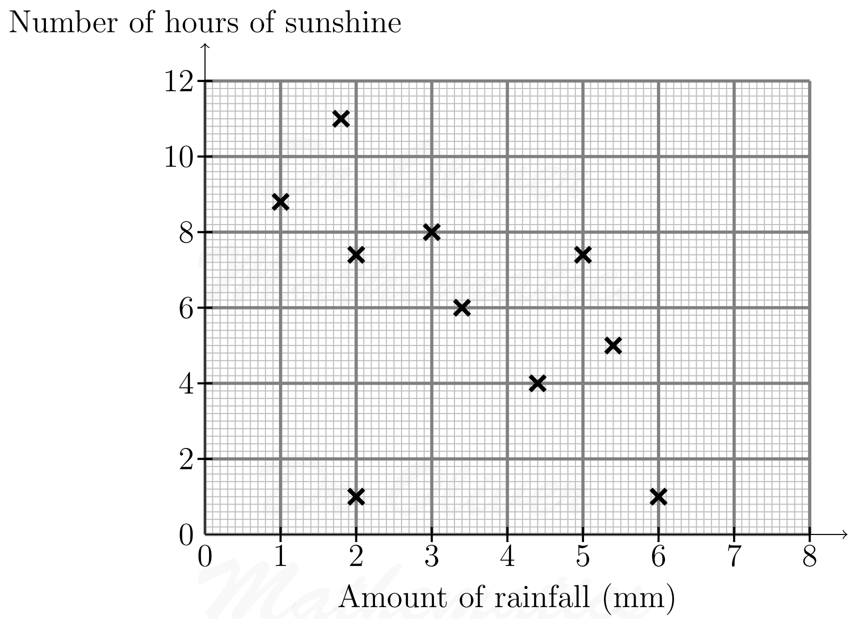
Dr Oliver Mathematics
GCSE Mathematics
2020 November Paper 2H: Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. The scatter graph shows information about the amount of rainfall, in mm, and the number of hours of sunshine for each of ten English towns on the same day.



One of the points is an outlier.

(a) Write down the coordinates of this point. (1)

Solution
It is the point $\underline{\underline{(2, 1)}}$.

(b) Ignoring the outlier, describe the relationship between the amount of rainfall and the number of hours of sunshine. (1)

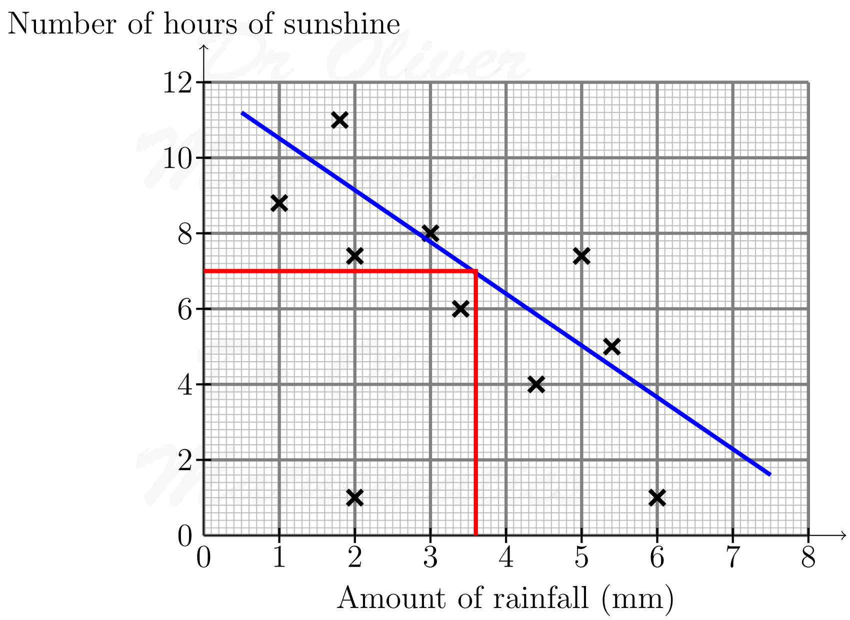
Solution
E.g., negative correlation, i.e., as the amount of rainfall decreases the number of hours of sunshine increases.

On the same day in another English town there were 7 hours of sunshine.

(c) Using the scatter graph, estimate the amount of rainfall in this town on this day. (2)

Solution

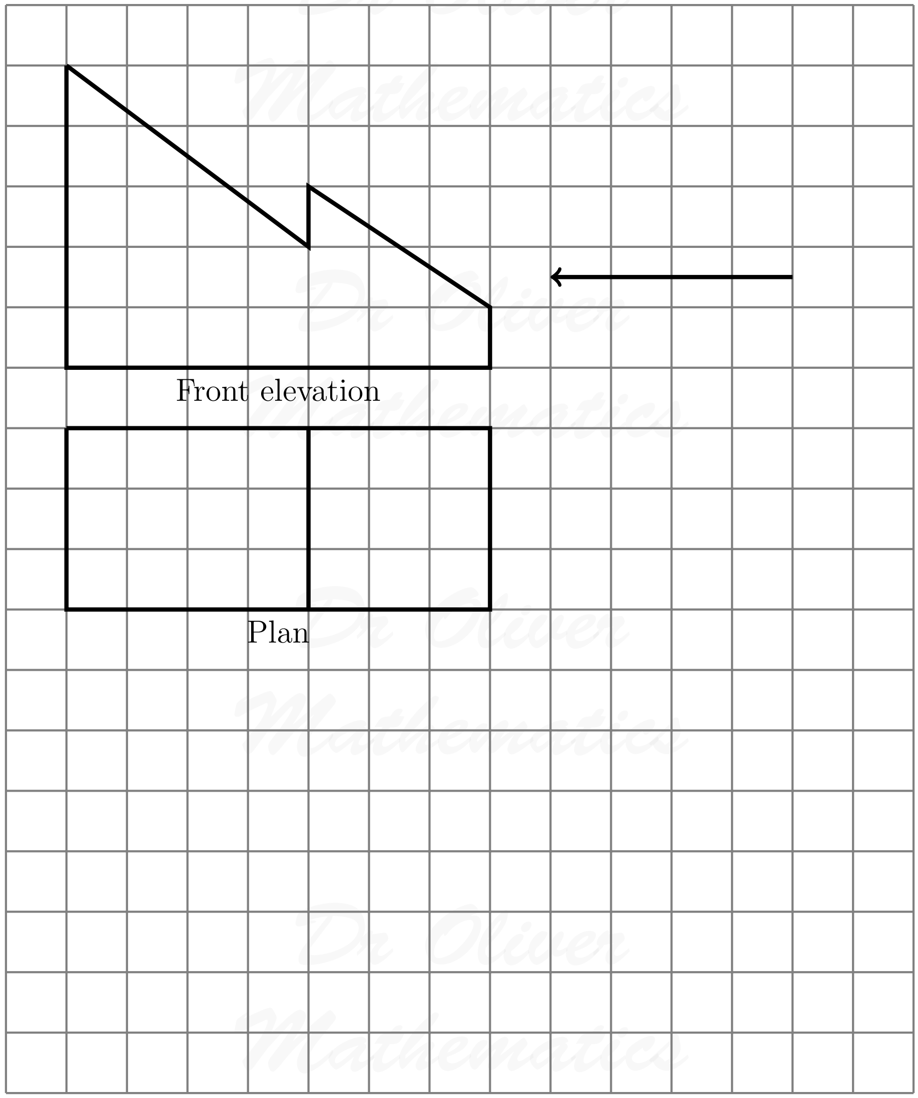
Insert a line of best fit:



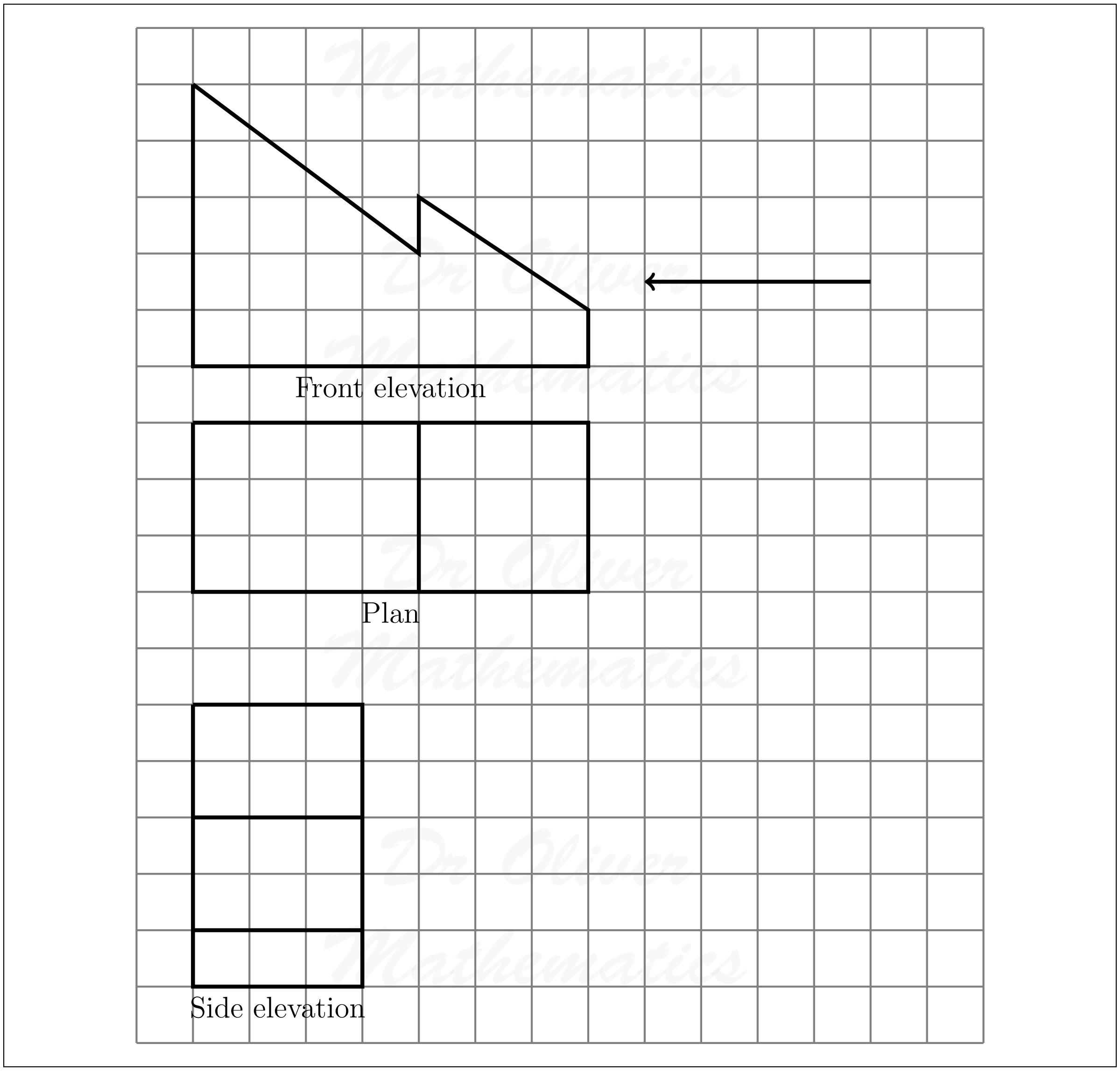
and correctly read off: e.g., 3.6 mm.

2. The front elevation and the plan of a solid are shown on the grid. (2)

On the grid, draw the side elevation of the solid from the direction of the arrow.



Solution



3. Here are the first five terms of an arithmetic sequence:

$$7 \quad 13 \quad 19 \quad 25 \quad 31.$$

(a) Find an expression, in terms of n, for the nth term of this sequence. (2)

Solution

Let the

$$n\text{th term} = an + b.$$

Now,

$$\begin{array}{ccccccc} 7 & & 13 & & 19 & & 25 \\ & 6 & & 6 & & 6 & \end{array}$$

$$\begin{array}{ccccccc} a+b & & 2a+b & & 3a+b & & 4a+b \\ & a & & a & & a & \end{array}$$

We compare terms:

$$a = 6$$

and

$$\begin{aligned} a + b = 7 &\Rightarrow a + b = 7 \\ &\Rightarrow b = 1. \end{aligned}$$

Hence,

$$n\text{th term} = \underline{\underline{6n + 1}}.$$

The nth term of a different sequence is

$$8 - 6n.$$

(b) Is -58 a term of this sequence? (2)
You must show how you get your answer.

Solution

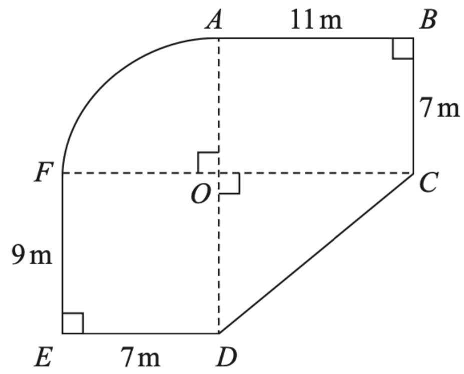
Well,

$$\begin{aligned} 8 - 6n = -58 &\Rightarrow -6n = -66 \\ &\Rightarrow n = 11; \end{aligned}$$

it is the $\underline{\underline{\text{11 term}}}$ is this series.

4. The diagram shows a plan of Jason's garden. (5)

- $ABCO$ and $DEFO$ are rectangles,
- CDO is a right-angled triangle,
- AFO is a sector of a circle with centre O, and
- angle $AOF = 90°$.



Jason is going to cover his garden with grass seed.

- Each bag of grass seed covers 14 m^2 of garden.
- Each bag of grass seed costs £10.95.

Work out how much it will cost Jason to buy all the bags of grass seed he needs.

Solution

He has to cover

$$\begin{aligned} & \text{big rectangle} + \text{small rectangle} + \text{triangle} + \text{quarter of the circle} \\ = \; & (11 \times 7) + (9 \times 7) + (\tfrac{1}{2} \times 9 \times 11) + (\tfrac{1}{4} \times \pi \times 7) \\ = \; & 77 + 63 + 49\tfrac{1}{2} + \tfrac{49}{4}\pi \\ = \; & 189\tfrac{1}{2} + \tfrac{49}{4}\pi. \end{aligned}$$

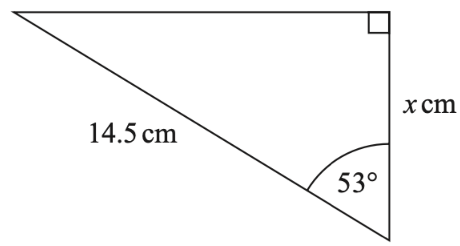
He has to buy

$$\frac{189\frac{1}{2} + \frac{49}{4}\pi}{14} = 16.284\,607\,86 \text{ (FCD)} \to 17$$

bags (we can't have a fraction of a bag!) and that will cost him

$$17 \times 10.95 = \underline{\underline{£186.15}}.$$

5. Work out the value of x. (2)



Give your answer correct to 3 significant figures.

Solution

$$\cos = \frac{\text{adj}}{\text{hyp}} \Rightarrow \cos 53^\circ = \frac{x}{14.5}$$
$$\Rightarrow x = 14.5\cos 53^\circ$$
$$\Rightarrow x = 8.726\,317\,836 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{x = 8.73 \text{ (3 sf)}}}.$$

6. Ella invests £7 000 for 2 years in an account paying compound interest. (3)

- In the first year, the rate of interest is 3%.
- In the second year, the rate of interest is 1.5%.

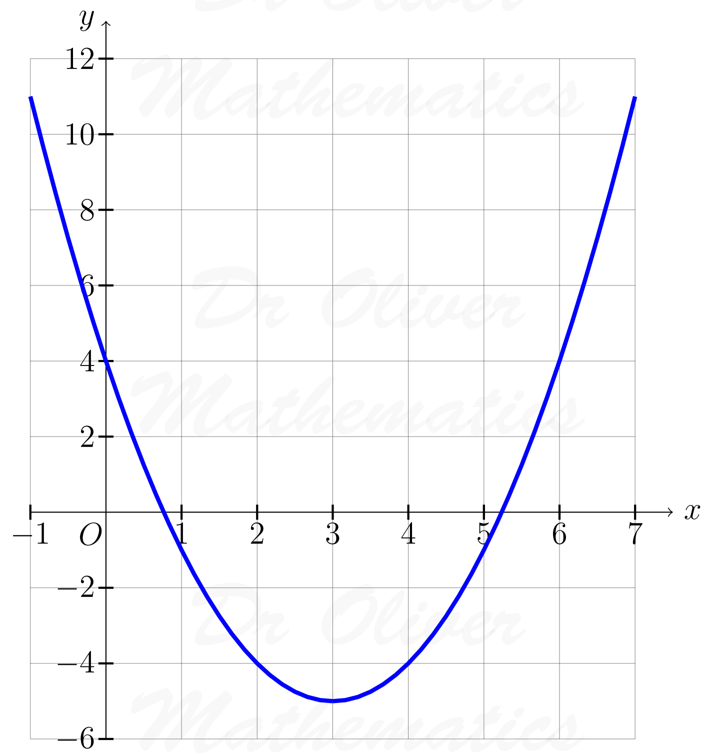
Work out the value of Ella's investment at the end of 2 years.

Solution

$$\text{Value} = 7\,000 \times 1.03 \times 1.015$$
$$= \underline{\underline{£7\,318.15}}.$$

7. Here is the graph of

$$y = x^2 - 6x + 4.$$



(a) Write down the y-intercept of the graph of **(1)**

$$y = x^2 - 6x + 4.$$

Solution
The y-intercept is $\underline{\underline{4}}$.

(b) Write down the coordinates of the turning point of the graph of **(1)**

$$y = x^2 - 6x + 4.$$

Solution
The turning point is $\underline{\underline{(3, -5)}}$.

(c) Use the graph to find estimates for the roots of **(2)**

$$x^2 - 6x + 4 = 0.$$

> **Solution**
> Correct read-off: approximately $\underline{\underline{x = 0.8}}$ and $\underline{\underline{x = 5.2}}$.

8. Chanda buys a necklace for £120. (3)
   She sells the necklace for £135.

   Work out her percentage profit.

> **Solution**
>
> $$\begin{aligned}\text{Percentage profit} &= \left(\frac{135-120}{120}\right) \times 100\% \\ &= \underline{\underline{12\tfrac{1}{2}\%}}.\end{aligned}$$

9. Here are the equations of two straight lines: (2)

   $$y = \tfrac{1}{2}x - 6 \text{ and } 6y = 3x + 7.$$

   Oscar says that these lines are parallel.

   Is Oscar correct?
   You must give a reason for your answer.

> **Solution**
> Well,
> $$6y = 3x + 7 \Rightarrow y = \tfrac{1}{2}x + \tfrac{7}{6};$$
> so, $\underline{\underline{\text{yes}}}$, they are $\underline{\underline{\text{parallel}}}$ because they both have the same $\underline{\underline{\text{gradient}}}$.

10. Aaliyah bought a car. (3)

    - In the first year after she bought the car, its value depreciated at a rate of 23% per annum.
    - In the second year after she bought the car, its value depreciated at a rate of 19% per annum.

At the end of the second year the car was worth £10 914.75.

What was the value of the car when Aaliyah bought it?

Solution

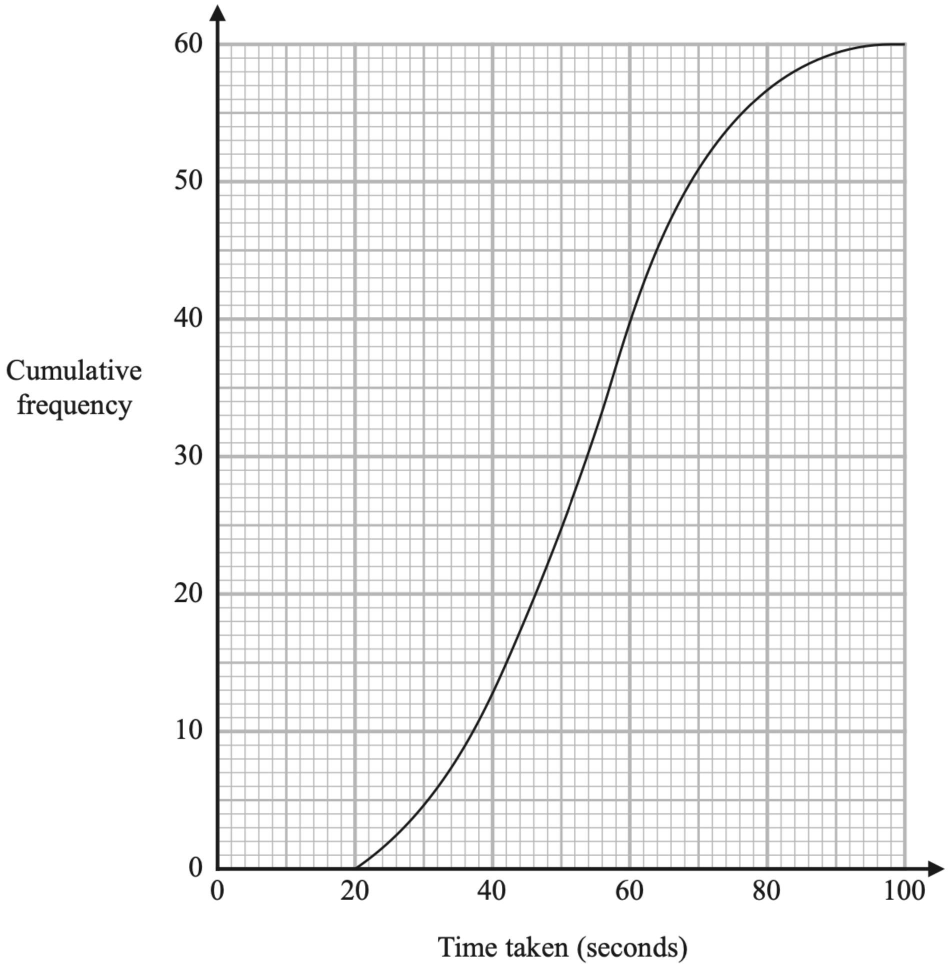
Let x be the price when it is new. Now,

$$\begin{aligned}(1-0.23)\times(1-0.19)\times x = 10\,914.75 &\Rightarrow 0.77\times 0.81\times x = 10\,914.75\\ &\Rightarrow x = \frac{10\,914.75}{0.77\times 0.81}\\ &\Rightarrow x = 17\,500;\end{aligned}$$

hence, it costs $\underline{\underline{\pounds 17\,500}}$ when new.

11. In an experiment, 60 students each completed a puzzle. (3)

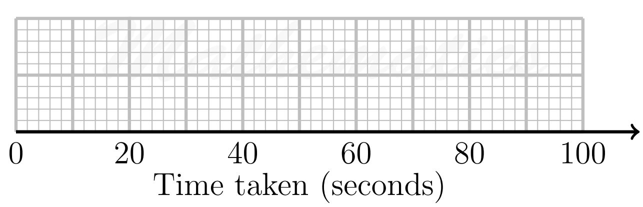
The cumulative frequency graph shows information about the times taken for the 60 students to complete the puzzle.



For these 60 students,

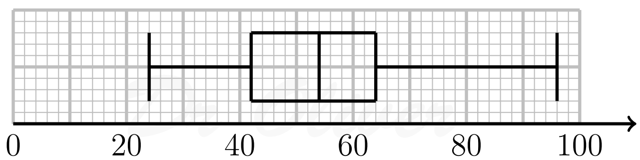
- the least time taken was 24 seconds and
- the greatest time taken was 96 seconds.

On the grid below, draw a box plot for the distribution of the times taken by the students.



Solution

Well, 15 (LQ), 30 (median) , and 45 (UQ) on the vertical axis go to 42, 54, and 64 respectively on the horizontal axis.



12. The number of insects in a population at the start of the Year n is P_n.

The number of insects in the population at the start of Year $(n+1)$ is P_{n+1} where

$$P_{n+1} = kP_n.$$

Given that k has a constant value of 1.13,

(a) find out how many years it takes for the number of insects in the population to double. (2)
You must show how you get your answer.

Solution

Well, $k^n = 2$ for some n:

Exponent	Power
k	1.13
k^2	1.276 9
k^3	1.442 ...
k^4	1.630 ...
k^5	1.842 ...
k^6	2.081 ...

So, it takes 6 years to double.

The value of k actually increases year on year from its value of 1.13 in Year 1.

(b) How does this affect your answer to part (a)? (1)

Solution

It will decrease.

13. A and B are points on a centimetre grid. (2)
A is the point with coordinates $(-7, 6)$.
B is the point with coordinates $(8, 5)$.

Work out the length of AB.
Give your answer correct to 1 decimal place.

Solution

$$\begin{aligned} AB &= \sqrt{[8-(-7)]^2 + (-5-6)^2} \\ &= \sqrt{15^2 + (-11)^2} \\ &= \sqrt{225+121} \\ &= \sqrt{346} \\ &= 18.601\,075\,24 \text{ (FCD)} \\ &= \underline{\underline{18.6 \text{ cm (1 dp)}}}. \end{aligned}$$

14. Using algebra, prove that $1.06\dot{2}$ can be written as $1\frac{14}{225}$. (3)

Solution

Let $x = 1.06\dot{2}$. Now,

$$\begin{aligned} 100x &= 106.\dot{2} \quad (1) \\ 1\,00x &= 1\,062.\dot{2} \quad (2). \end{aligned}$$

Do $(2) - (1)$:

$$\begin{aligned} 900x = 956 \Rightarrow 225x &= 239 \\ \Rightarrow x &= \tfrac{239}{225} \\ \Rightarrow \underline{\underline{x = 1\tfrac{14}{225}}}&, \end{aligned}$$

as required.

15. Faiza is studying the population of rabbits in a park.
She wants to estimate the number of rabbits in the park.

On Monday she catches a random sample of 20 rabbits in the park, marks each rabbit with a tag and releases them back into the park.

On Tuesday she catches a random sample of 42 rabbits in the park.
12 of the rabbits are marked with a tag.

(a) Find an estimate for the number of rabbits in the park. (3)

Solution

Let x be the number of rabbits in the park. Now,

$$\frac{x}{20} = \frac{42}{12} \Rightarrow x = \frac{42 \times 20}{12}$$
$$\Rightarrow \underline{\underline{x = 70}}.$$

Albie is studying the population of rabbits in a wood.

One day, he catches 55 rabbits and finds that 40 of these rabbits are marked with a tag.

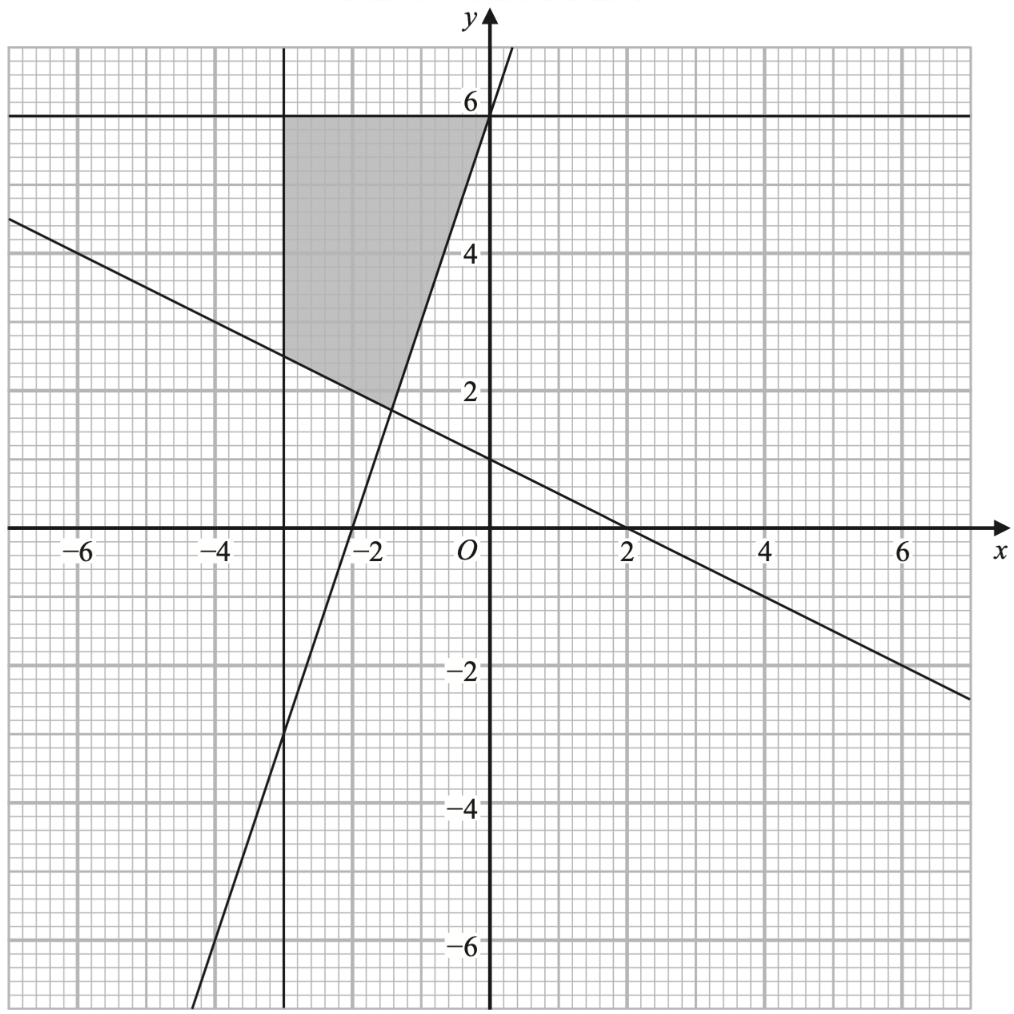
Albie estimates there are 50 rabbits in the wood.

(b) Explain why Albie's estimate cannot be correct. (1)

Solution

E.g., the sample size $\underline{\underline{\text{cannot}}}$ be larger than the actual population!

16. The shaded region shown on the grid is bounded by four straight lines. (4)



Find the four inequalities that define the shaded region.

Solution

The line that goes through $(-4,-6)$ and $(0,6)$:

$$\begin{aligned}\text{gradient} &= \frac{6-(-6)}{0-(-4)} \\ &= \frac{12}{4} \\ &= 3\end{aligned}$$

and the equation is $y = 3x + 6$.

The line that goes through $(2,0)$ and $(0,1)$:

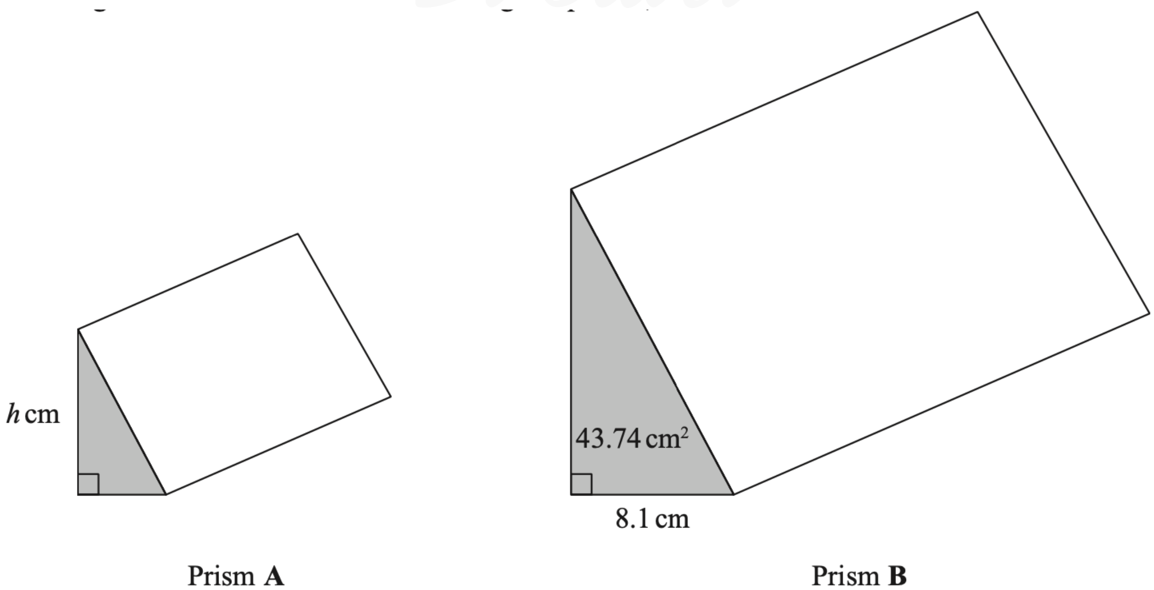
$$\begin{aligned}\text{gradient} &= \frac{1-0}{0-2}\\ &= -\tfrac{1}{2}\end{aligned}$$

and the equation is $y = -\frac{1}{2}x + 1$.

Hence, the four inequalities are

$$\underline{\underline{x \geqslant -3,\ y \leqslant 6,\ y \geqslant 3x+6,\ \text{and } y \geqslant -\tfrac{1}{2}x+1}}.$$

17. The diagram shows two similar solid triangular prisms, **A** and **B**. (4)



The volume of prism **A** is 58.806 cm^3.
The volume of prism **B** is 1 587.762 cm^3.

The cross section of each prism is a right-angled triangle.

For prism **B**,

- the length of the base of the triangle is 8.1 cm and
- the area of the triangle is 43.74 cm^2.

The height of the triangle for prism **A** is h cm.

Work out the value of h.

Solution

The Volume Scale Factor (VSF) is

$$\frac{1\,587.762}{58.806} = 27 = 3^3$$

and that makes Length Scale Factor (LSF) 3 and Area Scale Factor (ASF) $3^2 = 9$.

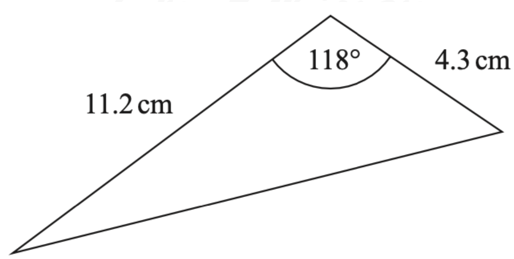
Let the vertical height of **B** be x cm. Then

$$\begin{aligned}\tfrac{1}{2} \times x \times 8.1 = 43.74 &\Rightarrow x = \frac{43.74}{4.05} \\ &\Rightarrow x = 10.8\end{aligned}$$

and, hence,

$$h = \frac{10.8}{3} = \underline{\underline{3.6}}.$$

18. Here is a triangle (2)



Work out the area of the triangle.
Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned}\text{Area} &= \tfrac{1}{2} \times 11.2 \times 4.3 \times \sin 118^\circ \\ &= 21.261\,378\,04 \text{ (FCD)} \\ &= \underline{\underline{21.3 \text{ cm}^2 \text{ (3 sf)}}}.\end{aligned}$$

19. Solve (3)

$$6x^2 + 5x - 6 = 0.$$

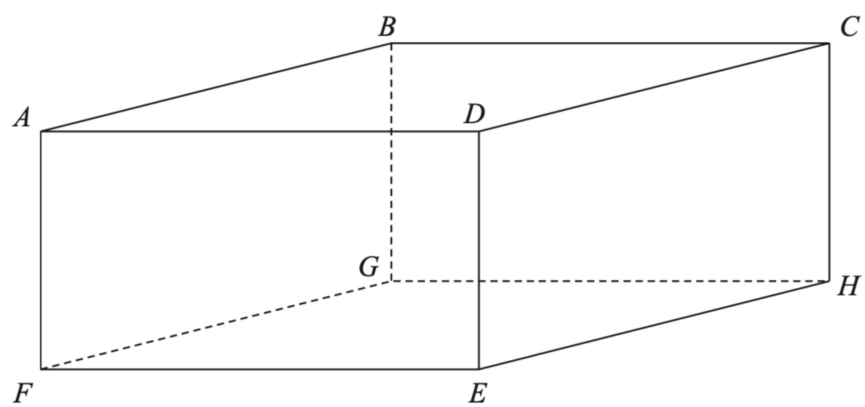
Solution

$$\left.\begin{array}{ll} \text{add to:} & +5 \\ \text{multiply to:} & (+6) \times (-6) = -36 \end{array}\right\} +9,\ -4$$

E.g.,

$$\begin{aligned} 6x^2 + 5x - 6 = 0 &\Rightarrow 6x^2 + 9x - 4x - 6 = 0 \\ &\Rightarrow 3x(2x+3) - 2(2x+3) = 0 \\ &\Rightarrow (3x-2)(2x+3) = 0 \\ &\Rightarrow 3x - 2 = 0 \text{ or } 2x + 3 = 0 \\ &\Rightarrow 3x = 2 \text{ or } 2x = -3 \\ &\Rightarrow \underline{\underline{x = \tfrac{2}{3} \text{ or } x = -1\tfrac{1}{2}}}. \end{aligned}$$

20. $ABCDEFGH$ is a cuboid. (4)



- $AD = 9$ cm.
- $FD = 13$ cm.
- Angle $GHF = 49°$.

Work out the size of angle FAH.
Give your answer correct to the nearest degree.

Solution

Well,

$$\begin{aligned} FD^2 = FA^2 + AD^2 &\Rightarrow 13^2 = FA^2 + 9^2 \\ &\Rightarrow 169 = FA^2 + 81 \\ &\Rightarrow FA^2 = 88 \\ &\Rightarrow FA = 2\sqrt{22} \end{aligned}$$

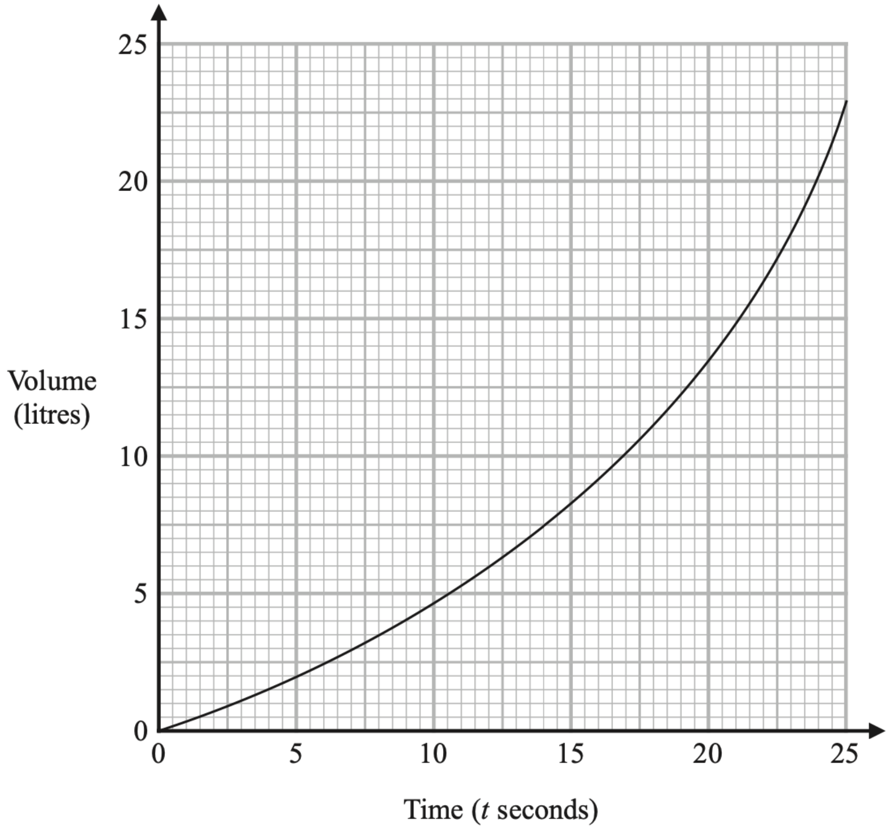
and

$$\begin{aligned} \cos = \frac{\text{adj}}{\text{hyp}} &\Rightarrow \cos 49^\circ = \frac{9}{FH} \\ &\Rightarrow FH = \frac{9}{\cos 49^\circ}. \end{aligned}$$

Finally,

$$\begin{aligned} \tan = \frac{\text{opp}}{\text{adj}} &\Rightarrow \tan FAH = \frac{\frac{9}{\cos 49^\circ}}{2\sqrt{22}} \\ &\Rightarrow \angle FAH = 55.634\,902\,12 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{\angle FAH = 56^\circ \text{ (nearest degree)}}}. \end{aligned}$$

21. The graph below gives the volume, in litres, of water in a container t seconds after the water starts to fill the container.



(a) Calculate an estimate for the gradient of the graph when $t = 17.5$. (3)
You must show how you get your answer.

Solution
Well, the gradient of the tangent to the curve passes through $(17.5, 10.5)$ and $(10, 2.5)$ and it is

$$\begin{aligned} \text{gradient} &= \frac{10.5 - 2.5}{17.5 - 10} \\ &= \frac{8}{7.5} \\ &= \underline{\underline{1\tfrac{1}{15}}}. \end{aligned}$$

(b) Describe fully what the gradient in part (a) represents. (1)

Solution
E.g., the rate at which the volume is <u>increasing</u> at $t = 17.5$.

22. $\mathrm{f}(x) = \sqrt[3]{x}$. (3)
$\mathrm{g}(x) = 2x + 3$.
$\mathrm{h}(x) = \mathrm{f}\,\mathrm{g}(x)$.

Find $\mathrm{h}^{-1}(x)$.

Solution

Well,

$$\begin{aligned}\mathrm{h}(x) &= \mathrm{f}\,\mathrm{g}(x)\\ &= \mathrm{f}(\mathrm{g}(x))\\ &= \mathrm{f}(2x+3)\\ &= \sqrt[3]{2x+3}.\end{aligned}$$

Now,

$$\begin{aligned}y = \sqrt[3]{2x+3} &\Rightarrow y^3 = 2x+3\\ &\Rightarrow y^3 - 3 = 2x\\ &\Rightarrow \frac{y^3-3}{2} = x\end{aligned}$$

and, hence,

$$\mathrm{h}^{-1}(x) = \underline{\underline{\frac{x^3-3}{2}}}.$$

23. A race is measured to have a distance of 10.6 km, correct to the nearest 0.1 km. (5)
Sam runs the race in a time of 31 minutes 48 seconds, correct to the nearest second.

Sam's average speed in this race is V km/hour.

By considering bounds, calculate the value of V to a suitable degree of accuracy.
You must show all your working and give a reason for your answer.

Solution

Well,

$$10.55 \leqslant \text{distance} < 10.65$$

and, converting

$$31 \text{ minutes } 48 \text{ seconds} = 1\,908 \text{ seconds},$$

we have

$$1\,907.5 \leqslant \text{time} < 1\,908.5.$$

Now, the upper bound is

$$\frac{10.65}{1\,907.5} \times 60 \times 60 = 20.099\,606\,82 \text{ (FCD)}$$

and the lower bound is

$$\frac{10.55}{1\,908.5} \times 60 \times 60 = 19.900\,445\,38 \text{ (FCD)}.$$

"To a suitable degree of accuracy"? Well, we need to start rounding ...

Number	Lower Bound	Upper Bound	Agree
1 sf	20	20	✓
2 sf	20	20	✓
3 sf	19.9	20.1	✗

Hence,

$$\underline{\underline{V = 20 \text{ km/hour (2 sf)}}}.$$

24. A circle has equation (4)

$$x^2 + y^2 = 12.25.$$

The point P lies on the circle.
The coordinates of P are $(2.1, 2.8)$.

The line **L** is the tangent to the circle at point P.

Find an equation of **L**.
Give your answer in the form

$$ax + by = c,$$

where a, b, and c are integers.

Solution

$$\begin{aligned} OP &= \frac{2.8}{2.1} \\ &= \tfrac{4}{3} \end{aligned}$$

and

$$m_{\text{normal}} = -\frac{1}{\frac{4}{3}} = -\tfrac{3}{4}.$$

Finally, an equation of **L** is

$$\begin{aligned} y - 2.8 = -\tfrac{3}{4}(x - 2.1) &\Rightarrow 4(y - 2.8) = -3(x - 2.1) \\ &\Rightarrow 4y - 11.2 = -3x + 6.3 \\ &\Rightarrow 3x + 4y = 17.5 \\ &\Rightarrow \underline{\underline{6x + 8y = 35}} \end{aligned}$$

with

$$\underline{\underline{a = 6,\ b = 8,\ \text{and } c = 35}}.$$