

Dr Oliver Mathematics
GCSE Mathematics
2012 November Paper 1H: Non-Calculator
1 hour 45 minutes

The total number of marks available is 100.
You must write down all the stages in your working.

1. Here are the ingredients needed to make 16 gingerbread men.

(3)

Ingredients
to make 16 gingerbread men

180 g flour
40 g ginger
110 g butter
30 g sugar

Hamish wants to make 24 gingerbread men.
Work out how much of each of the ingredients he needs.

Solution

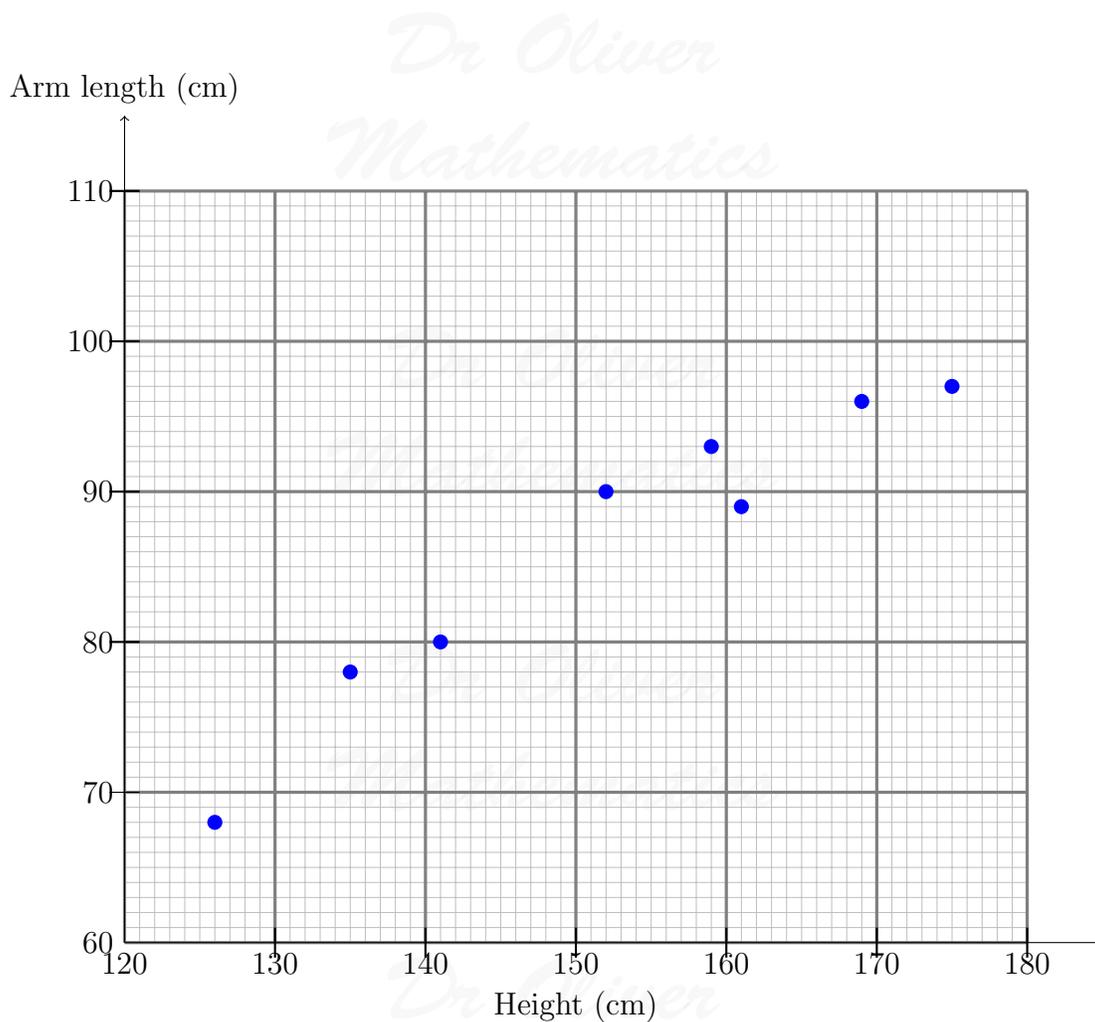
Flour: $\frac{3}{2} \times 180 = \underline{\underline{270 \text{ g}}}$

Ginger: $\frac{3}{2} \times 40 = \underline{\underline{60 \text{ g}}}$

Butterr: $\frac{3}{2} \times 110 = \underline{\underline{165 \text{ g}}}$

Sugar: $\frac{3}{2} \times 30 = \underline{\underline{45 \text{ g}}}$

2. The scatter graph shows information about the height and the arm length of each of 8 students in Year 11.



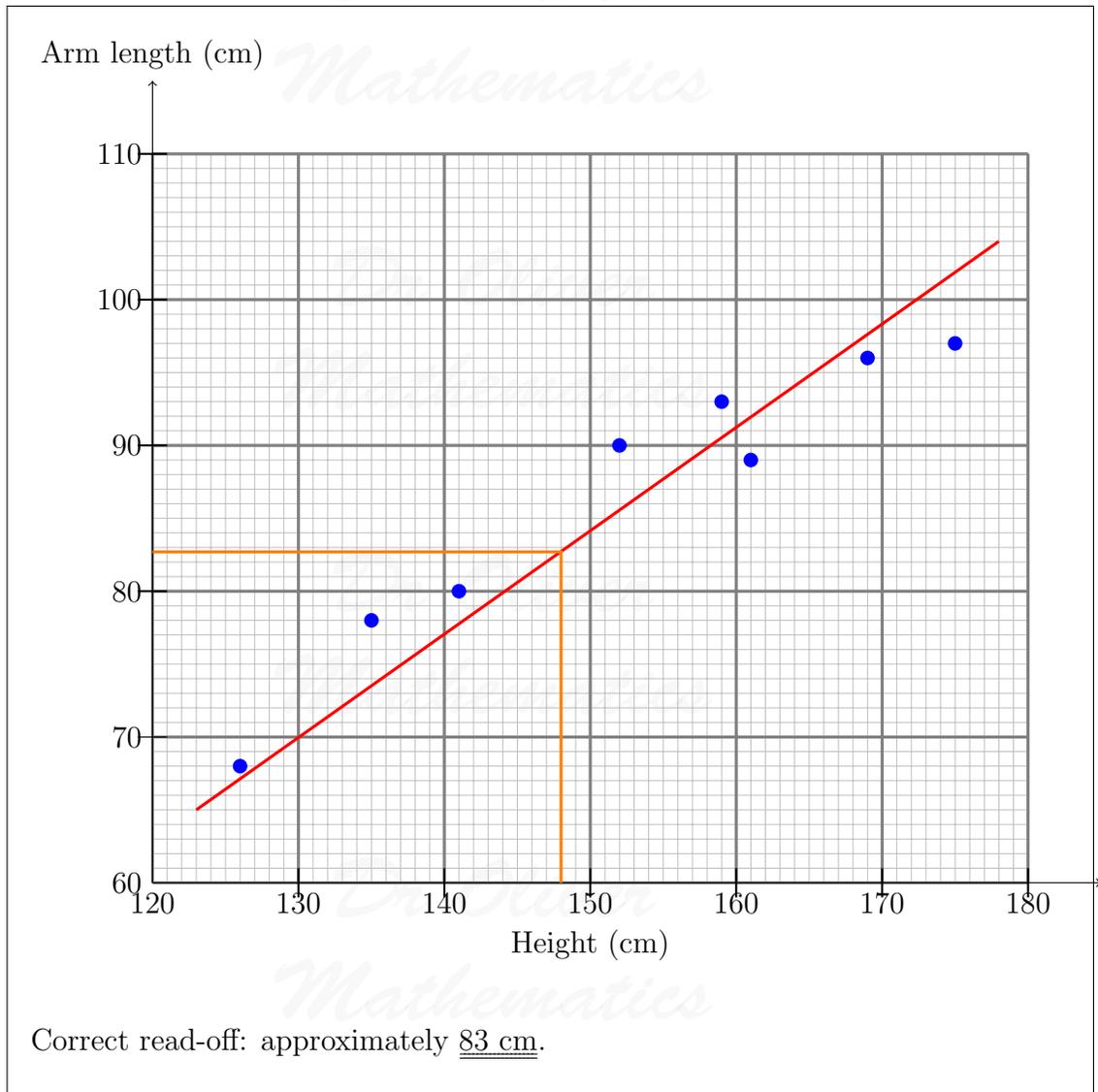
(a) What type of correlation does this scatter graph show? (1)

Solution
Positive correlation.

A different student in Year 11 has a height of 148 cm.

(b) Estimate the arm length of this student. (2)

Solution
 Draw a line of best fit:



3. Here is part of Gary's electricity bill.

(4)

Electricity bill	
New reading	7155 units
Old reading	7095 units
Price per unit 15p	

Work out how much Gary has to pay for the units of electricity he used.

Solution

He has used

$$7\,155 - 7\,095 = 60$$

units of electricity and the total amount he has to pay is

$$15 \times 60 = 900 \text{ p} = \underline{\underline{\pounds 9}}.$$

4. Alison wants to find out how much time people spend reading books. She is going to use a questionnaire. Design a suitable question for Alison to use in her questionnaire. (2)

Solution

A suitable question with a time frame, e.g., "Did you read today/last week/last month? Tick the appropriate box."

At least three exhaustive and non-overlapping tick boxes (best defined using inequality notation): for example, Did not read, $0 < x < 30$ min, $30 < x < 60$ min, $x \geq 60$ min.

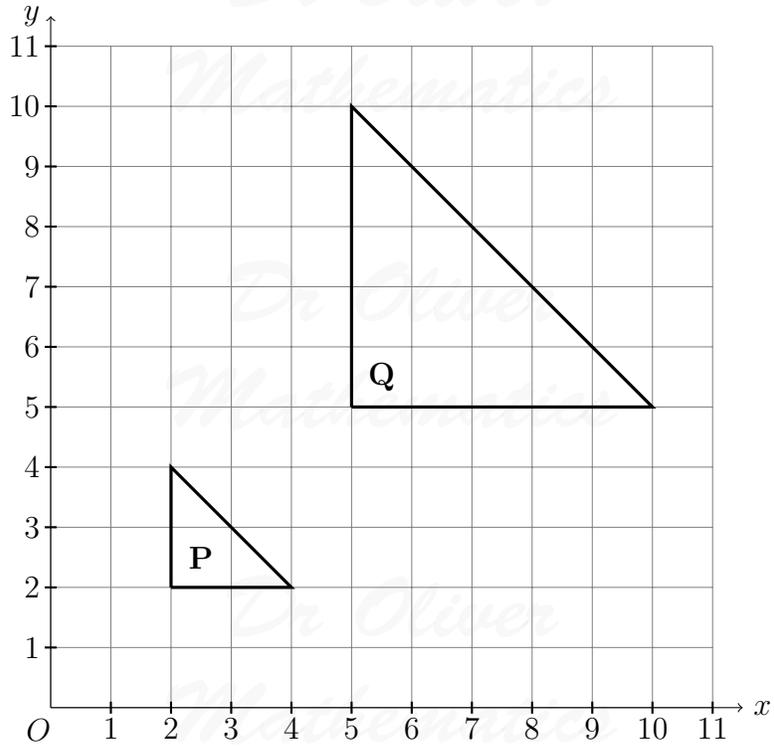
5. Work out an estimate for $\frac{31 \times 9.87}{0.509}$. (3)

Solution

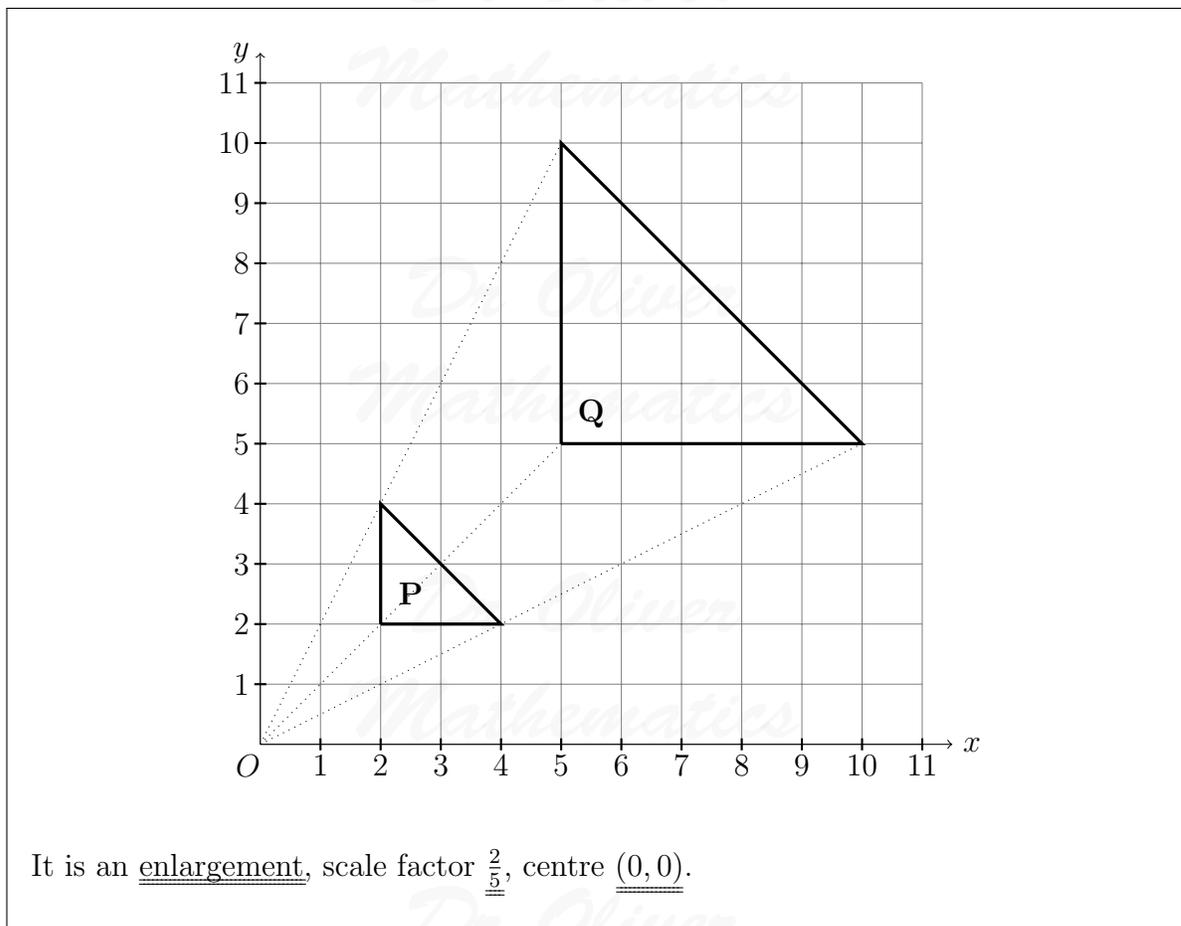
Round to 1 significant figure:

$$\begin{aligned} \frac{31 \times 9.87}{0.509} &\approx \frac{30 \times 10}{0.5} \\ &= \frac{300}{0.5} \\ &= \underline{\underline{600}}. \end{aligned}$$

6. Describe fully the single transformation that maps shape **P** onto shape **Q**. (3)

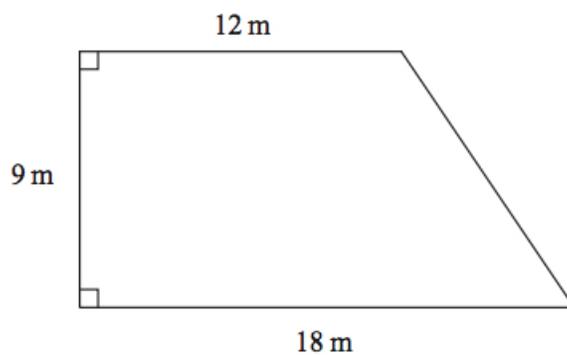


Solution



7. Here is a diagram of Jim's garden.

(4)



Jim wants to cover his garden with grass seed to make a lawn.

Grass seed is sold in bags.

There is enough grass seed in each bag to cover 20 m^2 of garden.

Each bag of grass seed costs £4.99.

Work out the least cost of putting grass seed on Jim's garden.

Solution

$$\begin{aligned}\text{Area} &= \frac{1}{2} \times 9 \times (12 + 18) \\ &= \frac{1}{2} \times 9 \times 30 \\ &= \frac{1}{2} \times 270 \\ &= 135 \text{ m}^2.\end{aligned}$$

How much garden seeds do he need?

$$\frac{135}{20} = 6.75;$$

hence, he needs 7 bags.

Finally, the least cost of putting grass seed on Jim's garden is

$$\begin{aligned}4.99 \times 7 &= (5 - 0.01) \times 7 \\ &= 5 \times 7 - 0.01 \times 7 \\ &= 35 - 0.07 \\ &= \underline{\underline{\pounds 34.93}}.\end{aligned}$$

8. There are only red counters, blue counters, white counters, and black counters in a bag. The table shows the probability that a counter taken at random from the bag will be red or blue.

Colour	Red	Blue	White	Black
Probability	0.2	0.5		

The number of white counters in the bag is the same as the number of black counters in the bag.

Tania takes at random a counter from the bag.

- (a) Work out the probability that Tania takes a white counter.

(2)

Solution

Let the probability that Tania takes a white counter be x . Then

$$\begin{aligned}0.2 + 0.5 + x + x &= 1 \Rightarrow 2x = 0.3 \\ &\Rightarrow \underline{\underline{x = 0.15}}.\end{aligned}$$

There are 240 counters in the bag.

(b) Work out the number of red counters in the bag.

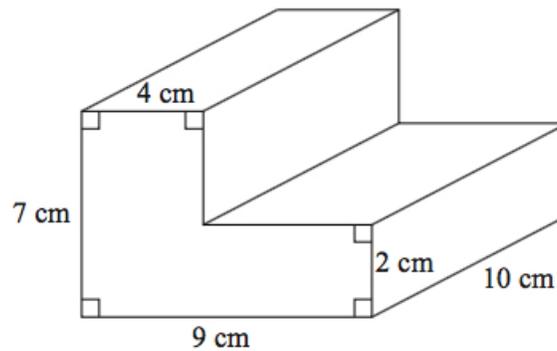
(2)

Solution

$$240 \times 0.2 = \underline{48 \text{ counters.}}$$

9. The diagram shows a prism.

(3)



Work out the volume of the prism.

Solution

$$\begin{aligned} \text{Area of the cross-section} &= (7 \times 9) - (5 \times 5) \\ &= 63 - 25 \\ &= 38 \text{ m}^2 \end{aligned}$$

and

$$\text{volume} = 38 \times 10 = \underline{380 \text{ m}^3}.$$

10. Here is a map.

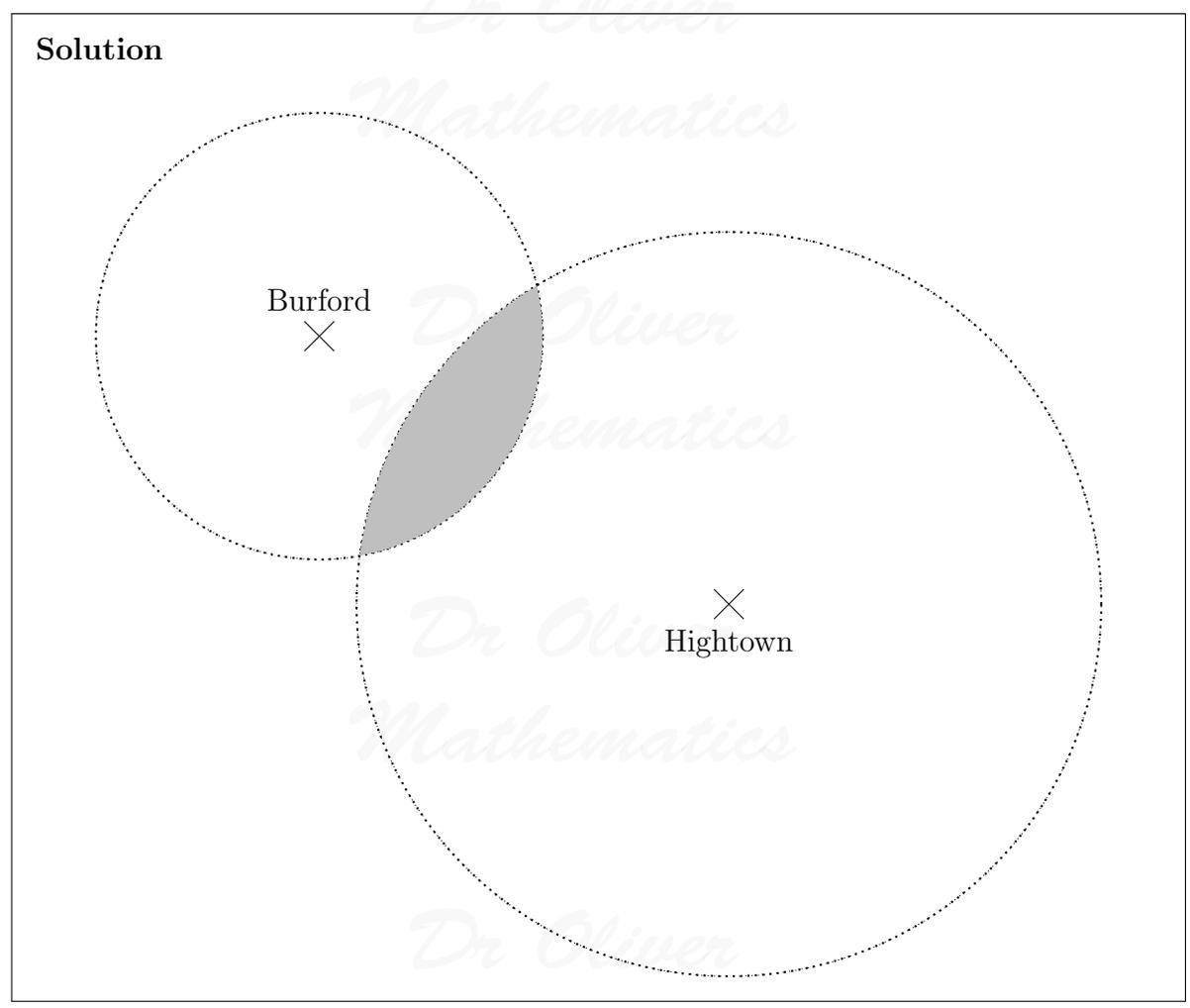
(3)

The map shows two towns, Burford and Hightown.

Dr Oliver
Burford
×
Mathematics

Dr Oliver
×
Hightown
Mathematics

A company is going to build a warehouse.
The warehouse will be less than 30 km from Burford and less than 50 km from Hightown.
Shade the region on the map where the company can build the warehouse.



The region is the intersection shaded in light gray.

11. (a) Expand $4(3x + 5)$. (1)

Solution

$$4(3x + 5) = \underline{\underline{12x + 20.}}$$

- (b) Expand and simplify (2)

$$2(x - 4) + 3(x + 5).$$

Solution

$$\begin{aligned} 2(x - 4) + 3(x + 5) &= 2x - 8 + 3x + 15 \\ &= \underline{\underline{5x + 7.}} \end{aligned}$$

- (c) Expand and simplify (2)

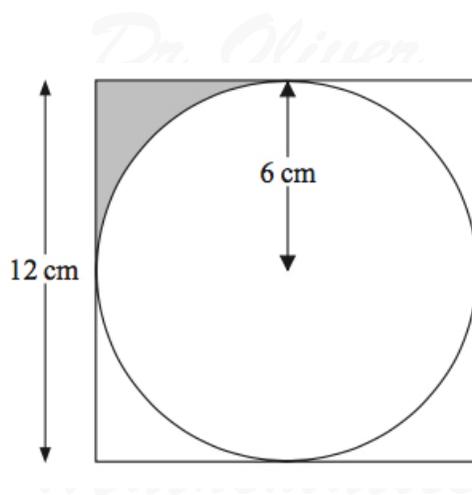
$$(x + 4)(x + 6).$$

Solution

×	x	+4
x	x^2	$+4x$
+6	$+6x$	$+24$

$$(x + 4)(x + 6) = \underline{\underline{x^2 + 10x + 24.}}$$

12. The diagram shows a circle drawn inside a square. (3)



The circle has a radius of 6 cm.
 The square has a side of length 12 cm.
 Work out the shaded area.
 Give your answer in terms of π .

Solution

$$\begin{aligned} \text{Shaded area} &= \frac{1}{4}(12^2 - \pi \times 6^2) \\ &= \frac{1}{4}(144 - 36\pi) \\ &= \underline{\underline{(36 - 9\pi) \text{ cm}^2}}. \end{aligned}$$

13. Talil is going to make some concrete mix. (4)
 He needs to mix cement, sand and gravel in the ratio 1 : 3 : 5 by weight.
 Talil wants to make 180 kg of concrete mix.
 Talil has 15 kg of cement, 85 kg of sand, and 100 kg of gravel.
 Does Talil have enough cement, sand, and gravel to make the concrete mix?

Solution

$$1 + 3 + 5 = 9.$$

Cement:

$$\frac{1}{9} \times 180 = 20 \text{ kg.}$$

Sand:

$$\frac{3}{9} \times 180 = 60 \text{ kg.}$$

Gravel:

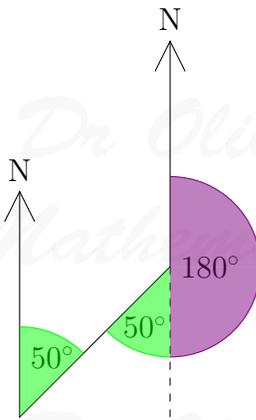
$$\frac{5}{9} \times 180 = 100 \text{ kg.}$$

No, he has does not have enough cement.

14. The bearing of a ship from a lighthouse is 050° .
Work out the bearing of the lighthouse from the ship.

(2)

Solution



So, the answer is

$$180 + 50 = \underline{\underline{230^\circ}}.$$

15. (a) Simplify $m^5 \div m^3$.

(1)

Solution

$$m^5 \div m^3 = \underline{\underline{m^2}}.$$

(b) Simplify $5x^4y^3 \times x^2y$.

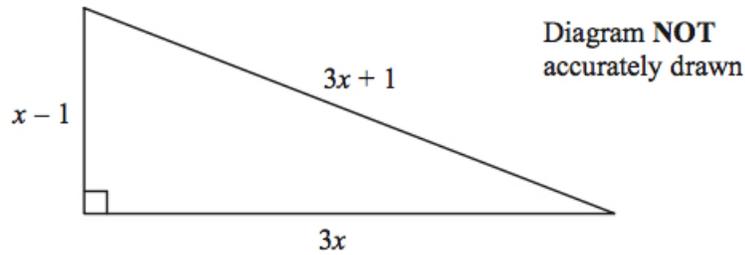
(2)

Solution

$$5x^4y^3 \times x^2y = \underline{\underline{5x^6y^4}}.$$

16. The diagram shows a triangle.

(4)



In the diagram, all the measurements are in metres.

The perimeter of the triangle is 56 m.

The area of the triangle is A m².

Work out the value of A .

Solution

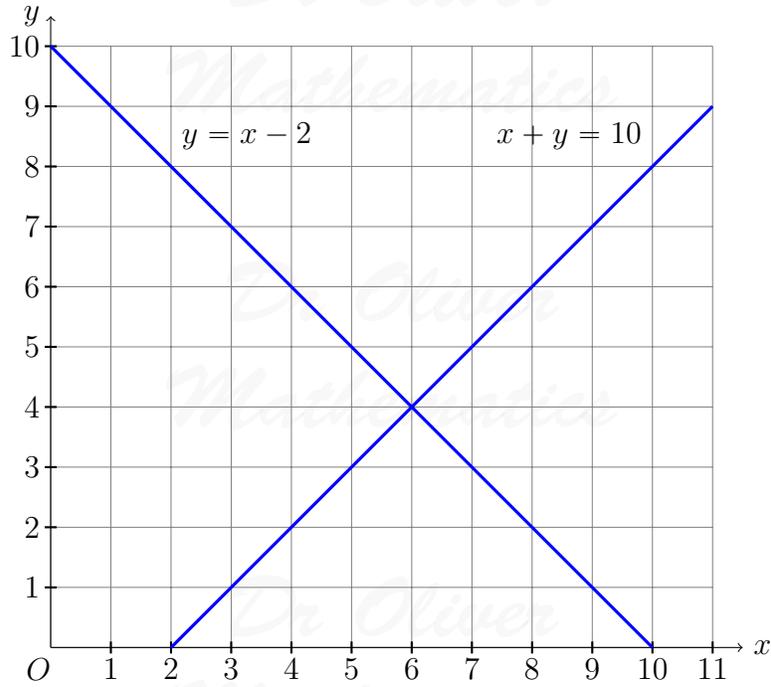
$$(x - 1) + 3x + (3x + 1) = 56 \Rightarrow 7x = 56 \\ \Rightarrow x = 8,$$

and we have three sides as 7, 24, and 25. Finally,

$$\text{area} = \frac{1}{2} \times 7 \times 24 \\ = 7 \times 12 \\ = \underline{\underline{84 \text{ m}^2}}.$$

17. The lines $y = x - 2$ and $x + y = 10$ are drawn on the grid.

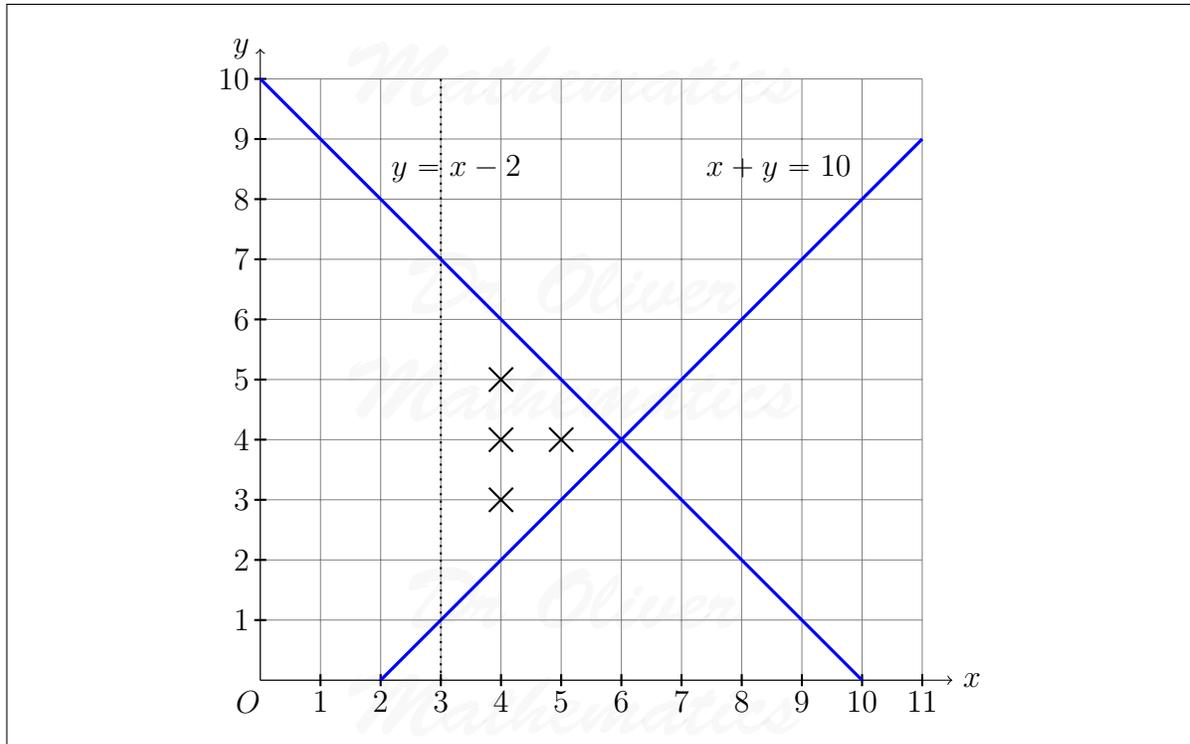
(3)



On the grid, mark with a cross (\times) each of the points with integer coordinates that are in the region defined by

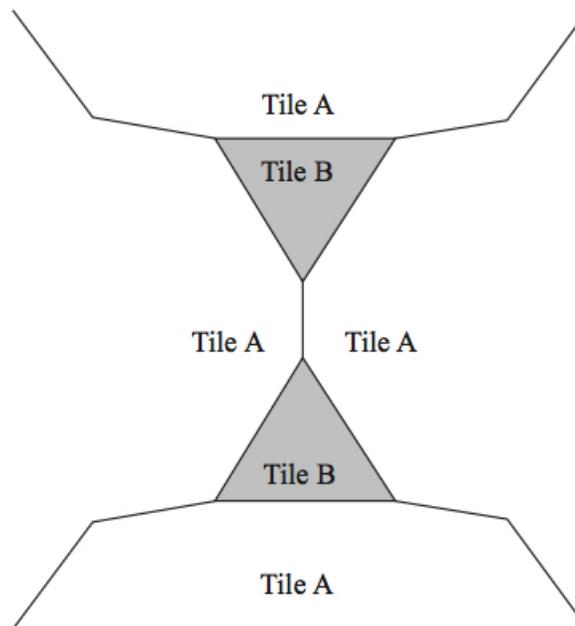
$$\begin{aligned}
 &y > x - 2 \\
 &x + y < 10 \\
 &x > 3.
 \end{aligned}$$

Solution



18. The diagram shows part of a pattern made from tiles.

(4)



The pattern is made from two types of tiles, tile A and tile B. Both tile A and tile B are regular polygons.

Work out the number of sides tile A has.

Solution

Tile B is a triangle. Now,

$$\frac{60}{2} = 30^\circ$$

and

$$\frac{360}{30} = \underline{\underline{12 \text{ sides}}}.$$

19. Sameena recorded the times, in minutes, some girls took to do a jigsaw puzzle. Sameena used her results to work out the information in this table.

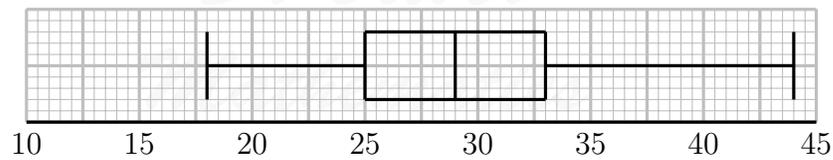
	Minutes
Shortest time	18
Lower quartile	25
Median	29
Upper quartile	33
Longest time	44

- (a) On the grid, draw a box plot to show the information in the table.

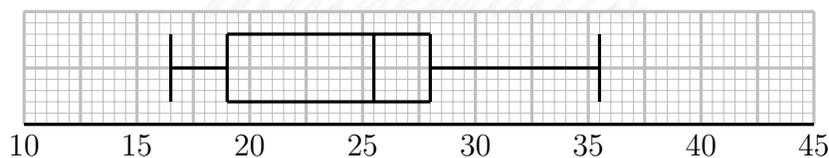
(2)



Solution



The box plot below shows information about the times, in minutes, some boys took to do the same jigsaw puzzle.



- (b) Compare the distributions of the girls' times and the boys' times. (2)

Solution

Average

Since the median for the boys' (25.5) is lower than the median for the girls' (29), the boys' scored more marks on average.

Spread

Since the range for the boys' ($35.5 - 16.5 = 19$) is smaller than the range for the girls' ($44 - 18 = 26$), the marks were more consistent in the boys'.

OR

Since the IQR for the girls' ($33 - 25 = 8$) is smaller than the IQR for the boys' ($28 - 19 = 9$), the marks were more consistent in the girls'.

20. Write the following numbers in order of size. (2)
Start with the smallest number.

$$0.038 \times 10^2 \quad 3800 \times 10^{-4} \quad 380 \quad 0.38 \times 10^{-1}$$

Solution

$$0.038 \times 10^2 = 3.8 \times 10^1$$

$$3800 \times 10^{-4} = 3.8 \times 10^{-1}$$

$$380 = 3.8 \times 10^2$$

$$0.38 \times 10^{-1} = 3.8 \times 10^{-2}$$

In increasing size,

$$\underline{\underline{0.38 \times 10^{-1}, 3800 \times 10^{-4}, 0.038 \times 10^2, \text{ and } 380.}}$$

21. The table shows information about the speeds of 100 lorries.

Speed (s) in km/h	Frequency
$0 < s \leq 20$	2
$20 < s \leq 40$	9
$40 < s \leq 60$	23
$60 < s \leq 80$	31
$80 < s \leq 100$	27
$100 < s \leq 120$	8

(a) Complete the cumulative frequency table for this information. (1)

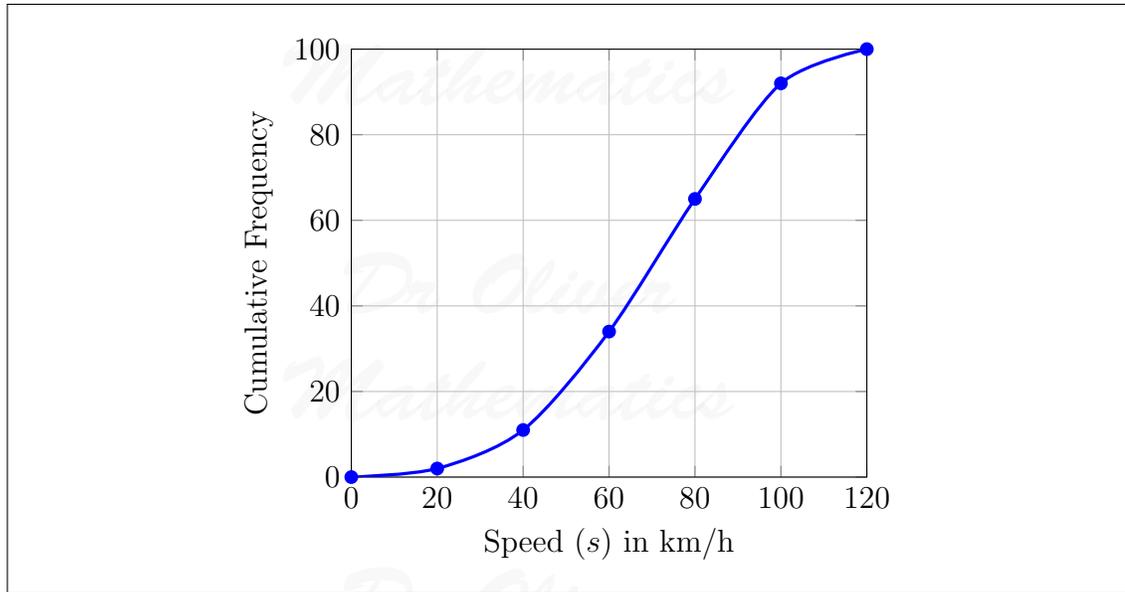
Speed (s) in km/h	Cumulative Frequency
$0 < s \leq 20$	2
$0 < s \leq 40$	
$0 < s \leq 60$	
$0 < s \leq 80$	
$0 < s \leq 100$	
$0 < s \leq 120$	

Solution

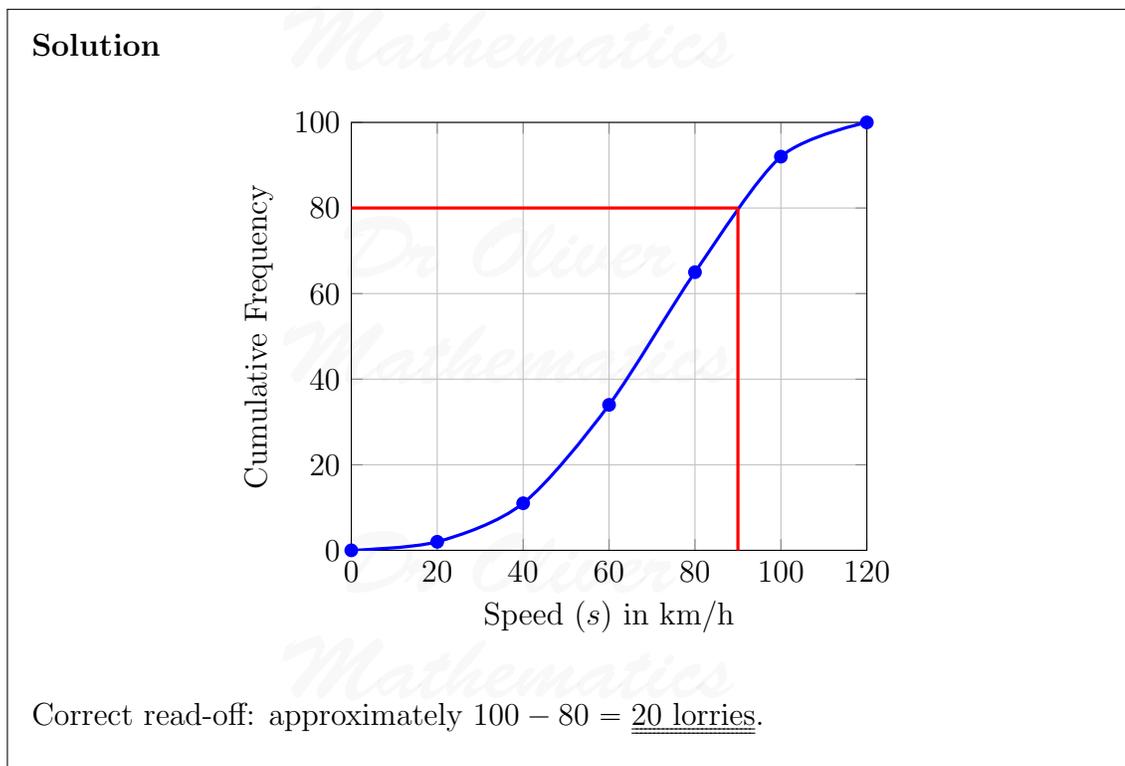
Speed (s) in km/h	Cumulative Frequency
$0 < s \leq 20$	2
$0 < s \leq 40$	$2 + 9 = \underline{\underline{11}}$
$0 < s \leq 60$	$11 + 23 = \underline{\underline{34}}$
$0 < s \leq 80$	$34 + 31 = \underline{\underline{65}}$
$0 < s \leq 100$	$65 + 27 = \underline{\underline{92}}$
$0 < s \leq 120$	$92 + 8 = \underline{\underline{100}}$

(b) On the grid, draw a cumulative frequency graph for your table. (2)

Solution



- (c) Find an estimate for the number of lorries with a speed of more than 90 km/h. (2)



22. Solve the simultaneous equations (4)

$$3x + 2y = 4$$

$$4x + 5y = 17.$$

Solution

$$3x + 2y = 4 \quad (1)$$

$$4x + 5y = 17 \quad (2)$$

$$4 \times (1) : \quad 12x + 8y = 16 \quad (3)$$

$$3 \times (2) : \quad 12x + 15y = 51 \quad (4)$$

$$(4) - (3):$$

$$7y = 35 \Rightarrow \underline{\underline{y = 5}}$$

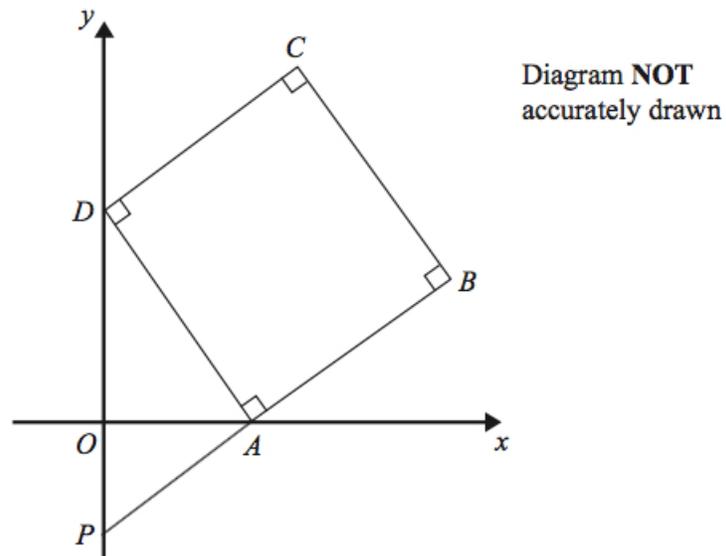
$$\Rightarrow 3x + 10 = 4$$

$$\Rightarrow 3x = -6$$

$$\Rightarrow \underline{\underline{x = -2.}}$$

23. $ABCD$ is a square.

(4)



P and D are points on the y -axis.

A is a point on the x -axis.

PAB is a straight line.

The equation of the line that passes through the points A and D is $y = -2x + 6$.

Find the length of PD .

Solution

$D(0, 6)$ and $A(3, 0)$.

The gradient of AD is -2 which means that the perpendicular gradient is $\frac{1}{2}$.

Now, the equation of AP is

$$y - 0 = \frac{1}{2}(x - 3) \Rightarrow y = \frac{1}{2}x - \frac{3}{2}.$$

Finally, the length of PD is

$$6 + \frac{3}{2} = \underline{\underline{7\frac{1}{2}}}.$$

24. Make t the subject of the formula

(4)

$$p = \frac{3 - 2t}{4 + t}.$$

Solution

$$p = \frac{3 - 2t}{4 + t} \Rightarrow p(4 + t) = 3 - 2t$$

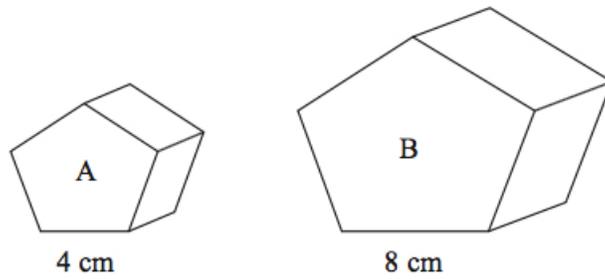
$$\Rightarrow 4p + pt = 3 - 2t$$

$$\Rightarrow 2t + pt = 3 - 4p$$

$$\Rightarrow t(2 + p) = 3 - 4p$$

$$\Rightarrow t = \underline{\underline{\frac{3 - 4p}{2 + p}}}.$$

25. The diagram shows two similar solids, A and B.



Solid A has a volume of 80 cm^3 .

- (a) Work out the volume of solid B.

(2)

Solution

The length scale ratio (LSR) is

$$\text{LSR}_{A \rightarrow B} = \frac{8}{4} = 2$$

and the volume scale ratio (VSR) is

$$\text{VSR}_{A \rightarrow B} = 2^3 = 8.$$

Finally, the volume of solid B is

$$80 \times 8 = \underline{\underline{640 \text{ cm}^3}}.$$

Solid B has a total surface area of 160 cm^2 .

- (b) Work out the total surface area of solid A.

(2)

Solution

$$\text{ASR}_{A \rightarrow B} = 2^2 = 4$$

and this means that the total surface area of solid A is

$$\frac{160}{4} = \underline{\underline{40 \text{ cm}^2}}.$$

26. (a) Rationalise the denominator of

$$\frac{5}{\sqrt{2}}.$$

(2)

Solution

$$\begin{aligned}\frac{5}{\sqrt{2}} &= \frac{5}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ &\Rightarrow \underline{\underline{\frac{5\sqrt{2}}{2}}}\end{aligned}$$

(b) Expand and simplify

$$(2 + \sqrt{3})^2 - (2 - \sqrt{3})^2.$$

(2)

Solution

$$\begin{array}{r|rr} \times & 2 & +\sqrt{3} \\ \hline 2 & 4 & +2\sqrt{3} \\ +\sqrt{3} & +2\sqrt{3} & +3 \\ \hline \end{array}$$

$$\begin{array}{r|rr} \times & 2 & -\sqrt{3} \\ \hline 2 & 4 & -2\sqrt{3} \\ -\sqrt{3} & -2\sqrt{3} & +3 \\ \hline \end{array}$$

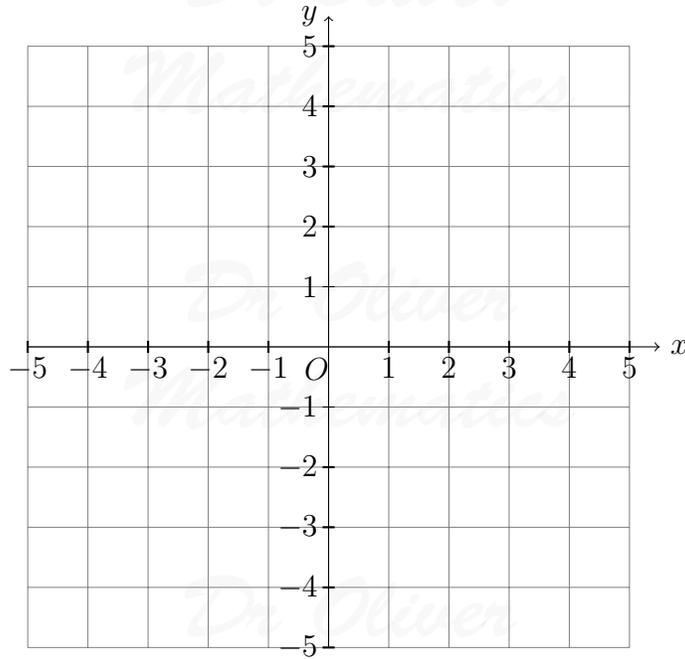
Finally,

$$\begin{aligned}(2 + \sqrt{3})^2 - (2 - \sqrt{3})^2 &= (7 + 4\sqrt{3}) - (7 - 4\sqrt{3}) \\ &= \underline{\underline{8\sqrt{3}}}.\end{aligned}$$

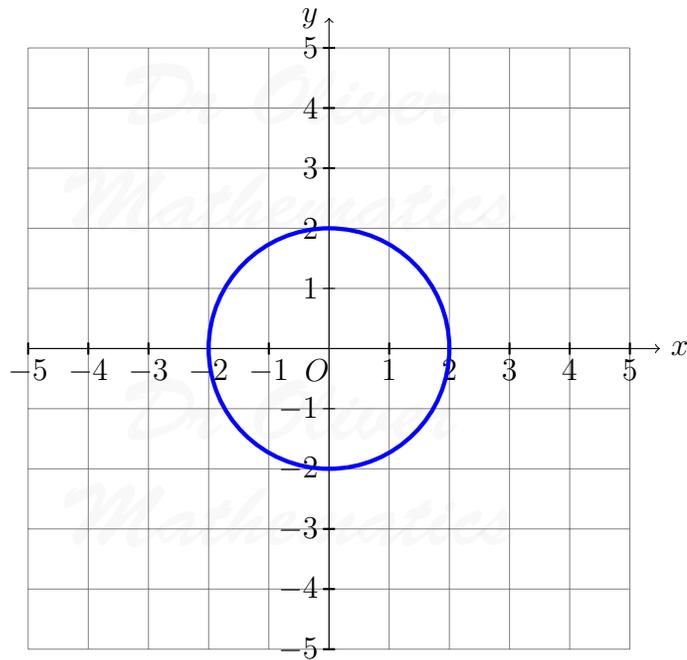
27. (a) On the grid, draw the graph of

$$x^2 + y^2 = 4.$$

(2)



Solution

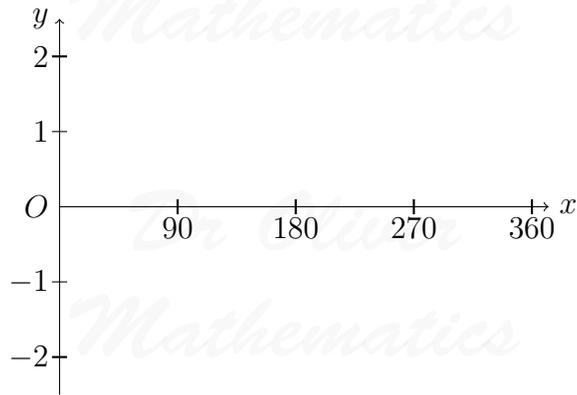


(b) On the grid, sketch the graph of

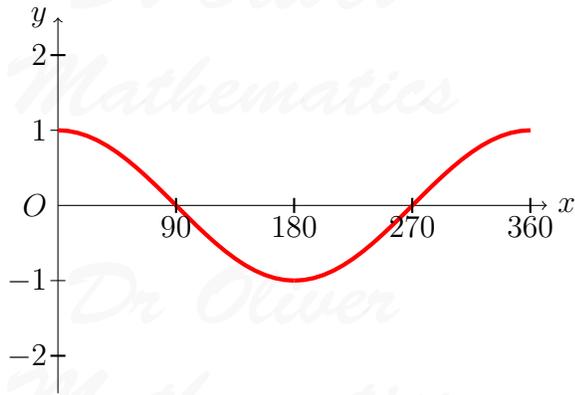
$$y = \cos x$$

(2)

for $0^\circ \leq x \leq 360^\circ$.



Solution



28. APB is a triangle.

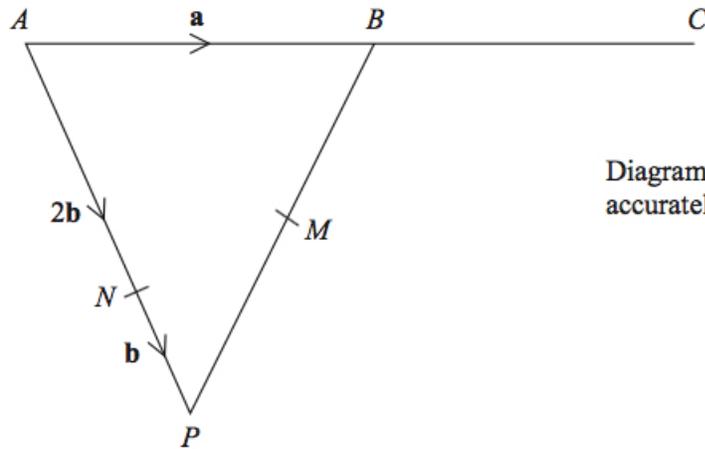


Diagram NOT accurately drawn

N is a point on AP .

$$\overrightarrow{AB} = \mathbf{a}.$$

$$\overrightarrow{AN} = 2\mathbf{b}.$$

$$\overrightarrow{NP} = \mathbf{b}.$$

- (a) Find the vector \overrightarrow{PB} , in terms of \mathbf{a} and \mathbf{b} . (1)

Solution

$$\begin{aligned}\overrightarrow{PB} &= \overrightarrow{PN} + \overrightarrow{NA} + \overrightarrow{AM} \\ &= -\mathbf{b} - 2\mathbf{b} + \mathbf{a} \\ &= \underline{\underline{\mathbf{a} - 3\mathbf{b}}}.\end{aligned}$$

B is the midpoint of AC .

M is the midpoint of PB .

- (b) Show that NMC is a straight line. (4)

Solution

$$\begin{aligned}\overrightarrow{NM} &= \overrightarrow{NP} + \overrightarrow{PM} \\ &= \overrightarrow{NP} + \frac{1}{2}\overrightarrow{PB} \\ &= \mathbf{b} + \frac{1}{2}(\mathbf{a} - 3\mathbf{b}) \\ &= \mathbf{b} + \frac{1}{2}\mathbf{a} - \frac{3}{2}\mathbf{b} \\ &= \frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{b}\end{aligned}$$

and

$$\begin{aligned}\overrightarrow{MC} &= \overrightarrow{MB} + \overrightarrow{BC} \\ &= \frac{1}{2}\overrightarrow{PB} + \overrightarrow{BC} \\ &= \frac{1}{2}(\mathbf{a} - 3\mathbf{b}) + \mathbf{a} \\ &= \frac{1}{2}\mathbf{a} - \frac{3}{2}\mathbf{b} + \mathbf{a} \\ &= \frac{3}{2}\mathbf{a} - \frac{3}{2}\mathbf{b} \\ &= 3 \times \left(\frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{b}\right) \\ &= 3\overrightarrow{NM};\end{aligned}$$

hence, NMC is a straight line.