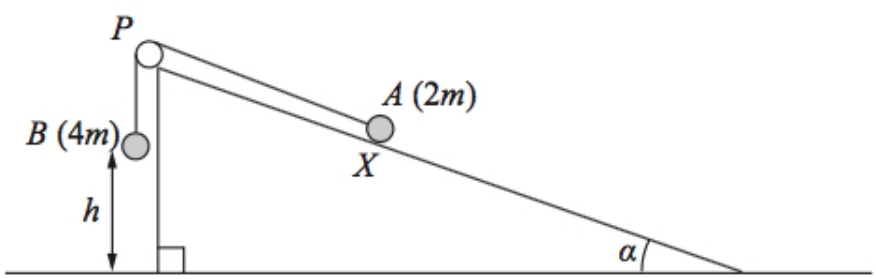


Figure 40: two particles A and B

Initially A is held at rest on a rough inclined plane which is fixed to horizontal ground. The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{5}$. The coefficient of friction between A and the plane is $\frac{1}{4}$. The string passes over a small smooth pulley P which is fixed at the top of the plane. The part of the string from A to P is parallel to a line of greatest slope of the plane and B hangs vertically below P. The system is released from rest with the string taut, with A at the point X and with B at a height h above the ground.

For the motion until B hits the ground,

(a) give a reason why the magnitudes of the accelerations of the two particles are the same, (1)

Solution
They are connected by a <u>light inextensible string</u>.

(b) write down an equation of motion for each particle, (4)

Solution

Well,

$$\sin\alpha = \tfrac{3}{5} \text{ and } \cos\alpha = \tfrac{4}{5}$$

and

$$\begin{aligned} A: &\quad \text{Parallel:} \quad \underline{\underline{T - 2mg\sin\alpha - F = 2ma}} \\ &\quad \text{Perpendicular:} \quad R = 2mg\cos\alpha \\ &\quad F = \mu R: \quad F = \tfrac{1}{4}R \\ B: &\quad \underline{\underline{4mg - T = 4ma}}. \end{aligned}$$

(c) find the acceleration of each particle. (5)

Solution

$$\begin{aligned} T - 2mg\sin\alpha - F = 2ma &\Rightarrow T - \tfrac{6}{5}mg - \tfrac{1}{4}R = 2ma \\ &\Rightarrow T - \tfrac{6}{5}mg - \tfrac{2}{5}mg = 2ma \\ &\Rightarrow T - \tfrac{8}{5}mg = 2ma \end{aligned}$$

and add:

$$\tfrac{12}{5}mg = 6ma \Rightarrow \underline{\underline{a = \tfrac{2}{5}g}}.$$

Particle B does not rebound when it hits the ground and A continues moving up the plane towards P. Given that A comes to rest at the point Y, without reaching P,

(d) find the distance XY in terms of h. (6)

Solution

$s = h, u = 0, v = ?, a = \frac{2}{5}g, t = ?$:

$$\begin{aligned} v^2 = u^2 + 2as &\Rightarrow v^2 = 0 + 2 \times \tfrac{2}{5}g \times h \\ &\Rightarrow v^2 = \tfrac{4}{5}gh \\ &\Rightarrow v = \sqrt{\tfrac{4}{5}gh}. \end{aligned}$$

Now,

$$-\tfrac{6}{5}mg - \tfrac{2}{5}mg = 2ma \Rightarrow a = -\tfrac{4}{5}g.$$

$s = ?, u = \sqrt{\frac{4}{5}gh}, v = 0, a = -\frac{4}{5}g, t = ?$:

$$\begin{aligned} v^2 = u^2 + 2as &\Rightarrow 0 = \tfrac{4}{5}gh + 2 \times (-\tfrac{4}{5}g) \times s \\ &\Rightarrow \tfrac{8}{5}gs = \tfrac{4}{5}gh \\ &\Rightarrow s = \tfrac{1}{2}h \end{aligned}$$

and the distance is

$$XY = h + \tfrac{1}{2}h = \underline{\underline{\tfrac{3}{2}h}}.$$

46. A woman travels in a lift. The mass of the woman is 50 kg and the mass of the lift is 950 kg. The lift is being raised vertically by a vertical cable which is attached to the top of the lift. The lift is moving upwards and has constant deceleration of 2 ms^{-2}. By modelling the cable as being light and inextensible, find

(a) the tension in the cable, (3)

Solution

Let T N be the tension. For the whole system,

$$T - 950g - 50g = 1000 \times (-2) \Rightarrow \underline{\underline{T = 7800}}.$$

(b) the magnitude of the force exerted on the woman by the floor of the lift. (3)

Solution

Let R N be the magnitude of the force exerted on the woman by the floor of the lift.

$$R - 50g = 50 \times (-2) \Rightarrow \underline{\underline{R = 390}}.$$

47. A box of mass 2 kg is held in equilibrium on a fixed rough inclined plane by a rope. The rope lies in a vertical plane containing a line of greatest slope of the inclined plane. The rope is inclined to the plane at an angle α, where $\tan\alpha = \frac{3}{4}$, and the plane is at an angle of 30° to the horizontal, as shown in Figure 41. (8)

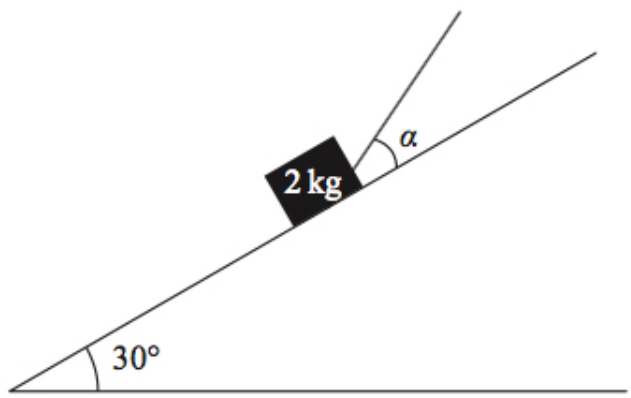


Figure 41: a box of mass 2 kg

The coefficient of friction between the box and the inclined plane is $\frac{1}{3}$ and the box is on the point of slipping up the plane. By modelling the box as a particle and the rope as a light inextensible string, find the tension in the rope.

Solution

Let F N be the friction and let R N be the normal reaction. Well,

$$\sin\alpha = \tfrac{3}{5} \text{ and } \cos\alpha = \tfrac{4}{5}$$

and

$$\begin{aligned} \text{Parallel:} \quad & T\cos\alpha = F + 2g\sin 30^\circ \\ \text{Perpendicular:} \quad & R + T\sin\alpha = 2g\cos 30^\circ \\ F = \mu R: \quad & F = \tfrac{1}{3}R. \end{aligned}$$

Now,

$$\begin{aligned} T\cos\alpha = F + 2g\sin 30^\circ &\Rightarrow \tfrac{4}{5}T = \tfrac{1}{3}R + g \\ &\Rightarrow \tfrac{4}{5}T = \tfrac{1}{3}(\sqrt{3}g - \tfrac{3}{5}T) + g \\ &\Rightarrow \tfrac{4}{5}T = \tfrac{\sqrt{3}}{3}g - \tfrac{1}{5}T + g \\ &\Rightarrow \underline{\underline{T = (\tfrac{\sqrt{3}+3}{3})g}}. \end{aligned}$$

48. Two particles A and B have masses $2m$ and $3m$ respectively. The particles are attached to the ends of a light inextensible string. Particle A is held at rest on a smooth horizontal table. The string passes over a small smooth pulley which is fixed at the edge of the

table. Particle B hangs at rest vertically below the pulley with the string taut, as shown in Figure 42.

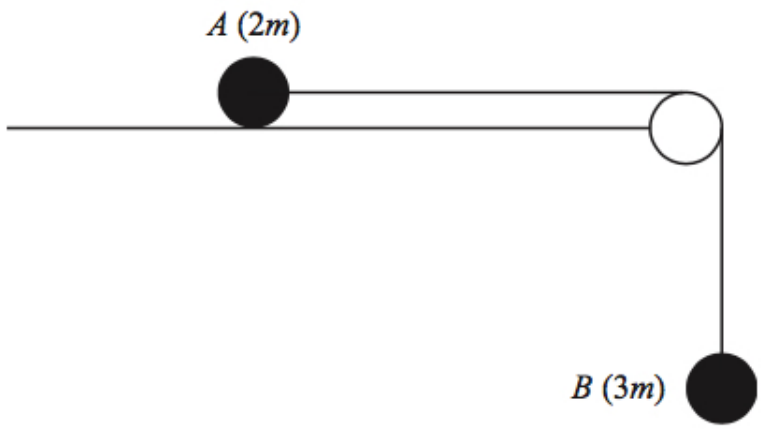


Figure 42: two particles A and B

Particle A is released from rest. Assuming that A has not reached the pulley, find

(a) the acceleration of B, (5)

Solution

$$\begin{aligned} A: &\quad \text{Parallel:} \quad T = 2ma \\ &\quad \text{Perpendicular:} \quad R = 2mg \\ B: &\quad 3mg - T = 3ma. \end{aligned}$$

Add:

$$3mg = 5ma \Rightarrow \underline{\underline{a = \tfrac{3}{5}g}}.$$

(b) the tension in the string, (1)

Solution

$$T = 2ma = \underline{\underline{\tfrac{6}{5}mg}}.$$

(c) the magnitude and direction of the force exerted on the pulley by the string. (4)

Solution

The magnitude is

$$\begin{aligned} F &= \sqrt{T^2 + T^2} \\ &= T\sqrt{2} \\ &= \underline{\underline{\tfrac{6\sqrt{2}}{5}mg}} \end{aligned}$$

and the direction is $\underline{\underline{45^\circ}}$, top right to bottom left ($\swarrow$).

49. A particle of weight 8 N is attached at C to the ends of two light inextensible strings AC and BC. The other ends, A and B, are attached to a fixed horizontal ceiling. The particle hangs at rest in equilibrium, with the strings in a vertical plane. The string AC is inclined at 35° to the horizontal and the string BC is inclined at 25° to the horizontal, as shown in Figure 43. (8)

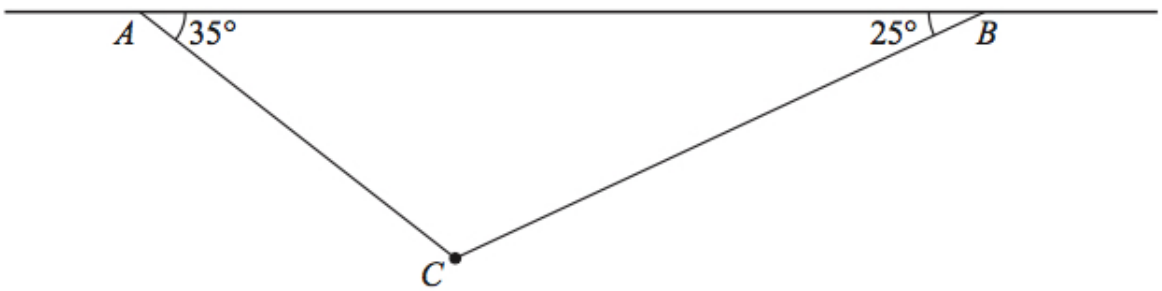


Figure 43: a particle of weight 8 N

Find

(a) the tension in the string AC,

Solution

Let S N be the tension in the 35°-string and T N be the tension in the 25°-string.

$$\begin{aligned} \text{Horizontal}: \quad & S\cos 35^\circ = T\cos 25^\circ \\ \text{Vertical}: \quad & S\sin 35^\circ + T\sin 25^\circ = 8. \end{aligned}$$

Now,

$$\begin{aligned}
S\cos 35^\circ = T\cos 25^\circ \Rightarrow T &= \frac{S\cos 35^\circ}{\cos 25^\circ} \\
&\Rightarrow S\sin 35^\circ + \frac{S\cos 35^\circ \sin 25^\circ}{\cos 25^\circ} = 8 \\
&\Rightarrow S(\sin 35^\circ + \cos 35^\circ \tan 25^\circ) = 8 \\
&\Rightarrow S = \frac{8}{\sin 35^\circ + \cos 35^\circ \tan 25^\circ} \\
&\Rightarrow S = 8.372\,112\,717 \text{ (FCD)} \\
&\Rightarrow \underline{\underline{S = 8.4 \text{ (2 sf)}}}.
\end{aligned}$$

(b) the tension in the string BC.

Solution

$$\begin{aligned}
T &= \frac{8.372\ldots\cos 35^\circ}{\cos 25^\circ} \\
&= 7.567\,002\,452 \text{ (FCD)} \\
&= \underline{\underline{7.6 \text{ (2 sf)}}}.
\end{aligned}$$

50. A fixed rough plane is inclined at 30° to the horizontal. A small smooth pulley P is fixed at the top of the plane. Two particles A and B, of mass 2 kg and 4 kg respectively, are attached to the ends of a light inextensible string which passes over the pulley P. The part of the string from A to P is parallel to a line of greatest slope of the plane and B hangs freely below P, as shown in Figure 44. (9)

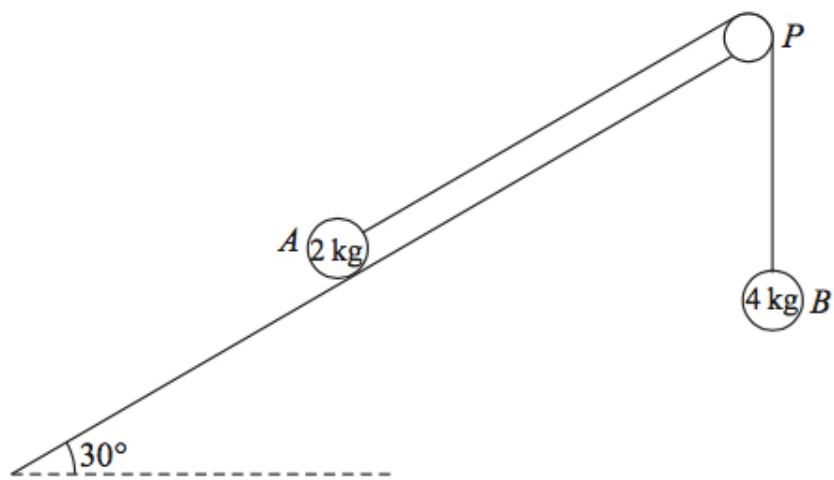


Figure 44: two particles A and B

The coefficient of friction between A and the plane is $\frac{1}{\sqrt{3}}$. Initially, A is held at rest on the plane. The particles are released from rest with the string taut and A moves up the plane.

Find the tension in the string immediately after the particles are released.

Solution

$$\begin{aligned} A: &\quad \text{Parallel:} \quad T - 2g\sin 30^\circ - F = 2a \\ &\quad \text{Perpendicular:} \quad R = 2g\cos 30^\circ \\ &\quad F = \mu R: \quad F = \tfrac{1}{\sqrt{3}}R \\ B: &\quad 4g - T = 4a. \end{aligned}$$

Now,

$$\begin{aligned} T - 2g\sin 30^\circ - F = 2a &\Rightarrow T - 2g\sin 30^\circ - \tfrac{1}{\sqrt{3}}R = 2a \\ &\Rightarrow T - 2g\sin 30^\circ - \tfrac{2}{\sqrt{3}}g\cos 30^\circ = 2a. \end{aligned}$$

Add:

$$
\begin{aligned}
& 4g - 2g\sin 30^\circ - \tfrac{2}{\sqrt{3}}g\cos 30^\circ = 4a + 2a \\
\Rightarrow \quad & a = \frac{4g - 2g\sin 30^\circ - \frac{2}{\sqrt{3}}g\cos 30^\circ}{6} \\
\Rightarrow \quad & a = \tfrac{1}{3}g \\
\Rightarrow \quad & T = (4 - \tfrac{4}{3})g \\
\Rightarrow \quad & \underline{\underline{T = \tfrac{8}{3}g}}.
\end{aligned}
$$

51. A particle P of mass 0.6 kg slides with constant acceleration down a line of greatest slope of a rough plane, which is inclined at 25° to the horizontal. The particle passes through two points, A and B, where $AB = 10$ m, as shown in Figure 45.

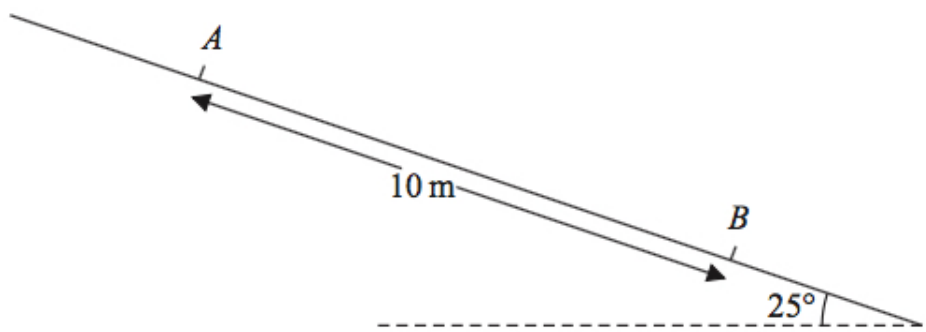


Figure 45: a particle P of mass 0.6 kg

The speed of P at A is 2 ms^{-1}. The particle P takes 3.5 s to move from A to B. Find

(a) the speed of P at B, (3)

Solution

$s = 10, u = 2, v = ?, a = ?, t = 3.5$:

$$
\begin{aligned}
s = \left(\frac{u+v}{2}\right)t \Rightarrow 10 = 3.5\left(\frac{2+v}{2}\right) \\
\Rightarrow \tfrac{20}{7} = \frac{2+v}{2} \\
\Rightarrow \tfrac{40}{7} = 2 + v \\
\Rightarrow \underline{\underline{v = \tfrac{26}{7}}}.
\end{aligned}
$$

(b) the acceleration of P, (2)

Solution

$s = 10, u = 2, v = \frac{26}{7}, a = ?, t = 3.5$:

$$\begin{aligned} v = u + at &\Rightarrow \tfrac{26}{7} = 2 + 3.5a \\ &\Rightarrow 3.5a = \tfrac{12}{7} \\ &\Rightarrow \underline{\underline{a = \tfrac{24}{49}}}. \end{aligned}$$

(c) the coefficient of friction between P and the plane. (5)

Solution

$$\begin{aligned} \text{Parallel:} \quad & 0.6g\sin 25^\circ - F = 0.6 \times \tfrac{24}{49} \\ \text{Perpendicular:} \quad & R = 0.6g\cos 25^\circ \\ F = \mu R & \end{aligned}$$

Now,

$$\begin{aligned} 0.6g\sin 25^\circ - F = \tfrac{72}{245} &\Rightarrow 0.6g\sin 25^\circ - \mu R = \tfrac{72}{245} \\ &\Rightarrow 0.6g\sin 25^\circ - 0.6\mu g\cos 25^\circ = \tfrac{72}{245} \\ &\Rightarrow 0.6\mu g\cos 25^\circ = 0.6g\sin 25^\circ - \tfrac{72}{245} \\ &\Rightarrow \mu = \frac{0.6g\sin 25^\circ - \frac{72}{245}}{0.6g\cos 25^\circ} \\ &\Rightarrow \mu = 0.411\,161\,739\,7 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{\mu = 0.41 \text{ (2 sf)}}}. \end{aligned}$$

52. A truck of mass 1750 kg is towing a car of mass 750 kg along a straight horizontal road. The two vehicles are joined by a light towbar which is inclined at an angle θ to the road, as shown in Figure 46.

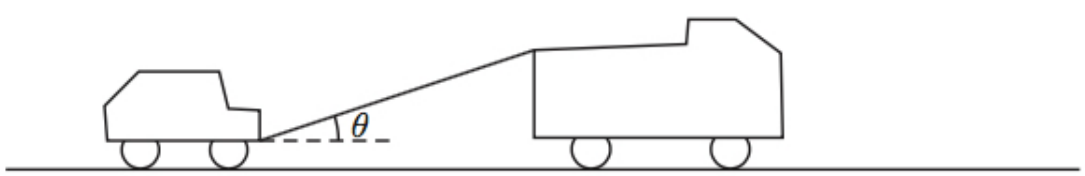


Figure 46: a truck and a car

The vehicles are travelling at 20 ms^{-1} as they enter a zone where the speed limit is 14 ms^{-1}. The truck's brakes are applied to give a constant braking force on the truck. The distance travelled between the instant when the brakes are applied and the instant when the speed of each vehicle is 14 ms^{-1} is 100 m.

(a) Find the deceleration of the truck and the car. (3)

Solution

$s = 100, u = 20, v = 14, a = ?, t = ?$:

$$v^2 = u^2 + 2as \Rightarrow 14^2 = 20^2 + 200a$$
$$\Rightarrow a = -1.02;$$

the deceleration is $\underline{\underline{1.02 \text{ ms}^{-2}}}$.

The constant braking force on the truck has magnitude R newtons. The truck and the car also experience constant resistances to motion of 500 N and 300 N respectively. Given that $\cos\theta = 0.9$, find

(b) the force in the towbar, (4)

Solution

For the car:

$$T\cos\theta - 300 = -765 \Rightarrow 0.9T = -465$$
$$\Rightarrow T = -516.\dot{6};$$

hence, the tension is $\underline{\underline{520 \text{ N (2 sf)}}}$.

(c) the value of R. (4)

Solution

The whole system:

$$500 + 300 + R = 2500 \times 1.02 \Rightarrow \underline{\underline{R = 1750}}.$$

53. A particle of weight W newtons is attached at C to two light inextensible strings AC and BC. The other ends of the strings are attached to fixed points A and B on a horizontal ceiling. The particle hangs in equilibrium with AC and BC inclined to the horizontal at 30^circ and 50^circ respectively, as shown in Figure 47.

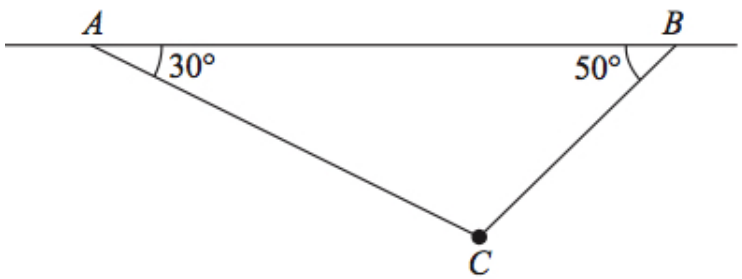


Figure 47: a particle of weight W newtons

Given that the tension in BC is 6 N, find

(a) the tension in AC, (3)

Solution

Let T N be the tension in the 30°-string.

$$\begin{aligned} \text{Horizontal}: \quad & T\cos 30^\circ = 6\cos 50^\circ \\ \text{Vertical}: \quad & T\sin 30^\circ + 6\sin 50^\circ = W. \end{aligned}$$

Now,

$$\begin{aligned} T\cos 30^\circ = 6\cos 50^\circ \Rightarrow T &= \frac{6\cos 50^\circ}{\cos 30^\circ} \\ \Rightarrow T &= 4.453\,363\,194 \text{ (FCD)} \\ \Rightarrow \underline{\underline{T = 4.5 \text{ (2 sf)}}}. \end{aligned}$$

(b) the value of W. (3)

Solution

$$\begin{aligned} W &= 4.453\ldots\sin 30^\circ + 6\sin 50^\circ \\ &= 6.822\,948\,256 \text{ (FCD)} \\ &= \underline{\underline{W = 6.8 \text{ (2 sf)}}}. \end{aligned}$$

54. A rough plane is inclined at 40° to the horizontal. Two points A and B are 3 metres apart and lie on a line of greatest slope of the inclined plane, with A above B, as shown in Figure 48.

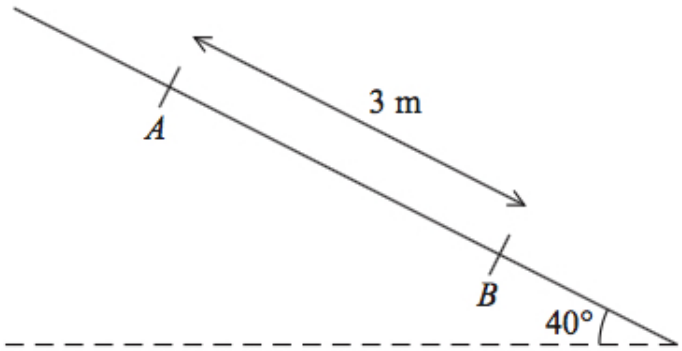


Figure 48: two points A and B

A particle P of mass m kg is held at rest on the plane at A. The coefficient of friction between P and the plane is $\frac{1}{2}$. The particle is released.

(a) Find the acceleration of P down the plane. (5)

Solution

$$\begin{aligned} \text{Parallel:} \quad & mg\sin 40^\circ - F = ma \\ \text{Perpendicular:} \quad & R = mg\cos 40^\circ \\ F = \mu R \quad & F = \tfrac{1}{2}R \end{aligned}$$

Now,

$$\begin{aligned} mg\sin 40^\circ - F = ma &\Rightarrow mg\sin 40^\circ - \tfrac{1}{2}R = ma \\ &\Rightarrow ma = mg\sin 40^\circ - \tfrac{1}{2}mg\cos 40^\circ \\ &\Rightarrow a = g\sin 40^\circ - \tfrac{1}{2}g\cos 40^\circ \\ &\Rightarrow a = 2.545\,700\,804 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{a = 2.5 \text{ (2 sf)}}}. \end{aligned}$$

(b) Find the speed of P at B. (2)

Solution

$s = 3, u = 0, v = ?, a = 2.545\ldots, t = ?$:

$$\begin{aligned} v^2 = u^2 + 2as &\Rightarrow v^2 = 0 + 2 \times 2.545\ldots \times 3 \\ &\Rightarrow v^2 = 15.274\,204\,82 \text{ (FCD)} \\ &\Rightarrow v = 3.908\,222\,719 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{v = 3.9 \text{ (2 sf)}}}. \end{aligned}$$

55. Three particles A, B, and C have masses $3m$, $2m$, and $2m$ respectively. Particle C is attached to particle B. Particles A and B are connected by a light inextensible string which passes over a smooth light fixed pulley. The system is held at rest with the string taut and the hanging parts of the string vertical, as shown in Figure 49.

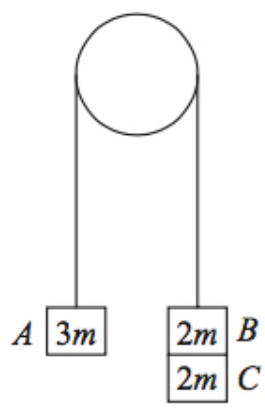


Figure 49: three particles A, B, and C

The system is released from rest and A moves upwards.

(a) (i) Show that the acceleration of A is $\frac{1}{7}g$. (7)

Solution
Let T N be the tension.

$$\begin{aligned} A: &\quad T - 3mg = 3ma \\ B \text{ and } C: &\quad 4mg - T = 4ma. \end{aligned}$$

Add:

$$mg = 7ma \Rightarrow \underline{\underline{a = \tfrac{1}{7}g}}.$$

(ii) Find the tension in the string as A ascends.

Solution

$$T = 3m(g + a) = \underline{\underline{\tfrac{24}{7}mg}}.$$

At the instant when A is 0.7 m above its original position, C separates from B and falls away. In the subsequent motion, A does not reach the pulley.

(b) Find the speed of A at the instant when it is 0.7 m above its original position. (2)

Solution

$s = 0.7, u = 0, v = ?, a = \frac{1}{7}g, t = ?$:

$$\begin{aligned} v^2 = u^2 + 2as &\Rightarrow v^2 = 0 + 2 \times \tfrac{1}{7}g \times 0.7 \\ &\Rightarrow v^2 = 1.96 \\ &\Rightarrow \underline{\underline{v = 1.4}}. \end{aligned}$$

(c) Find the acceleration of A at the instant after C separates from B. (4)

Solution

Let S N be the tension.

$$\begin{aligned} A: &\quad 3mg - S = 3ma \\ C: &\quad S - 2mg = 2ma. \end{aligned}$$

Add:

$$mg = 5ma \Rightarrow \underline{\underline{a = \tfrac{1}{5}g}}.$$

(d) Find the greatest height reached by A above its original position. (3)

Solution

$s = ?, u = 1.4, v = 0, a = -\frac{1}{5}g, t = ?$:

$$\begin{aligned} v^2 = u^2 + 2as &\Rightarrow 0 = 1.4^2 + 2 \times (-\tfrac{1}{5}g) \times s \\ &\Rightarrow s = 0.5; \end{aligned}$$

so, in total, the greatest height reached by A is

$$0.7 + 0.5 = \underline{\underline{1.2 \text{ m}}}.$$

56. A particle P of weight W newtons is attached to one end of a light inextensible string. The other end of the string is attached to a fixed point O. A horizontal force of magnitude 5 N is applied to P. The particle P is in equilibrium with the string taut and with OP making an angle of $25°$ to the downward vertical, as shown in Figure 50.

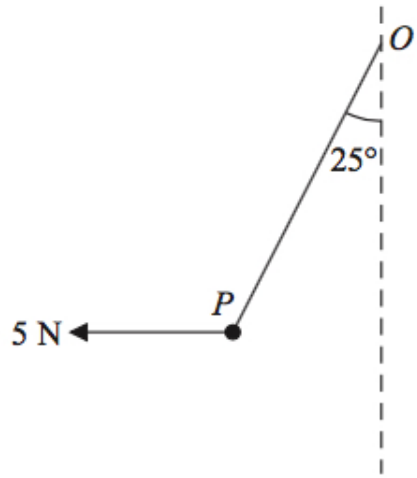


Figure 50: a particle P of weight W newtons

Find

(a) the tension in the string, (3)

Solution

Let T N be the tension.

$$\begin{aligned} \text{Horizontal}: &\quad 5 = T\cos 65° \\ \text{Vertical}: &\quad T\sin 65° = W. \end{aligned}$$

Now,

$$\begin{aligned} 5 = T\cos 65° &\Rightarrow T = \frac{5}{\cos 65°} \\ &\Rightarrow T = 11.831\,007\,92 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{T = 12 \text{ (2 sf)}}}. \end{aligned}$$

(b) the value of W. (3)

Solution

$$\begin{aligned} W &= 11.831\ldots \sin 65^\circ \\ &= 10.722\,534\,6 \text{ (FCD)} \\ &= \underline{\underline{11 \text{ (2 sf)}}}. \end{aligned}$$

57. Two particles A and B have masses $2m$ and $3m$ respectively. The particles are connected by a light inextensible string which passes over a smooth light fixed pulley. The system is held at rest with the string taut. The hanging parts of the string are vertical and A and B are above a horizontal plane, as shown in Figure 51.

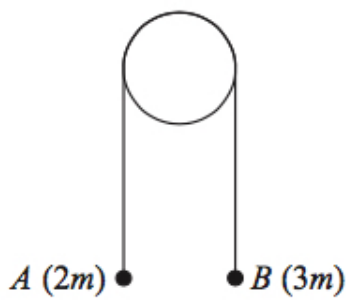


Figure 51: two particles A and B

The system is released from rest.

(a) Show that the tension in the string immediately after the particles are released is $\frac{12}{5}mg$. (6)

Solution

Let T N be the tension.

$$\begin{aligned} A: &\quad T - 2mg = 2ma \\ B: &\quad 3mg - T = 3ma. \end{aligned}$$

Add:

$$\begin{aligned} mg = 5ma &\Rightarrow a = \tfrac{1}{5}g \\ &\Rightarrow T = 2mg + \tfrac{2}{5}mg \\ &\Rightarrow \underline{\underline{T = \tfrac{12}{5}mg}}. \end{aligned}$$

After descending 1.5 m, B strikes the plane and is immediately brought to rest. In the subsequent motion, A does not reach the pulley.

(b) Find the distance travelled by A between the instant when B strikes the plane and the instant when the string next becomes taut. (6)

Solution

For B, $s = 1.5, u = 0, v = ?, a = \frac{1}{5}g, t = ?$:

$$\begin{aligned} v^2 = u^2 + 2as &\Rightarrow v^2 = 0 + 2 \times \tfrac{1}{5}g \times 1.5 \\ &\Rightarrow v^2 = 5.88 \\ &\Rightarrow v = \sqrt{5.88}. \end{aligned}$$

For A, $s = ?, u = \sqrt{5.88}, v = -\sqrt{5.88}, a = -g, t = ?$:

$$\begin{aligned} v^2 = u^2 + 2as &\Rightarrow (-5.88)^2 = 5.88^2 + 2 \times (-g) \times s \\ &\Rightarrow 19.6s = 11.76 \\ &\Rightarrow \underline{\underline{s = 0.6 \text{ m}}}. \end{aligned}$$

Given that $m = 0.5$ kg,

(c) find the magnitude of the impulse on B due to the impact with the plane. (2)

Solution

$$\begin{aligned} \text{Impulse} &= m(v - u) \\ &= 1.5(0 - \sqrt{5.88}) \\ &= -1.5\sqrt{5.88} \end{aligned}$$

and so

$$\begin{aligned} \text{magnitude} &= \left|-1.5\sqrt{5.88}\right| \\ &= 3.637\,306\,696 \text{ (FCD)} \\ &= \underline{\underline{3.6 \text{ N (2 sf)}}}. \end{aligned}$$

58. A particle P of mass 2.7 kg lies on a rough plane inclined at 40° to the horizontal. The particle is held in equilibrium by a force of magnitude 15 N acting at an angle of 540° to the plane, as shown in Figure 52.

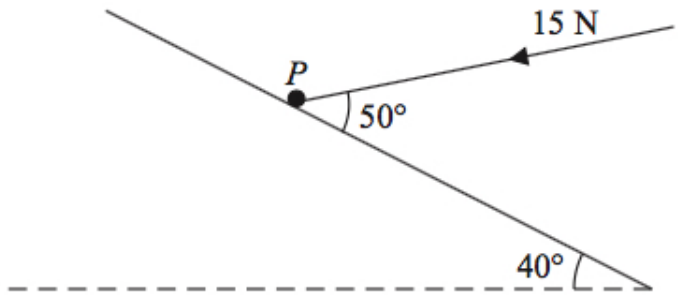


Figure 52: a particle P of mass 2.7 kg

The force acts in a vertical plane containing a line of greatest slope of the plane. The particle is in equilibrium and is on the point of sliding down the plane. Find

(a) the magnitude of the normal reaction of the plane on P, (4)

Solution

$$\begin{aligned} \text{Parallel:} \quad & F + 15\cos 50^\circ - 2.7g\sin 40^\circ = 0 \\ \text{Perpendicular:} \quad & R = 15\sin 50^\circ + 2.7g\cos 40^\circ \\ F = \mu R: & \end{aligned}$$

Now,

$$\begin{aligned} R &= 15\sin 50^\circ + 2.7g\cos 40^\circ \\ &= 31.760\,020\,261 \text{ (FCD)} \\ &= \underline{\underline{32 \text{ (2 sf)}}}. \end{aligned}$$

(b) the coefficient of friction between P and the plane. (5)

Solution

$$
\begin{aligned}
& F + 15\cos 50^\circ - 2.7g\sin 40^\circ = 0 \\
\Rightarrow\quad & \mu R + 15\cos 50^\circ - 2.7g\sin 40^\circ = 0 \\
\Rightarrow\quad & \mu(15\sin 50^\circ + 2.7g\cos 40^\circ) + 15\cos 50^\circ - 2.7g\sin 40^\circ = 0 \\
\Rightarrow\quad & \mu(15\sin 50^\circ + 2.7g\cos 40^\circ) = 2.7g\sin 40^\circ - 15\cos 50^\circ \\
\Rightarrow\quad & \mu = \frac{2.7g\sin 40^\circ - 15\cos 50^\circ}{15\sin 50^\circ + 2.7g\cos 40^\circ} \\
\Rightarrow\quad & \mu = 0.231\,936\,366\,9 \text{ (FCD)} \\
\Rightarrow\quad & \underline{\underline{\mu = 0.23 \text{ (2 sf)}}}.
\end{aligned}
$$

The force of magnitude 15 N is removed.

(c) Determine whether P moves, justifying your answer. (4)

Solution

$$
\begin{aligned}
\text{Parallel:}\quad & F = 2.7g\sin 40^\circ \\
\text{Perpendicular:}\quad & R = 2.7g\cos 40^\circ \\
F = \mu R:\quad &
\end{aligned}
$$

Now,

$$F = 2.7g\sin 40^\circ = 17.008\,160\,15 \text{ (FCD)}$$

and

$$F_{\max} = 2.7\mu g\cos 40^\circ = 4.701\,242\,153 \text{ (FCD)};$$

hence, <u>the particle moves</u>.

59. A particle of mass 2 kg is suspended from a horizontal ceiling by two light inextensible strings, PR and QR. The particle hangs at R in equilibrium, with the strings in a vertical plane. The string PR is inclined at 55° to the horizontal and the string QR is inclined at 35° to the horizontal, as shown in Figure 53.

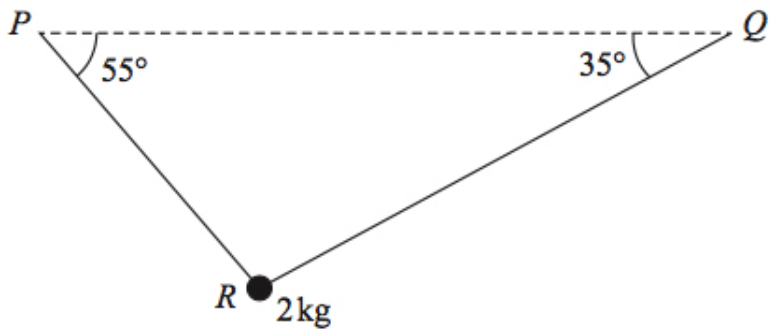


Figure 53: a particle of mass 2 kg

Find

(a) the tension in the string PR,

Solution

Let S N be the tension in the 55°-string and let T N be the tension in the 35°-string.

$$\begin{aligned} \text{Horizontal}: &\quad S\cos 55^\circ = T\cos 35^\circ \\ \text{Vertical}: &\quad S\sin 55^\circ + T\sin 35^\circ = 2g. \end{aligned}$$

Now,

$$\begin{aligned} S\cos 55^\circ = T\cos 35^\circ \Rightarrow T &= \frac{S\cos 55^\circ}{\cos 35^\circ} \\ &\Rightarrow S\sin 55^\circ + S\cos 55^\circ \tan 35^\circ = 2g \\ &\Rightarrow S(\sin 55^\circ + \cos 55^\circ \tan 35^\circ) = 2g \\ &\Rightarrow S = \frac{2g}{\sin 55^\circ + \cos 55^\circ \tan 35^\circ} \\ &\Rightarrow S = 16.055\,380\,07 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{S = 16 \text{ (2 sf)}}}. \end{aligned}$$

(b) the tension in the string QR.

Solution

$$
\begin{aligned}
T &= \frac{S\cos 55^\circ}{\cos 35^\circ} \\
&= 11.242\,098\,15 \text{ (FCD)} \\
&= \underline{\underline{11 \text{ (2 sf)}}}.
\end{aligned}
$$

60. A lift of mass 200 kg is being lowered into a mineshaft by a vertical cable attached Figure 54.

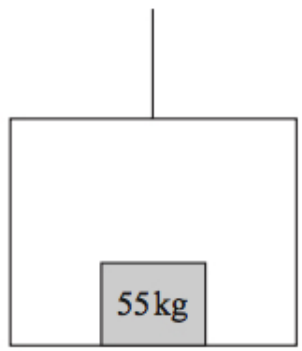


Figure 54: a lift of mass 200 kg

The lift descends vertically with constant acceleration. There is a constant upwards resistance of magnitude 150 N on the lift. The crate experiences a constant normal reaction of magnitude 473 N from the floor of the lift.

(a) Find the acceleration of the lift. (3)

Solution

Let a ms^{-2} be the acceleration. For the crate,

$$
\begin{aligned}
55g - 473 = 55a \Rightarrow 55a &= 66 \\
\Rightarrow \underline{\underline{a = 1.2}}.
\end{aligned}
$$

(b) Find the magnitude of the force exerted on the lift by the cable. (4)

Solution

Let T N be the tension. For the whole system,

$$\begin{aligned} 200g + 55g - T - 150 = (200+55)a &\Rightarrow 2349 - T = 306 \\ &\Rightarrow T = 2043 \\ &\Rightarrow \underline{\underline{T = 2000 \text{ (2 sf)}}}. \end{aligned}$$

61. Two particles P and Q have mass 4 kg and 0.5 kg respectively. The particles are attached to the ends of a light inextensible string. Particle P is held at rest on a fixed rough plane, which is inclined to the horizontal at an angle α where $\tan\alpha = \frac{4}{3}$. The coefficient of friction between P and the plane is 0.5. The string lies along the plane and passes over a small smooth light pulley which is fixed at the top of the plane. Particle Q hangs freely at rest vertically below the pulley. The string lies in the vertical plane which contains the pulley and a line of greatest slope of the inclined plane, as shown in Figure 55.

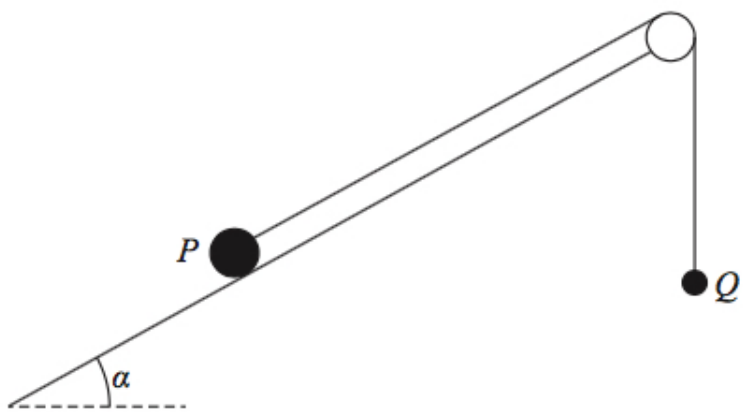


Figure 55: two particles P and Q have mass 4 kg and 0.5 kg respectively

Particle P is released from rest with the string taut and slides down the plane. Given that Q has not hit the pulley, find

(a) the tension in the string during the motion, (11)

Solution

Let T N be the tension and let a ms^{-2} be the acceleration. Well,

$$\sin\alpha = \tfrac{4}{5} \text{ and } \cos\alpha = \tfrac{3}{5}$$

and

$$\begin{aligned} P: &\quad \text{Parallel:} \quad 4g\sin\alpha - T - F = 4a \\ &\quad \text{Perpendicular:} \quad R = 4g\cos\alpha \\ &\quad F = \mu R: \quad F = 0.5R \\ Q: &\quad T - 0.5g = 0.5a. \end{aligned}$$

Now,

$$\begin{aligned} 4g\sin\alpha - T - F = 4a &\Rightarrow \tfrac{16}{5}g - T - 0.5R = 4a \\ &\Rightarrow \tfrac{16}{5}g - T - \tfrac{6}{5}g = 4a \\ &\Rightarrow 2g - T = 4a. \end{aligned}$$

Add:

$$\begin{aligned} 2g - \tfrac{1}{2}g = \tfrac{9}{2}a &\Rightarrow \tfrac{3}{2}g = \tfrac{9}{2}a \\ &\Rightarrow a = \tfrac{1}{3}g \\ &\Rightarrow T = \tfrac{1}{2}g + \tfrac{1}{2} \times \tfrac{1}{3}g \\ &\Rightarrow \underline{\underline{T = \tfrac{2}{3}g}}. \end{aligned}$$

(b) the magnitude of the resultant force exerted by the string on the pulley. (4)

Solution

We need $\cos\left(\frac{90-\alpha}{2}\right)$ (why?):

$$\begin{aligned} & \cos(90-\alpha) = 2\cos^2\left(\tfrac{90-\alpha}{2}\right) - 1 \\ \Rightarrow \quad & \sin\alpha + 1 = 2\cos^2\left(\tfrac{90-\alpha}{2}\right) \\ \Rightarrow \quad & \tfrac{9}{5} = 2\cos^2\left(\tfrac{90-\alpha}{2}\right) \\ \Rightarrow \quad & \tfrac{9}{10} = \cos^2\left(\tfrac{90-\alpha}{2}\right) \\ \Rightarrow \quad & \cos\left(\tfrac{90-\alpha}{2}\right) = \tfrac{3\sqrt{10}}{10} \end{aligned}$$

as $\cos\left(\frac{90-\alpha}{2}\right)$ is positive. The magnitude is

$$\begin{aligned} F &= 2T\cos\left(\tfrac{90-\alpha}{2}\right) \\ &= 2 \times \tfrac{2}{3}g \times \tfrac{3\sqrt{10}}{10} \\ &= \underline{\underline{\tfrac{2\sqrt{10}}{5}g}}. \end{aligned}$$

62. A vertical rope AB has its end B attached to the top of a scale pan. The scale pan has mass 0.5 kg and carries a brick of mass 1.5 kg, as shown in Figure 56.

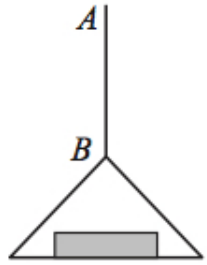


Figure 56: a vertical rope AB

The scale pan is raised vertically upwards with constant acceleration 0.5 ms^{-2} using the rope AB. The rope is modelled as a light inextensible string.

(a) Find the tension in the rope AB. (3)

Solution

Let T N be the tension and we consider the whole system:

$$\begin{aligned} T - 0.5g - 1.5g = (0.5 + 1.5) \times 0.5 &\Rightarrow T - 2g = 1 \\ &\Rightarrow T = 20.6 \\ &\Rightarrow \underline{\underline{T = 20 \text{ (2 sf)}}}. \end{aligned}$$

(b) Find the magnitude of the force exerted on the scale pan by the brick. (3)

Solution

Let S N be the the magnitude of the force exerted on the scale pan by the brick and we consider the brick:

$$\begin{aligned} S - 1.5g = 1.5 \times 0.5 &\Rightarrow S = 15.45 \\ &\Rightarrow \underline{\underline{S = 15 \text{ (2 sf)}}}. \end{aligned}$$

63. A particle P of mass 2 kg is held at rest in equilibrium on a rough plane by a constant force of magnitude 40 N. The direction of the force is inclined to the plane at an angle of 30°. The plane is inclined to the horizontal at an angle of 20°, as shown in Figure 57. (10)

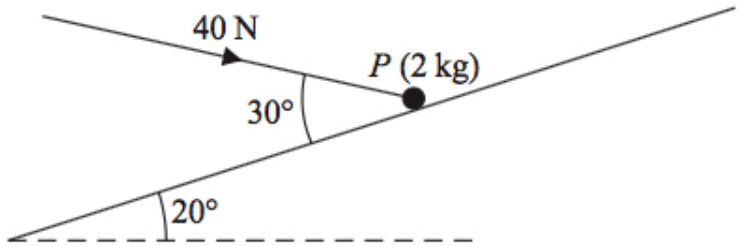


Figure 57: a particle P of mass 2 kg

The line of action of the force lies in the vertical plane containing P and a line of greatest slope of the plane. The coefficient of friction between P and the plane is μ. Given that P is on the point of sliding up the plane, find the value of μ.

Solution

$$\begin{aligned} \text{Parallel:} \quad & 40\cos 30^\circ - F = 2g\sin 20^\circ \\ \text{Perpendicular:} \quad & R = 2g\cos 20^\circ + 40\sin 30^\circ \\ F = \mu R: & \end{aligned}$$

Now,

$$\begin{aligned} 40\cos 30^\circ - F = 2g\sin 20^\circ &\Rightarrow 40\cos 30^\circ - \mu R = 2g\sin 20^\circ \\ &\Rightarrow 40\cos 30^\circ - \mu(2g\cos 20^\circ + 40\sin 30^\circ) = 2g\sin 20^\circ \\ &\Rightarrow \mu(2g\cos 20^\circ + 40\sin 30^\circ) = 40\cos 30^\circ - 2g\sin 20^\circ \\ &\Rightarrow \mu = \frac{40\cos 30^\circ - 2g\sin 20^\circ}{2g\cos 20^\circ + 40\sin 30^\circ} \\ &\Rightarrow \mu = 0.727\,196\,607\,2 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{\mu = 0.73 \text{ (2 sf)}}}. \end{aligned}$$

64. Two particles P and Q have masses 1.5 kg and 3 kg respectively. The particles are attached to the ends of a light inextensible string. Particle P is held at rest on a fixed rough horizontal table. The coefficient of friction between P and the table is $\frac{1}{5}$. The string is parallel to the table and passes over a small smooth light pulley which is fixed at the edge of the table. Particle Q hangs freely at rest vertically below the pulley, as shown in Figure 58.

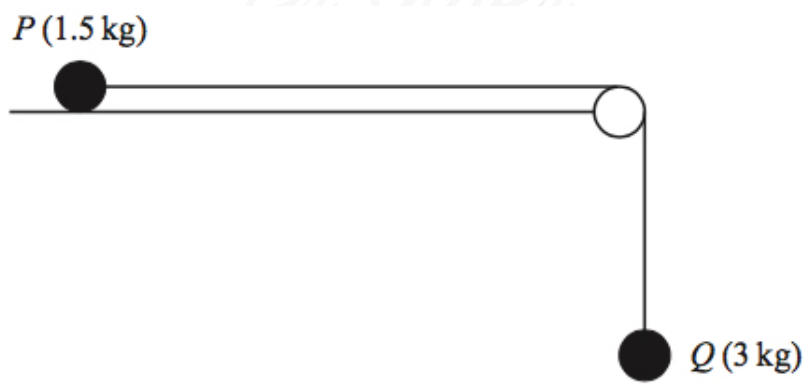


Figure 58: two particles P and Q

Particle P is released from rest with the string taut and slides along the table. Assuming that P has not reached the pulley, find

(a) the tension in the string during the motion, (8)

Solution

$$\begin{aligned} P: &\quad \text{Parallel:} \quad T - F = 1.5a \\ &\quad \text{Perpendicular:} \quad R = 1.5g \\ &\quad F = \mu R: \quad F = \tfrac{1}{5}R \\ Q: &\quad 3g - T = 3a. \end{aligned}$$

Now,

$$\begin{aligned} T - F = 1.5a &\Rightarrow T - \tfrac{1}{5}R = 1.5a \\ &\Rightarrow T - \tfrac{3}{10}g = 1.5a. \end{aligned}$$

Add:

$$\begin{aligned} 3g - \tfrac{3}{10}g = 1.5a + 3a &\Rightarrow \tfrac{27}{10}g = 4.5a \\ &\Rightarrow a = \tfrac{3}{5}g \\ &\Rightarrow T = \tfrac{3}{10}g + 1.5 \times \tfrac{3}{5}g \\ &\Rightarrow \underline{\underline{T = \tfrac{6}{5}g}}. \end{aligned}$$

(b) the magnitude and direction of the resultant force exerted on the pulley by the string. (4)

Solution

The magnitude is

$$\begin{aligned} F &= \sqrt{T^2 + T^2} \\ &= T\sqrt{2} \\ &= \underline{\underline{\tfrac{6\sqrt{2}}{5}mg}} \end{aligned}$$

and the direction is $\underline{\underline{45^\circ}}$, top right to bottom left ($\swarrow$).

65. Three forces, $(15\mathbf{i} + \mathbf{j})$ N, $(5q\mathbf{i} - p\mathbf{j})$ N, and $(-3p\mathbf{i} - q\mathbf{j})$ N, where p and q are constants, act on a particle. Given that the particle is in equilibrium, find the value of p and the value of q. (6)

Solution

$$\begin{aligned} \text{Horizontal:} \quad & 15 + 5q - 3p = 0 \\ \text{Vertical:} \quad & 1 - p - q = 0. \end{aligned}$$

Now,

$$\begin{aligned} q = 1 - p &\Rightarrow 15 + 5(1 - p) - 3p = 0 \\ &\Rightarrow 15 + 5 - 5p - 3p = 0 \\ &\Rightarrow 8p = 20 \\ &\Rightarrow \underline{\underline{p = \tfrac{5}{2}}} \\ &\Rightarrow \underline{\underline{q = -\tfrac{3}{2}}}. \end{aligned}$$

66. A particle P of mass 5 kg is held at rest in equilibrium on a rough inclined plane by a horizontal force of magnitude 10 N. The plane is inclined to the horizontal at an angle α where $\tan\alpha = \frac{3}{4}$, as shown in Figure 59. (9)

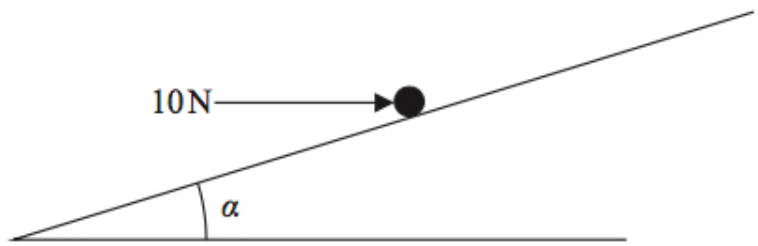


Figure 59: a particle P of mass 5 kg

The line of action of the force lies in the vertical plane containing P and a line of greatest slope of the plane. The coefficient of friction between P and the plane is μ. Given that P is on the point of sliding down the plane, find the value of μ.

Solution

Well,

$$\sin\alpha = \tfrac{3}{5} \text{ and } \cos\alpha = \tfrac{4}{5}$$

and

$$\begin{aligned} \text{Parallel:} \quad & 10\cos\alpha - 5g\sin\alpha + F = 0 \\ \text{Perpendicular:} \quad & R = 5g\cos\alpha + 10\sin\alpha \\ F = \mu R: & \end{aligned}$$

Now,

$$\begin{aligned} 10\cos\alpha - 5g\sin\alpha + F = 0 &\Rightarrow 6 - 3g + \mu R = 0 \\ &\Rightarrow 8 - 3g + \mu(4g+6) = 0 \\ &\Rightarrow \mu(4g+6) = 3g - 8 \\ &\Rightarrow \mu = \frac{3g-8}{4g+6} \\ &\Rightarrow \mu = \tfrac{107}{226} \\ &\Rightarrow \underline{\underline{\mu = 0.47 \text{ (2 sf)}}}. \end{aligned}$$

67. A vertical light rod PQ has a particle of mass 0.5 kg attached to it at P and a particle of mass 0.75 kg attached to it at Q, to form a system, as shown in Figure 60. (6)

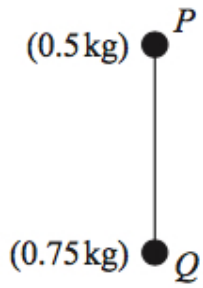


Figure 60: a vertical light rod PQ

The system is accelerated vertically upwards by a vertical force of magnitude 15 N applied to the particle at Q. Find the thrust in the rod.

Solution

Let T N be the thrust and let a ms^{-2} be the acceleration.

$$\begin{aligned} P: &\quad T - 0.5g = 0.5a \\ Q: &\quad 15 - T - 0.75g = 0.75a. \end{aligned}$$

Add:

$$\begin{aligned} 5 - 1.25g = 1.25a &\Rightarrow 1.25a = 2.75 \\ &\Rightarrow a = 2.2 \\ &\Rightarrow T = 0.5g + 0.5 \times 2.2 \\ &\Rightarrow \underline{\underline{T = 6}}. \end{aligned}$$

68. Two particles, A and B, have masses $2m$ and m respectively. The particles are attached to the ends of a light inextensible string. Particle A is held at rest on a fixed rough horizontal table at a distance d from a small smooth light pulley which is fixed at the edge of the table at the point P. The coefficient of friction between A and the table is μ, where $\mu < \frac{1}{2}$. The string is parallel to the table from A to P and passes over the pulley. Particle B hangs freely at rest vertically below P with the string taut and at a height h, $(h < d)$, above a horizontal floor, as shown in Figure 61.

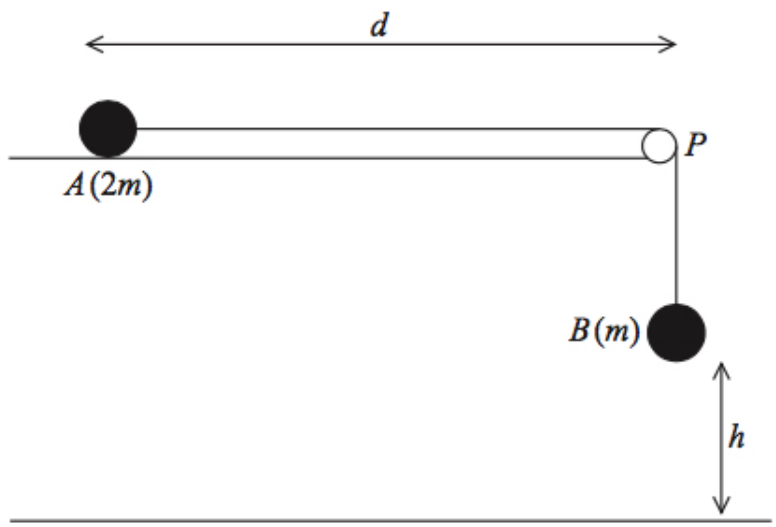


Figure 61: two particles, A and B

Particle A is released from rest with the string taut and slides along the table.

(a) (i) Write down an equation of motion for A. (4)

Solution

$$\text{Parallel:} \quad \underline{\underline{T - F = 2ma}}$$

$$\text{Perpendicular:} \quad R = 2mg$$

$$F = \mu R.$$

(ii) Write down an equation of motion for B.

Solution

$$\underline{\underline{mg - T = ma.}}$$

(b) Hence show that, until B hits the floor, the acceleration of A is (3)

$$\tfrac{1}{3}g(1 - 2\mu).$$

Solution

$$T - F = 2ma \Rightarrow T - \mu R = 2ma$$
$$\Rightarrow T - 2\mu mg = 2ma.$$

Add:

$$mg - 2\mu mg = 2ma + ma \Rightarrow mg(1-2\mu) = 3ma$$
$$\Rightarrow \underline{\underline{a = \tfrac{1}{3}g(1-2\mu)}}.$$

(c) Find, in terms of g, h, and μ, the speed of A at the instant when B hits the floor. (2)

Solution

For B, $s = h, u = 0, v = ?, a = \frac{1}{3}g(1-2\mu), t = ?$:

$$v^2 = u^2 + 2as \Rightarrow v^2 = 0 + 2 \times \tfrac{1}{3}g(1-2\mu) \times h$$
$$\Rightarrow v^2 = \tfrac{2}{3}gh(1-2\mu)$$
$$\Rightarrow \underline{\underline{v = \sqrt{\tfrac{2}{3}gh(1-2\mu)}}}.$$

After B hits the floor, A continues to slide along the table. Given that $\mu = \frac{1}{3}$ and that A comes to rest at P,

(d) find d in terms of h. (5)

Solution

For A,

$$-F = 2ma \Rightarrow -\tfrac{2}{3}mg = 2ma \Rightarrow a = -\tfrac{1}{3}g$$

and

$$\sqrt{\tfrac{2}{3}gh(1 - 2 \times \tfrac{1}{3})} = \sqrt{\tfrac{2}{9}gh}.$$

$s = ?, u = \sqrt{\frac{2}{9}gh}, v = 0, a = -\frac{1}{3}g, t = ?$:

$$v^2 = u^2 + 2as \Rightarrow 0 = \tfrac{2}{9}gh + 2 \times (-\tfrac{1}{3}g) \times s$$
$$\Rightarrow \tfrac{2}{3}gs = \tfrac{2}{9}gh$$
$$\Rightarrow s = \tfrac{1}{3}h;$$

and

$$d = \tfrac{1}{3}h + h = \underline{\underline{\tfrac{4}{3}h}}.$$

(e) Describe what would happen if $\mu = \frac{1}{2}$. (1)

Solution

A and B would remain in limiting equilibrium.